Gear Technical Reference

The Role Gears are Playing

Gears are used in various types of machinery as a transmission component. The reasons why gears are so widely used to this day can best be described by these facts:

- Gears range in size from miniature instrument installations, such as watches, to large, powerful gears used in turbine drives for ocean liners.
- Gears offer positive transmission of power
- Transmission ratio can be freely controlled with high accuracy by changing the number of gear teeth.
- By increasing or decreasing the number of paired gears, enables you to adjust position transmission with very high angular or linear accuracy.
- Gears can couple power and motion between shafts whose axis are parallel, intersecting or skew.

This technical reference provides the fundamentals of both theoretical and practical information. When you select KHK products for your applications, please make use of the KHK 3010 catalog and this technical reference.

KOHARA GEAR INDUSTRY CO., LTD.

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1 Gear Types and Terminology

Gears are identified by many types and there are many specific technical words to describe their definition. This section introduces those technical words along with commonly used gears and their features.

1.1 Types of Gears

The most common way to classify gears is by category type and by the orientation of axes.

Gears are classified into 3 categories; parallel axes gears, intersecting axes gears, and nonparallel and nonintersecting axes gears.

Spur and helical gears are parallel axes gears. Bevel gears are intersecting axes gears. Screw or crossed helical, worm and hypoid gears belong to the third category. Table 1.1 lists the gear types by axes orientation.

Table 1.1 Types of Gears and Their Categories

Categories of Gears	Types of Gears	Efficiency(%)
Parallel Axes Gears	Spur gear	98.0—99.5
	Spur rack	
	Internal gear	
	Helical gear	
	Helical rack	
	Double helical gear	
Intersecting Axes Gears	Straight bevel gear	98.0—99.0
	Spiral bevel gear	
	Zerol bevel gear	
Nonparallel and Nonintersecting	Screw gear	70.0—95.0
	Worm gear	30.0—90.0

Also, included in table 1.1 is the theoretical efficiency range of various gear types. These figures do not include bearing and lubricant losses.

Since meshing of paired parallel axis gears or intersecting axis gears involves simple rolling movements, they produce relatively minimal slippage and their efficiency is high.

Nonparallel and nonintersecting gears, such as screw gears or worm gears, rotate with relative slippage and by power transmission, which tends to produce friction and makes the efficiency lower when compared to other types of gears.

Efficiency of gears is the value obtained when the gears are installed and working accurately. Particularly for bevel gears, it is assumed that the efficiency will decrease if improperly mounted from off-position on the cone-top.

(1) Parallel Axes Gears

① Spur Gear

This is a cylindrical shaped gear, in which the teeth are parallel to the axis. It is the most commonly used gear with a wide range of applications and is the easiest to manufacture.

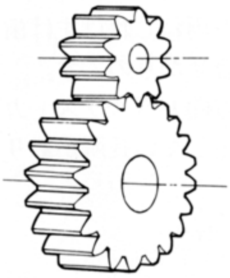


Fig.1.1 Spur Gear

② Spur Rack

This is a linear shaped gear which can mesh with a spur gear with any number of teeth. The spur rack is a portion of a spur gear with an infinite radius.

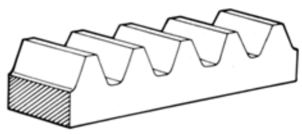


Fig.1.2 Spur Rack

③ Internal Gear

This is a cylindrical shaped gear, but with the teeth inside the circular ring. It can mesh with a spur gear. Internal gears are often used in planetary gear systems.

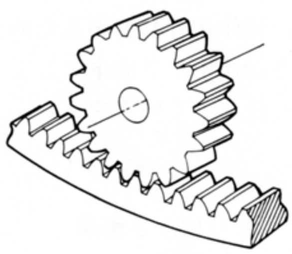


Fig.1.3 Internal Gear and Spur Gear

④ Helical Gear

This is a cylindrical shaped gear with helicoid teeth. Helical gears can bear more load than spur gears, and work more quietly. They are widely used in industry. A disadvantage is the axial thrust force caused by the helix form.

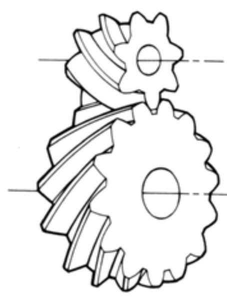


Fig.1.4 Helical Gear

⑤ Helical Rack

This is a linear shaped gear that meshes with a helical gear. A Helical Rack can be regarded as a portion of a helical gear with infinite radius.

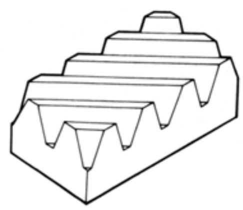


Fig.1.5 Helical Rack

⑥ Double Helical Gear

A gear with both left-hand and right-hand helical teeth. The double helical form balances the inherent thrust forces.

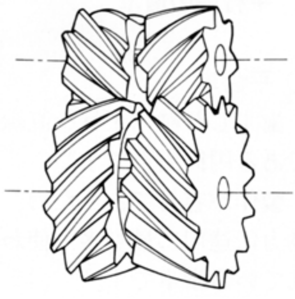


Fig1.6 Double Helical Gear

(2) Intersecting Axes

① Straight Bevel Gear

This is a gear in which the teeth have tapered conical elements that have the same direction as the pitch cone base line (generatrix). The straight bevel gear is both the simplest to produce and the most widely applied in the bevel gear family.

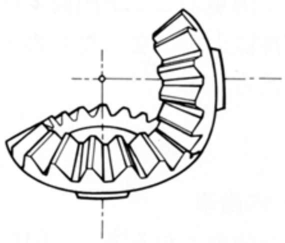


Fig.1.7 Straight Bevel Gear

② Spiral Bevel Gear

This is a bevel gear with a helical angle of spiral teeth. It is much more complex to manufacture, but offers higher strength and less noise.

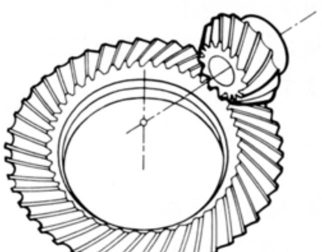


Fig.1.8 Spiral Bevel Gear

③ Zerol Bevel Gear

This is a special type of spiral bevel gear, where the spiral angle is zero degree. It has the characteristics of both the straight and spiral bevel gears. The forces acting upon the tooth are the same as for a straight bevel gear.

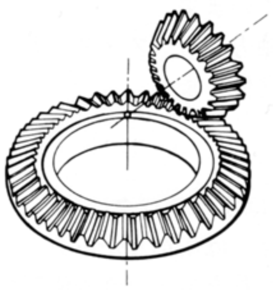


Fig.1.9 Zerol Bevel Gear

(3) Nonparallel and Nonintersecting Axes Gears

① Cylindrical Worm Gear Pair

Worm gear pair is the name for a meshed worm and worm wheel. An outstanding feature is that it offers a very large gear ratio in a single mesh. It also provides quiet and smooth action. However, transmission efficiency is poor.

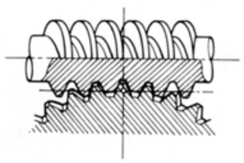


Fig.1.10 Worm Gear pair

② Screw Gear (Crossed Helical Gear)

A pair of cylindrical gears used to drive non-parallel and nonintersecting shafts where the teeth of one or both members of the pair are of screw form. Screw gears are used in the combination of screw gear / screw gear, or screw gear / spur gear. Screw gears assure smooth, quiet operation. However, they are not suitable for transmission of high horsepower.

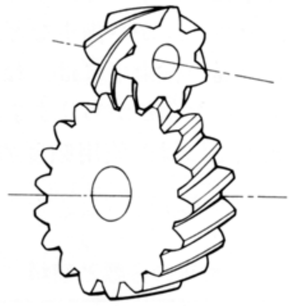


Fig.1.11 Screw Gear

(4) Other Special Gears

① Face Gear

A pseudo bevel gear that is limited to 90°intersecting axes. The face gear is a circular disc with a ring of teeth cut in its side face; hence the name Face Gear

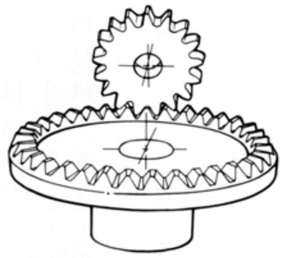


Fig.1.12 Face Gear

② Enveloping Gear Pair

This worm set uses a special worm shape that partially envelops the worm gear as viewed in the direction of the worm gear axis. Its big advantage over the standard worm is much higher load capacity. However, the worm gear is very complicated to design and produce.

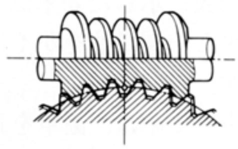


Fig.1.13 Enveloping Gear Pair

③ Hypoid Gear

This gear is a slight deviation from a bevel gear that originated as a special development for the automobile industry. This permitted the drive to the rear axle to be nonintersecting, and thus allowed the auto body to be lowered. It looks very much like the spiral bevel gear. However, it is complicated to design and is the most difficult to produce on a bevel gear generator.

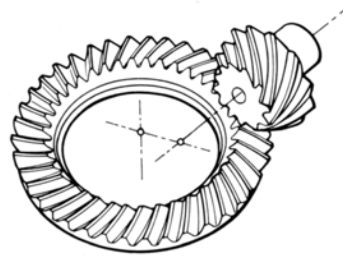


Fig.1.14 Hypoid Gear

1.2 Symbols and Terminology

Symbols and technical words used in this catalog are listed in Table 1.2 to Table 1.4. The formerly used JIS B 0121 and JIS B 0102 Standards were revised to JIS B 0121:1999 and JIS B 0102:1999 conforming to the International Standard Organization (ISO) Standard. In accordance with the revision, we have unified the use of words and symbols conforming to the ISO standard.

Table 1.2 Linear and Circular Dimensions

Terms	Symbols
Centre distance	a
Reference pitch	p
Transverse pitch	p_t
Normal pitch	p_n
Axial pitch	p_x
Base pitch	p_b
Transverse base pitch	p_{bt}
Normal base pitch	p_{bn}
Tooth depth	h
Addendum	h_a
Dedendum	h_f
Chordal height	$\bar{h}_a$
Constant chord height	$\bar{h}_c$
Working depth	h_w
Tooth thickness	s
Normal tooth thickness	s_n
Transverse tooth thickness	s_t
Crest width	s_a
Base thickness	s_b
Chordal tooth thickness	$\bar{s}$
Constant chord	$\bar{s}_c$
Span measurement over k teeth	W
Tooth space	e
Tip and root clearance	c
Circumferential backlash	j_t
Normal backlash	j_n
Radial backlash	j_r
Axial backlash (Radial play) NOTE 1	j_x
Angular backlash	j_θ
Facewidth	b
Effective facewidth	b_w
Lead	p_z
Length of path of contact	g_α
Length of approach path	g_f
Length of recess path	g_a
Overlap length	g_β
Reference diameter	d
Pitch diameter	d_w
Tip diameter	d_a
Base diameter	d_b
Root diameter	d_f
Center reference diameter	d_m
Inner tip diameter	d_i
Reference radius	r
Pitch radius	r_w
Tip radius	r_a
Base radius	r_b
Root radius	r_f
Radius of curvature of tooth profile	ρ
Cone distance	R
Back cone distance	R_v

*NOTE 1.
"Axial backlash" is not a word defined by JIS.

Table 1.3 Angular Dimensions

Terms	Symbols
Reference pressure angle	α
Working pressure angle	α_w
Cutter pressure angle	α_0
Transverse pressure angle	α_t
Normal pressure angle	α_n
Axial pressure angle	α_x
Transverse working pressure angle	α_{wt}
Tip pressure angle	α_a
Normal working pressure angle	α_n
Reference cylinder helix angle	β
Pitch cylinder helix angle	β'
Mean spiral angle NOTE 2	β_m
Tip cylinder helix angle	β_a
Base cylinder helix angle	β_b
Reference cylinder lead angle	γ
Pitch cylinder lead angle	γ_w
Tip cylinder lead angle	γ_a
Base cylinder lead angle	γ_b
Shaft angle	Σ
Reference cone angle	δ
Pitch angle NOTE 3	δ_w
Tip angle NOTE 4	δ_a
Root angle NOTE 5	δ_f
Addendum angle	θ_a
Dedendum angle	θ_f
Transverse angle of transmission	ζ_α
Overlap angle	ζ_β
Total angle of transmission	ζ_γ
Tooth thickness half angle	ψ
Tip tooth thickness half angle	ψ_a
Spacewidth half angle	η
Angular pitch of crown gear	τ
Involute function (Involute α)	invα

NOTE 2. The spiral angle of spiral bevel gears was defined as the helix angle by JIS B 0102

NOTE 3. This must be Pitch Angle, according to JIS B 0102.

NOTE 4. This must be Tip Angle, according to JIS B 0102.

NOTE 5. This must be Root Angle, according to JIS B 0102.

Table 1.4 Others

Terms	Symbols
Number of teeth	z
Equivalent number of teeth	z_v
Number of threads, or number of teeth in pinion	z_1
Gear ratio	u
Transmission ratio	i
Module	m
Transverse module	m_t
Normal module	m_n
Axial module	m_x
Diametral pitch	P
Transverse contact ratio	ε_α
Overlap ratio	ε_β
Total contact ratio	ε_γ
Angular speed	ω
Tangential speed	v
Rotational speed	n
Profile shift coefficient	x
Normal profile shift coefficient	x_n
Transverse profile shift coefficient	x_t
Center distance modification coefficient	y
Tangential force (Circumference)	F_t
Axial force (Thrust)	F_x
Radial force	F_r
Pin diameter	d_p
Ideal pin diameter	d'_p
Measurement over rollers (pin)	M
Pressure angle at pin center	φ
Coefficient of friction	μ
Circular thickness factor	K
Single pitch deviation	f_{pt}
Pitch deviation	f_υ or f_{pu}
Total cumulative pitch deviation	F_p
Total profile deviation	F_α
Runout	F_r
Total helix deviation	F_β

A numerical subscript is used to distinguish "pinion" from "gear" (Example z_1 and z_2), "worm" from "worm wheel", "drive gear" from "driven gear", and so forth. (To find an example, see next page Fig. 2.1).

Table 1.5 indicates the Greek alphabet, the international phonetic alphabet.

Table 1.5 The Greek alphabet

Upper Case Letters	Lower Case Letters	Spelling
Α	α	Alpha
Β	β	Beta
Γ	γ	Gamma
Δ	δ	Delta
Ε	ε	Epsilon
Ζ	ζ	Zeta
Η	η	Eta
Θ	θ	Theta
Ι	ι	Iota
Κ	κ	Kappa
Λ	λ	Lambda
Μ	μ	Mu
Ν	ν	Nu
Ξ	ξ	Xi
Ο	ο	Omicron
Π	π	Pi
Ρ	ρ	Rho
Σ	σ	Sigma
Τ	τ	Tau
Υ	υ	Upsilon
Φ	φ	Phi
Χ	χ	Chi
Ψ	ψ	Psi
Ω	ω	Omega

2 Gear Trains

Gears cannot work singularly to transmit power. At least two or more gears must be meshed to work. This section introduces a simple gear train "Single-Stage Gear Train" and its use in pairs for a "Two-Stage Gear Train".

2.1 Single-Stage Gear Train

A pair of meshed gears is the basic form of a single-stage gear train. Figure 2.1 shows forms of the single-stage gear train

In a single-stage gear train, which consists of z_1 and z_2 numbers of teeth on the driver and driven gears, and their respective rotations, n_1 & n_2. The speed ratio is:

$$\text{Speed Ratio } i = \frac{z_2}{z_1} = \frac{n_1}{n_2} \tag{2.1}$$

Gear trains can be classified by three types, in accordance with the value of the speed ratio i:

Speed ratio $i < 1$, Increasing : $n_1 < n_2$
Speed ratio $i = 1$, Equal speeds : $n_1 = n_2$
Speed ratio $i > 1$, Reducing : $n_1 > n_2$

For the very common cases of spur and bevel gear meshes, see Figures 2.1 (A) and (B), the direction of rotation of driver and driven gears are reversed. In the case of an internal gear mesh, see Figure 2.1 (C), both gears have the same direction of rotation. In the case of a worm mesh, see Figure 2.1 (D), the rotation direction of z_2 is determined by its helix hand.

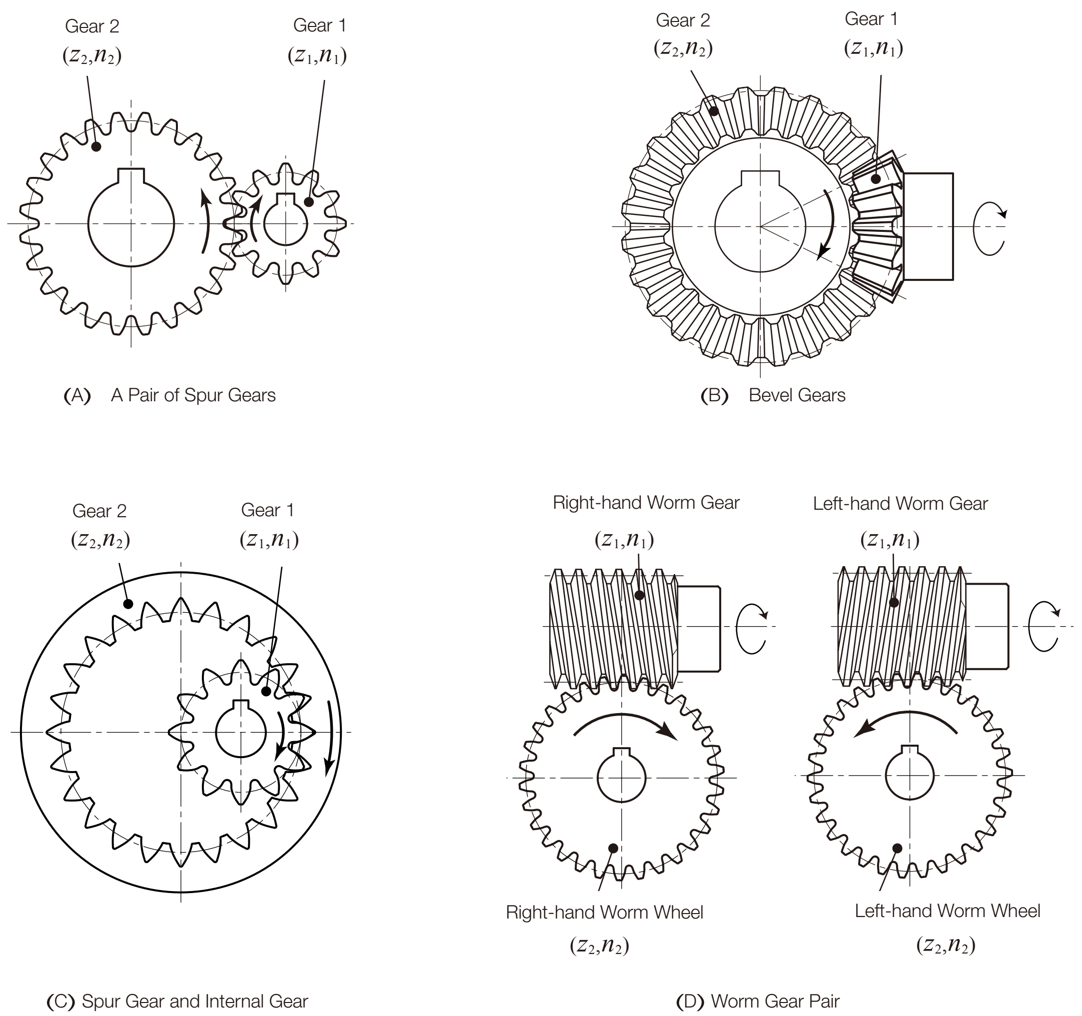


Fig. 2.1 Single-Stage Gear Trains

In addition to these four basic forms, the combination of a rack and pinion can be considered as a specific type. The displacement of a rack, ℓ, for rotation θ of the mating pinion is:

$$\ell = \frac{z_1 \theta}{360} \times \pi m \qquad (2.2)$$

Where: πm is the reference pitch

z_1 is the number of teeth of the pinion

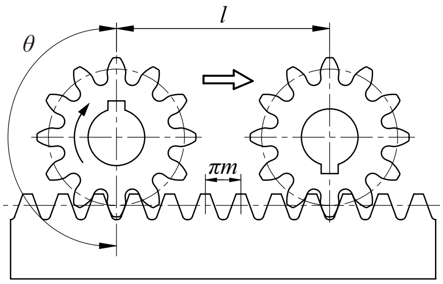


Fig. 2.2 Rack and Pinion

2.2 Two-Stage Gear Train

A two-stage gear train uses two single-stages in series. Figure 2.3 represents the basic form of an external gear two-stage gear train. Let the first gear in the first stage be the driver. Then the speed ratio of the two-stage gear train is:

$$\text{Speed ratio } i = \frac{z_2}{z_1} \times \frac{z_4}{z_3} = \frac{n_1}{n_2} \times \frac{n_3}{n_4} \qquad (2.3)$$

In this arrangement, $n_2 = n_3$

In the two-stage gear train, Fig. 2.3, Gear 1 rotates in the same direction as gear 4.

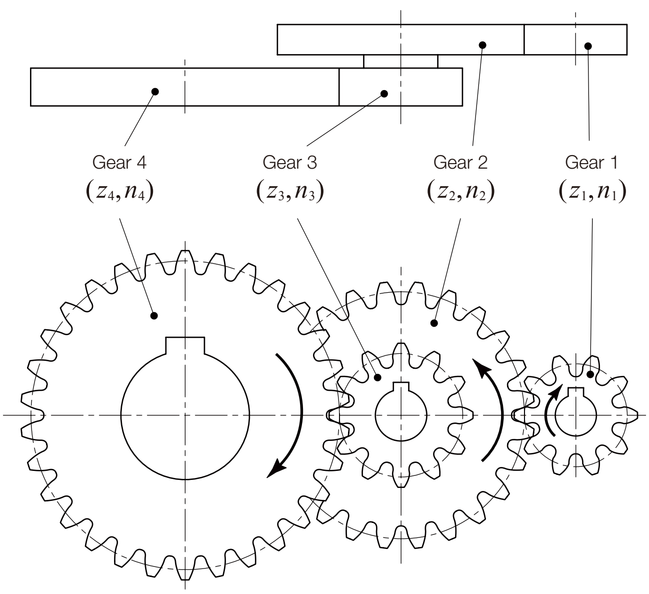


Fig. 2.3 Two-Stage Gear Train

If gears 2 and 3 have the same number of teeth, then the train is simplified as shown in Figure 2.4. In this arrangement, gear 2 is known as an idler, which has no effect on the speed ratio. The speed ratio is then:

$$\text{Speed ratio } i = \frac{z_2}{z_1} \times \frac{z_3}{z_2} = \frac{z_3}{z_1} \qquad (2.4)$$

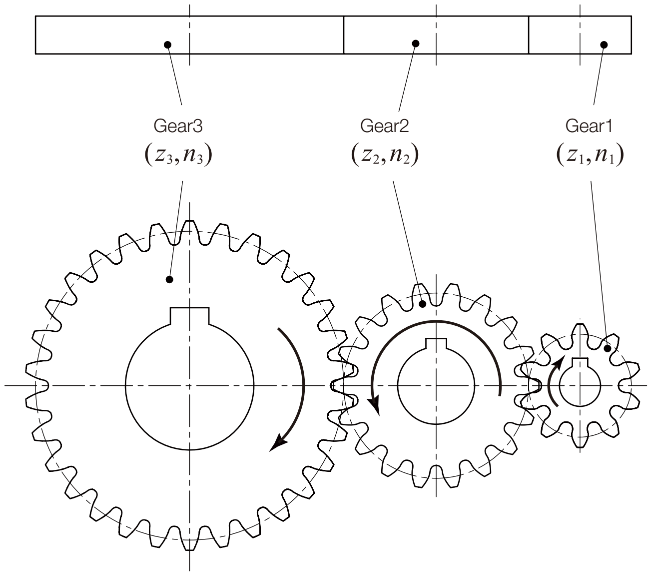


Fig. 2.4 Single-stage gear train with an idler

Table 2.1 introduces calculation examples for two-stage gear trains in Fig.2.3.

Table 2.1 Speed Ratio of Two-Stage Gear Trains

No.	Term	Symbols	Formula	Calculation Example	
				Pinion	Gear
1	No. of Teeth (First Gear)	z_1 , z_2	Set Value	10	24
2	No. of Teeth (Second Gear)	z_3 , z_4		12	30
3	RPM (Gear 1)	n_1		1200	–
4	Speed ratio (First Stage)	i_1	$\frac{z_2}{z_1}$	2.4	
5	Speed ratio (Second Stage)	i_2	$\frac{z_4}{z_3}$	2.5	
6	Final speed ratio	i	$i_1 \times i_2$	6	
7	RPM (Gear2 and 3)	n_2	$\frac{n_1}{i_1}$	500	
8	RPM (Gear4)	n_4	$\frac{n_1}{i}$	–	200

RPM: Revolution per Minute
Set value here stands for the values pre-designated by the designer.

3 Gear Tooth Profiles

For power transmission gears, the tooth form most commonly used today is the involute profile. Involute gears can be manufactured easily, and the gearing has a feature that enables smooth meshing despite the misalignment of center distance to some degree.

3.1 Module Sizes and Standards

Fig. 3.1 shows the tooth profile of a rack, which is the standard involute gear profile.
Table 3.1 lists terms, symbols, formulas and definitions related to gear tooth profiles.
The tooth profile shown in Fig 3.1, where the tooth depth is 2.25 times the module, is called a full depth tooth. This type of full depth tooth is most common, but other types with shorter or longer tooth depths are also used in some applications. Although the pressure angle is usually set to 20 degrees, can be 14.5 or 17.5 degrees in specific applications.

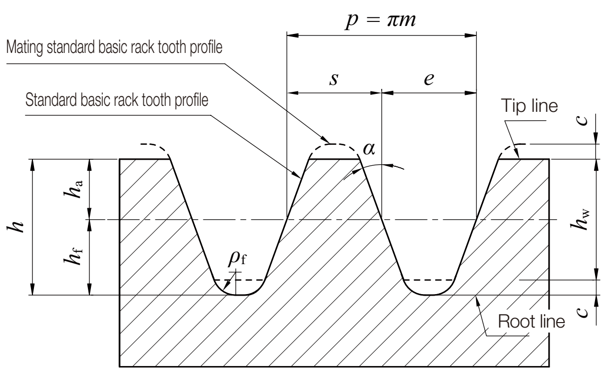


Fig. 3.1 Standard basic rack tooth profile

Table.3.1 Symbols related to Gear Tooth Profile

Terms	Symbols	Formula	Definition
Module	m	$\frac{p}{\pi}$	Module is the unit size indicated in millimeter (mm). The value is calculated from dividing the reference pitch by Pi (π).
Pitch	p	πm	Reference Pitch is the distance between corresponding points on adjacent teeth. The value is calculated from multiplying Module m by Pi(π).
Pressure Angle	α	(Degree)	The angle of a gear tooth leaning against a normal reference line.
Addendum	h_a	$1.00m$	The distance between reference line and tooth tip.
Dedendum	h_f	$1.25m$	The distance between reference line and tooth root.
Tooth Depth	h	$2.25m$	The distance between tooth tip and tooth root.
Working Depth	h_w	$2.00m$	Depth of tooth meshed with the mating gear.
Tip and Root Clearance	c	$0.25m$	The distance (clearance) between tooth root and the tooth tip of mating gear.
Dedendum Fillet Radius	ρ_f	$0.38m$	The radius of curvature between tooth surface and the tooth root.

The data in table 3.2 is extracted from JIS B 1701-2: 1999 which defines the tooth profile and dimensions of involute spur and helical gears. It is recommended to use the values in the series I and not to use with Module 6.5, if possible.

Table 3.2 Standard values of module Unit: mm

I	II
0.1	0.15
0.2	0.25
0.3	0.35
0.4	0.45
0.5	0.55
0.6	0.7
	0.75
0.8	0.9
1	1.125
1.25	1.375
1.5	1.75
2	2.25
2.5	2.75

I	II
3	3.5
4	4.5
5	5.5
6	(6.5)
	7
8	9
10	11
12	14
16	18
20	22
25	28
32	36
40	45
50	

*Extracted from JIS B 1701-2: 1999 Cylindrical Gears foy general engineering and for heavy engineering - Part 2 : Modules.

Figure 3.2 shows the comparative size of various rack teeth.

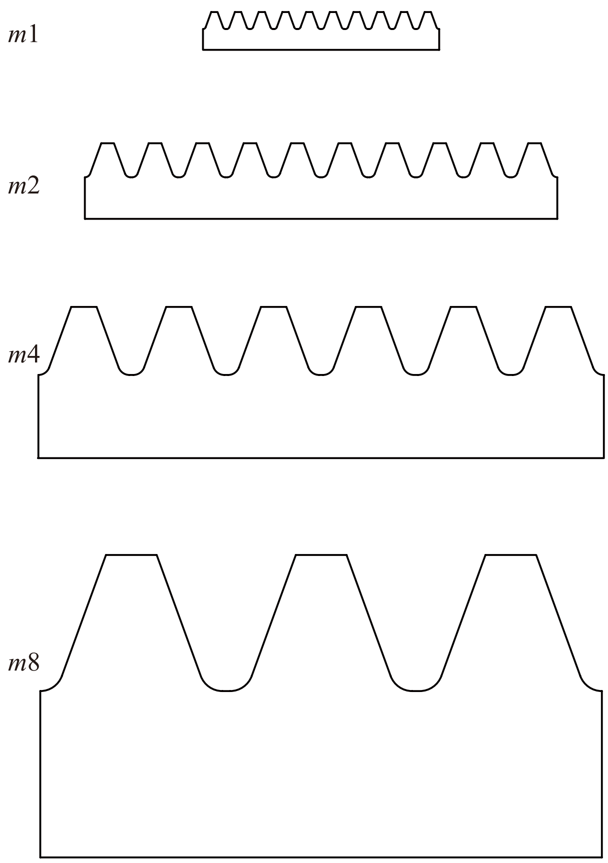


Fig. 3.2 Comparative size of various rack teeth

To indicate tooth size, there are two other units as well as the Module; Circular Pitch (CP) and Diametral Pitch P (D.P). Circular Pitch denotes the reference pitch P. If the reference pitch is set to an integer value, an integral feed in a mechanism is easily obtained.

Diametral Pitch P (D.P.), the unit to denote the size of the gear tooth, is used in the USA, the UK, and other countries. The transformation from Diametral Pitch P (D.P.) to module m is accomplished by the following equation:

$$m = \frac{25.4}{P} \qquad (3.1)$$

Table 3.3 shows the pitch comparisons

Table 3.3 Pitch comparisons

Module m	Circular Pitch CP	Diametral Pitch DP
0.39688	1.24682	64
0.5	1.57080	50.8
0.52917	1.66243	48
0.6	1.88496	42.33333
0.79375	2.49364	32
0.79577	2.5	31.91858
0.8	2.51327	31.75
1	3.14159	25.4
1.05833	3.32485	24
1.25	3.92699	20.32
1.27000	3.98982	20
1.5	4.71239	16.93333
1.59155	5	15.95929
1.58750	4.98728	16
2	6.28319	12.7
2.11667	6.64970	12
2.5	7.85398	10.16
2.54000	7.97965	10
3	9.42478	8.46667
3.17500	9.97456	8
3.18310	10	7.97965
4	12.56637	6.35
4.23333	13.29941	6
4.77465	15	5.31976
5	15.70796	5.08
5.08000	15.95929	5
6	18.84956	4.23333
6.35000	19.94911	4
6.36620	20	3.98982
8	25.13274	3.175
8.46667	26.59882	3
10	31.41593	2.54

3.2 The Involute Curve

Figure 3.3 shows an element of involute curve. The definition of involute curve is the curve traced by a point on a straight line which rolls without slipping on the circle. The circle is called the base circle of the involutes.

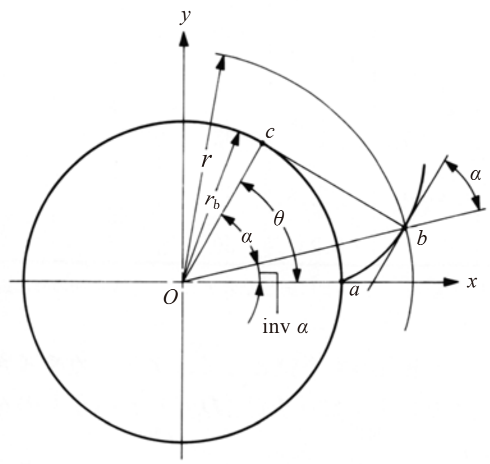


Fig. 3.3 The Involute Curve

In Fig.3.3, invα stands for Involute Angle (Involute α). The units for inv α is radians. θ is called involute rolling angle.

$$\mathrm{inv}\,\alpha = \tan\alpha - \alpha \quad (\mathrm{rad}) \qquad (3.2)$$

With the center of the base circle O at the origin of a coordinate system, the involute curve can be expressed by values of x and y as follows:

$$\left.\begin{aligned} \alpha &= \cos^{-1}\frac{r_b}{r} \\ x &= r\cos(\mathrm{inv}\,\alpha) \\ y &= r\sin(\mathrm{inv}\,\alpha) \end{aligned}\right\} \qquad (3.3)$$

Table 3.4 Calculation Example: The coordinates of an Involute Curve

Gear Specifications	Set Value	Gear Specifications	Set Value
Module	5	Reference diameter	150
Pressure Angle	20	Base diameter	140.95389
No. of Teeth	30	Tip diameter	160

r (Radius)	α (Pressure Angle)	x -coordinate	y -coordinate
70.47695	0.00000	70.4769	0.0000
72	11.80586	71.9997	0.2136
74	17.75087	73.9961	0.7628
76	21.97791	75.9848	1.5192
78	25.37123	77.9615	2.4494
80	28.24139	79.9218	3.5365

The following method was used to create the above table:

① Determine Radius (r)

② Calculate the coordinate of pressure angle α, x/y using the formulas (3.3)

3.3 Meshing of Involute Gear

Figure 3.4 shows a pair of standard gears meshing together. The contact point of the two involutes, as Figure 3.4 shows, slides along the common tangent of the two base circles as rotation occurs. The common tangent is called the line of contact, or line of action.

A pair of gears can only mesh correctly if the pitches and the pressure angle are the same. The requirement that the pressure angles must be identical becomes obvious from the following equation for base pitch p_b:

$$p_b = \pi m \cos\alpha \qquad (3.4)$$

Thus, if the pressure angles are different, the base pitches cannot be identical.

The contact length ab shown in Figure 3.4 is described as the "Length of the path of contact". The contact ratio can be expressed by the following equation:.

$$\text{Transverse Contact Ratio } \varepsilon_\alpha = \frac{\text{Length of path of contact } ab}{\text{Base pitch } p_b} \qquad (3.5)$$

It is good practice to maintain a transverse contact ratio of 1 or greater. Module m and the pressure angle α are the key items in the meshing of gears.

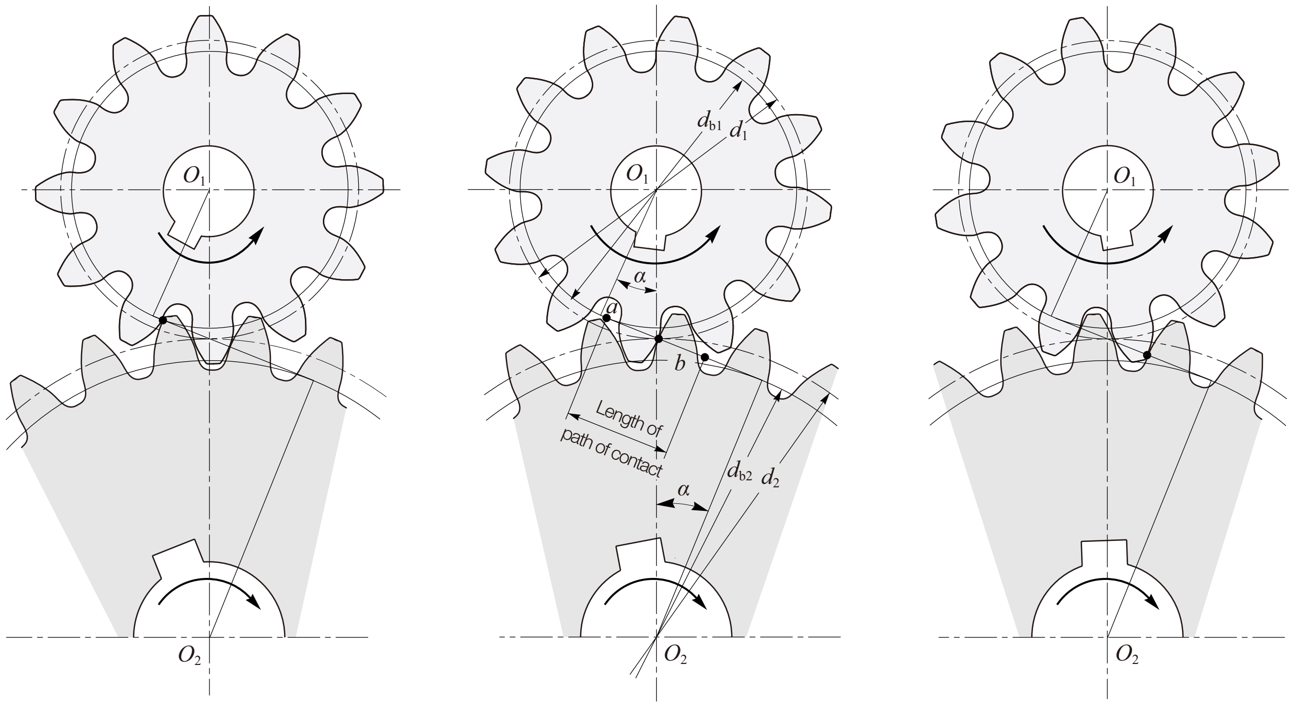


Fig. 3.4 Meshing of Involute Gear

3.4 The Generating of a Spur Gear

Involute gears can be easily generated by rack type cutters. The hob is in effect a rack cutter. Gear generation is also produced with gear type cutters using a shaper or planer machine. Figure 3.5 illustrates how an involute gear tooth profile is generated. It shows how the pitch line of a rack cutter rolling on a pitch circle generates a spur gear. Gear shapers with pinion cutters can also be used to generate involute gears. Gear shapers can not only generate external gears but also generate internal gear teeth.

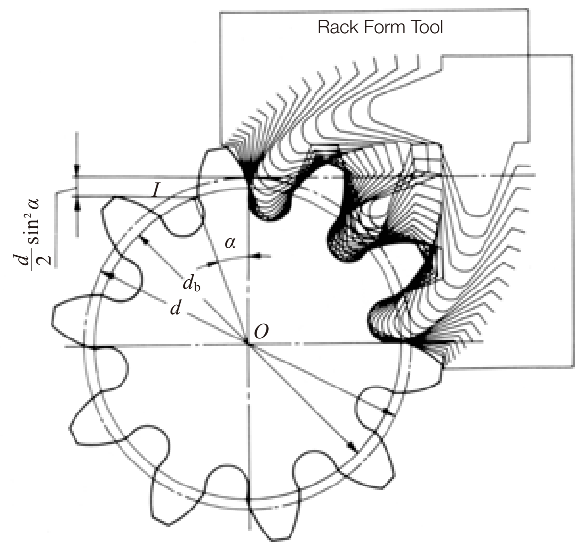


Fig. 3.5 Generation of a Standard Spur Gear
($\alpha = 20°$, $z = 10$, $x = 0$)

3.5 Undercutting

When cutting a stock spur pinion like the gear shown in Fig. 3.5, undercutting occurs if you cut deeper than the interfering point. I. Undercutting is a phenomenon that occurs when some part of tooth dedendum is unexpectedly cut by the edge of the generating tool (h_c). The condition for no undercutting in a standard spur gear is given by the expression:

Max addendum

$$m \leqq \frac{mz}{2} \sin^2 \alpha \qquad (3.6)$$

and the minimum number of teeth (z) is:

$$z = \frac{2}{\sin^2\alpha} \qquad (3.7)$$

For pressure angle 20 degrees, the minimum number of teeth free of undercutting is 17. However, gears with 16 teeth or less can be usable if their strength and contact ratio pose any ill effect.

3.6 Profile Shifting

As Figure 3.5 shows, a gear with 20 degrees of pressure angle and 10 teeth will have a huge undercut volume. To prevent undercut, a positive correction must be introduced. A positive correction, as in Figure 3.6, can prevent undercut. Undercutting will get worse if a negative correction is applied. See Figure 3.7. The extra feed of gear cutter (xm) in Figures 3.6 and 3.7 is the amount of shift or correction. And x is the profile shift coefficient.

The condition to prevent undercut in a spur gear is:

$$m - xm \leqq \frac{zm}{2} \sin^2 \alpha \qquad (3.8)$$

The number of teeth without undercut (z) will be:

$$z = \frac{2(1-x)}{\sin^2\alpha} \qquad (3.9)$$

The profile shift coefficient without undercut (x) is:

$$x = 1 - \frac{z}{2} \sin^2 \alpha \qquad (3.10)$$

Profile shift is not merely used to prevent undercut, it can also be used to adjust the center distance between two gears. If a positive correction is applied, such as to prevent undercut in a pinion, the tooth tip is sharpened.

Table 3.5 presents the calculation of top land thickness (Crest width).

Table 3.5 Calculations of Top Land Thickness (Crest Width)

No.	Item	Symbols	Symbol	Formula	Example
1	Module	m	mm		2
2	Pressure angle	α	Degree		20
3	No. of Teeth	z	–	Set Value	16
4	Profile Shift Coefficient	x	–		0.3
5	Reference Diameter	d		zm	32
6	Base Diameter	d_b	mm	$d\cos\alpha$	30.07016
7	Tip Diameter	d_a		$d + 2m\,(1+x)$	37.2
8	Tip Pressure Angle	α_a	Degree	$\cos^{-1}\frac{d_b}{d_a}$	36.06616
9	Involute α	invα		$\tan\alpha - \alpha$	0.014904
10	Involute α_a	invα_a	Radian	$\tan\alpha_a - \alpha_a$	0.098835
11	Tip Tooth Thickness Half Angle	ψ_a		$\frac{\pi}{2z} + \frac{2x\tan\alpha}{z} + (\mathrm{inv}\,\alpha - \mathrm{inv}\,\alpha_a)$	0.027893
12	Crest Width	s_a	mm	$\psi_a d_a$	1.03762

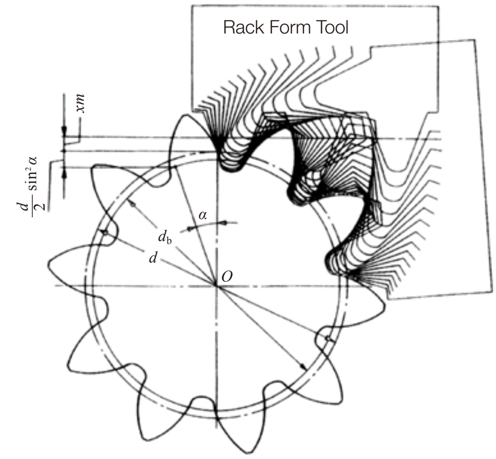


Fig. 3.6 Generation of Positive Shifted Spur Gear
($\alpha = 20°$, $z = 10$, $x = +0.5$)

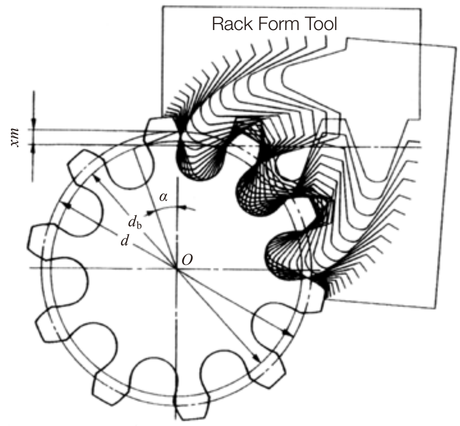


Fig. 3.7 Generation of Negative Shifted Spur Gear
($\alpha = 20°$, $z = 10$, $x = -0.5$)

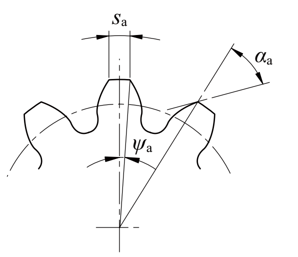


Fig. 3.8 Top Land Thickness

3.7 Gear Tooth Modifications

There are many unique technical words related to gearing. Also, there are various unique ways of modifying gears. This section introduces some of most common methods.

(1) Tooth Profile Modification

Tooth Profile Modification generally means adjusting the addendum. Tooth profile adjustment is done by chamfering the tooth surface in order to make the incorrect involute profile on purpose. This adjustment, enables the tooth to vault when it gets the load, so it can avoid interfering with the mating gear. This is effective for reducing noise and longer surface life. However, too much adjustment may create bad tooth contact as it is functions the same as a large tooth profile error.

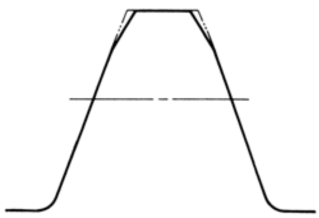


Fig. 3.9 Tooth Profile Modification

(2) Crowning and End Relief

Crowning is the removal of a slight amount of the tooth from the center on out to the reach edge, making the tooth surface slightly convex. This method allows the gear to maintain contact in the central region of the tooth and permits avoidance of edge contact. Crowning should not be larger than necessary as it will reduce the tooth contact area, thus weakening the gears strength.

End relief is the chamfering of both ends of tooth surface.

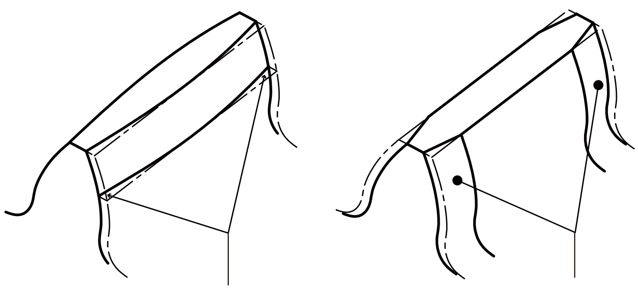


Fig. 3.10 Crowning and End Relief

(3) Topping and Semitopping

In topping, often referred to as top hobbing, the top or tip diameter of the gear is cut simultaneously with the generation of the teeth. Fig. 3.5, 3.6 and 3.7 indicate topping and generating of the gear by rack type cutters. An advantage is that there will be no burrs on the tooth top. Also, the tip diameter is highly concentric with the pitch circle.

Semitopping is the chamfering of the tooth's top corner, which is accomplished simultaneously with tooth generation. Fig.3.11 shows a semitopping cutter and the resultant generated semitopped gear. Such a tooth end prevents corner damage and has no burrs.

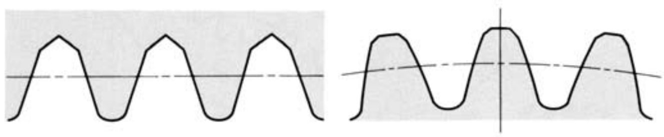


Fig. 3.11 Semitopping Cutter and the Gear Profile

The magnitude of semitopping should not go beyond a proper limit as otherwise it would significantly shorten addendum and contact ratio. Fig. 3.12 shows a recommended magnitude of semitopping.
Topping and semitopping are independent modifications but, if desired, can be applied simultaneously.

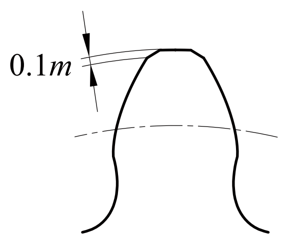


Fig. 3.12 Magnitude of Semitopping

4 Calculation of Gear Dimensions

Gear dimensions are determined in accordance with their specifications, such as Module (m), Number of teeth (z), Pressure angle (a), and Profile shift coefficient (x). This section introduces the dimension calculations for spur gears, helical gears, racks, bevel gears, screw gears, and worm gear pairs. Calculations of external dimensions (eg. Tip diameter) are necessary for processing the gear blanks. Tooth dimensions such as root diameter or tooth depth are considered when gear cutting.

4.1 Spur Gears

Spur Gears are the simplest type of gear. The calculations for spur gears are also simple and they are used as the basis for the calculations for other types of gears. This section introduces calculation methods of standard spur gears, profile shifted spur gears, and linear racks. The standard spur gear is a non-profile-shifted spur gear.

(1) Standard Spur Gear

Figure 4.1 shows the meshing of standard spur gears. The meshing of standard spur gears means the reference circles of two gears contact and roll with each other. The calculation formulas are in Table 4.1.

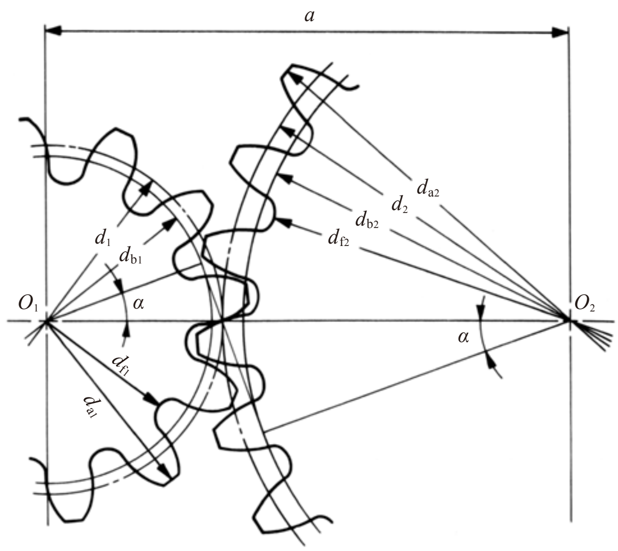


Fig. 4.1 The Meshing of Standard Spur Gears
($\alpha = 20°, z_1 = 12, z_2 = 24, x_1 = x_2 = 0$)

Table 4.1 Calculations for Standard Spur Gears

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Module	m	Set Value	3	
2	Reference Pressure Angle	α		20°	
3	Number of Teeth	z		12	24
4	Center Distance	a	$\frac{(z_1 + z_2)\,m}{2}$ NOTE 1	54.000	
5	Reference Diameter	d	zm	36.000	72.000
6	Base Diameter	d_b	$d \cos \alpha$	33.829	67.658
7	Addendum	h_a	$1.00m$	3.000	3.000
8	Tooth Depth	h	$2.25m$	6.750	6.750
9	Tip Diameter	d_a	$d + 2m$	42.000	78.000
10	Root Diameter	d_f	$d - 2.5m$	28.500	64.500

NOTE 1 : The subscripts 1 and 2 of z_1 and z_2 denote pinion and gear.

All calculated values in Table 4.1 are based upon given module m and number of teeth (z_1 and z_2). If instead, the module m, center distance a and speed ratio i are given, then the number of teeth, z_1 and z_2, would be calculated using the formulas as shown in Table 4.2.

Table 4.2 The Calculations for Number of Teeth

No.	Item	Symbol	Formula		Example	
					Pinion (1)	Gear (2)
1	Module	m	Set Value		3	
2	Center Distance	a			54.000	
3	Speed Ratio	i			1.25	
4	Sum of No. of Teeth	$z_1 + z_2$	$\dfrac{2a}{m}$		36	
5	Number of Teeth	z	$\dfrac{z_1 + z_2}{i + 1}$	$\dfrac{i\,(z_1 + z_2)}{i + 1}$	16	20

Note, that the number of teeth will probably not be integer values when using the formulas in Table 4.2. In this case, it will be necessary to resort to profile shifting or to employ helical gears to obtain as near a transmission ratio as possible.

(2) Profile Shifted Spur Gear

Figure 4.2 shows the meshing of a pair of profile shifted gears. The key items in profile shifted gears are the operating (working) pitch diameters (d_w) and the working (operating) pressure angle (α_w). These values are obtainable from the modified center distance and the following formulas:

$$\left.\begin{aligned} d_{w1} &= 2a \frac{z_1}{z_1 + z_2} \\ d_{w2} &= 2a \frac{z_2}{z_1 + z_2} \\ \alpha_w &= \cos^{-1}\left(\frac{d_{b1} + d_{b2}}{2a}\right) \end{aligned}\right\} \quad (4.1)$$

In the meshing of profile shifted gears, it is the operating pitch circle that is in contact and roll on each other that portrays gear action. Table 4.3 presents the calculations where the profile shift coefficient has been set at x_1 and x_2 at the beginning. This calculation is based on the idea that the amount of the tip and root clearance should be 0.25 m.

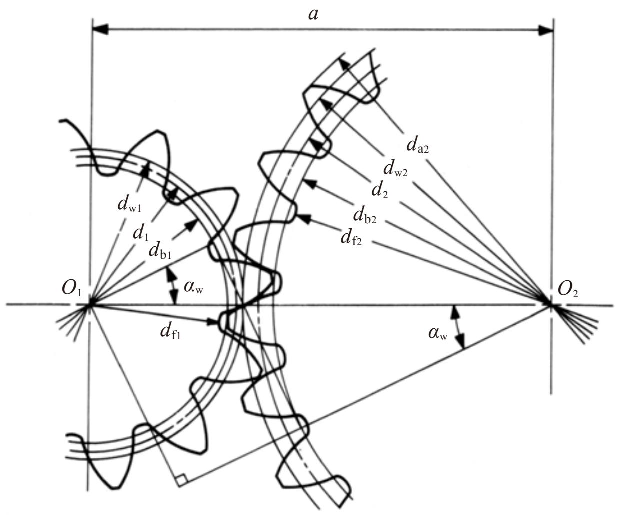


Fig. 4.2 The Meshing of Profile Shifted Gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = +0.6$, $x_2 = +0.36$)

Table 4.3 The Calculations for Profile Shifted Spur Gears (1)

No.	Item	Symbol	Formula	Example: Pinion (1)	Example: Gear (2)
1	Module	m	Set Value	3	
2	Reference Pressure Angle	α		20°	
3	Number of Teeth	z		12	24
4	Profile Shift Coefficient	x		0.6	0.36
5	Involute α_w	$\mathrm{inv}\,\alpha_w$	$2\tan\alpha\left(\frac{x_1 + x_2}{z_1 + z_2}\right) + \mathrm{inv}\,\alpha$	0.034316	
6	Working Pressure Angle	α_w	Find from Involute Function Table	26.0886°	
7	Center Distance Modification Coefficient	y	$\frac{z_1 + z_2}{2}\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right)$	0.83329	
8	Center Distance	a	$\left(\frac{z_1 + z_2}{2} + y\right)m$	56.4999	
9	Reference Diameter	d	zm	36.000	72.000
10	Base Diameter	d_b	$d\cos\alpha$	33.8289	67.6579
11	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	37.667	75.333
12	Addendum	h_{a1} h_{a2}	$(1 + y - x_2)m$ $(1 + y - x_1)m$	4.420	3.700
13	Tooth Depth	h	$\{2.25 + y - (x_1 + x_2)\}m$	6.370	
14	Tip Diameter	d_a	$d + 2h_a$	44.840	79.400
15	Root Diameter	d_f	$d_a - 2h$	32.100	66.660

A standard spur gear is, according to Table 4.3, a profile shifted gear with 0 coefficient of shift; that is , $x_1 = x_2 = 0$.

Table 4.4 is the inverse formula of items from 4 to 8 of Table 4.3.

Table 4.4 The Calculations for Profile Shifted Spur Gears (2)

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Center Distance	a	Set Value	56.4999	
2	Center Distance Modification Coefficient	y	$\frac{a}{m} - \frac{z_1 + z_2}{2}$	0.8333	
3	Working Pressure Angle	α_w	$\cos^{-1}\left(\frac{\cos\alpha}{\frac{2y}{z_1+z_2}+1}\right)$	26.0886°	
4	Sum of Profile Shift Coefficient	$x_1 + x_2$	$\frac{(z_1+z_2)(\mathrm{inv}\,\alpha_w - \mathrm{inv}\,\alpha)}{2\tan\alpha}$	0.9600	
5	Profile Shift Coefficient	x	—	0.6000	0.3600

There are several theories concerning how to distribute the sum of profile shift coefficient ($x_1 + x_2$) into pinion (x_1) and gear (x_2) separately. BSS (British) and DIN (German) standards are the most often used. In the example above, the 12 tooth pinion was given sufficient correction to prevent undercut, and the residual profile shift was given to the mating gear.

(3) Rack and Spur Gear

Table 4.5 presents the method for calculating the mesh of a rack and spur gear.

Figure 4.3 (1) shows the the meshing of standard gear and a rack. In this mesh, the reference circle of the gear touches the pitch line of the rack.

Figure 4.3 (2) shows a profile shifted spur gear, with positive correction xm, meshed with a rack. The spur gear has a larger pitch radius than standard, by the amount xm. Also, the pitch line of the rack has shifted outward by the amount xm.

Table 4.5 presents the calculation of a meshed profile shifted spur gear and rack. If the profile shift coefficient x_1 is 0, then it is the case of a standard gear meshed with the rack.

Table 4.5 The calculations of dimensions of a profile shifted spur gear and a rack

No.	Item	Symbol	Formula	Example	
				Spur gear	Rack
1	Module	m	Set Value	3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		12	—
4	Profile shift coefficient	x		0.6	
5	Height of pitch line	H		—	32.000
6	Working pressure angle	α_w		20°	
7	Mounting distance	a	$\frac{zm}{2}+H+xm$	51.800	
8	Reference diameter	d	zm	36.000	—
9	Base diameter	d_b	$d\cos\alpha$	33.829	
10	Working pitch diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	36.000	
11	Addendum	h_a	$m(1+x)$	4.800	3.000
12	Tooth depth	h	$2.25m$	6.750	
13	Tip diameter	d_a	$d+2h_a$	45.600	—
14	Root diameter	d_f	d_a-2h	32.100	

One rotation of the spur gear will displace the rack l one circumferential length of the gear's reference circle, per the formula:

$$l=\pi mz \tag{4.2}$$

The rack displacement, l, is not changed in any way by the profile shifting. Equation (4.2) remains applicable for any amount of profile shift.

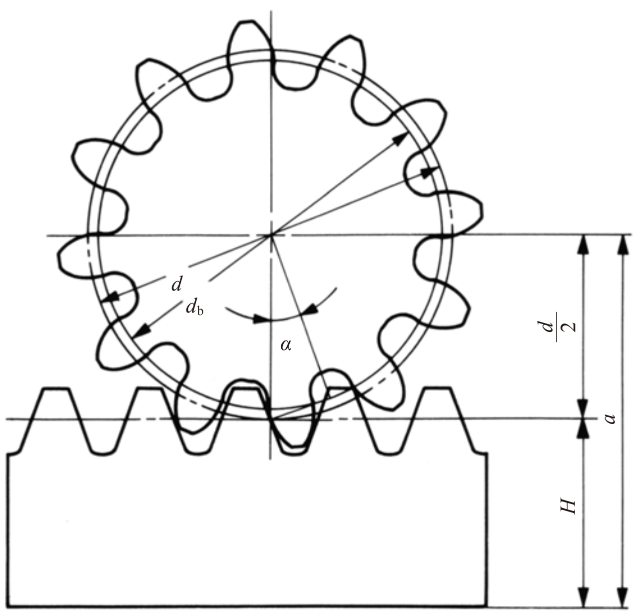


Fig. 4.3 (1) The meshing of standard spur gear and rack
(α = 20°, z_1 = 12, x_1 = 0)

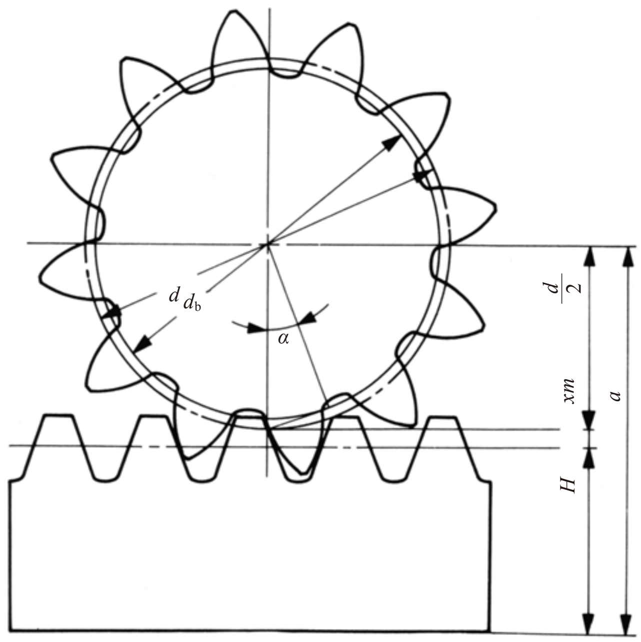


Fig. 4.3 (2) The meshing of profile shifted spur gear and rack
(α = 20°, z_1 = 12, x_1 = +0.6)

4.2 Internal Gears

Internal Gears are composed of a cylindrical shaped gear having teeth inside a circular ring. Gear teeth of the internal gear mesh with the teeth space of a spur gear. Spur gears have a convex shaped tooth profile and internal gears have reentrant shaped tooth profile; this characteristic is opposite of Internal gears. Here are the calculations for the dimensions of internal gears and their interference.

(1) Internal Gear Calculations

Figure 4.4 presents the mesh of an internal gear and external gear. Of vital importance is the working pitch diameters (d_w) and working pressure angle (α_w). They can be derived from center distance (a) and Equations (4.3).

$$\left.\begin{aligned} d_{w1} &= 2a\frac{z_1}{z_2 - z_1} \\ d_{w2} &= 2a\frac{z_2}{z_2 - z_1} \\ \alpha_w &= \cos^{-1}\left(\frac{d_{b2} - d_{b1}}{2a}\right) \end{aligned}\right\} \qquad (4.3)$$

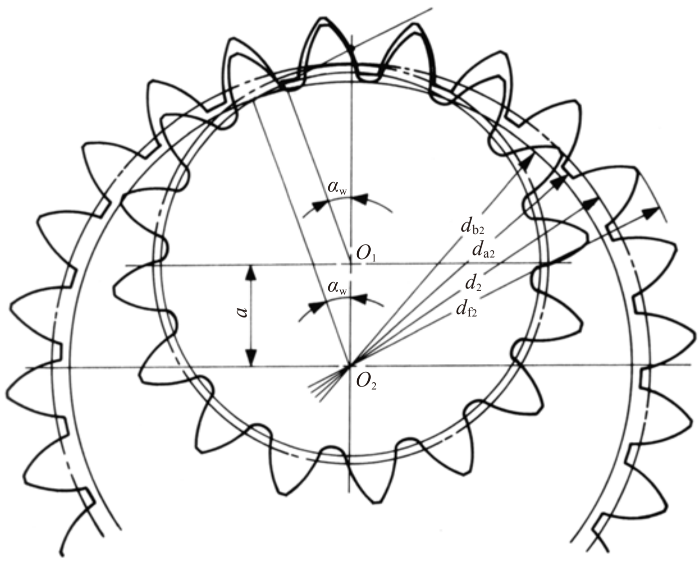


Fig.4.4 The meshing of internal gear and external gear

($\alpha = 20°$, $z_1 = 16$, $z_2 = 24$, $x_1 = x_2 = +0.5$)

Table 4.6 shows the calculation steps. It will become a standard gear calculation if $x_1 = x_2 = 0$.

Table 4.6 The calculations of a profile shifted internal gear and external gear (1)

No.	Item	Symbol	Formula	Example	
				External gear (1)	Internal gear (2)
1	Module	m	Set Value	3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		16	24
4	Profile shift coefficient	x		0	+ 0.516
5	Involute function α_w	inv α_w	$2\tan\alpha\left(\frac{x_2 - x_1}{z_2 - z_1}\right) + \text{inv}\,\alpha$	0.061857	
6	Working pressure angle	α_w	Find from involute Function Table	31.321258°	
7	Center distance modification coefficient	y	$\frac{z_2 - z_1}{2}\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right)$	0.4000	
8	Center distance	a	$\left(\frac{z_2 - z_1}{2} + y\right)m$	13.2	
9	Reference diameter	d	zm	48.000	72.000
10	Base diameter	d_b	$d\cos\alpha$	45.105	67.658
11	Working pitch diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	52.7998	79.1997
12	Addendum	h_{a1} h_{a2}	$(1 + x_1)m$ $(1 - x_2)m$	3.000	1.452
13	Tooth depth	h	$2.25m$	6.75	
14	Tip diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 - 2h_{a2}$	54.000	69.096
15	Root diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_{a2} + 2h$	40.500	82.596

If the center distance (a) is given, x_1 and x_2 would be obtained from the inverse calculation from item 4 to item 8 of Table 4.6. These inverse formulas are in Table 4.7.

Table 4.7 The calculations of profile shifted internal gear and external gear (2)

No.	Item	Symbol	Formula	Example	
				External gear (1)	Internal gear (2)
1	Center distance	a	Set Value	13.1683	
2	Center distance modification coefficient	y	$\frac{a}{m}-\frac{z_2-z_1}{2}$	0.38943	
3	Working pressure angle	α_w	$\cos^{-1}\left(\frac{\cos\alpha}{\frac{2y}{z_2-z_1}+1}\right)$	31.0937°	
4	Difference of profile shift coefficients	x_2-x_1	$\frac{(z_2-z_1)(\text{inv}\,\alpha_w-\text{inv}\,\alpha)}{2\tan\alpha}$	0.5	
5	Profile shift coefficient	x	—	0	0.5

Pinion cutters are often used in cutting internal gears and external gears. The actual value of tooth depth and root diameter, after cutting, will be slightly different from the calculation. That is because the cutter has a profile shift coefficient. In order to get a correct tooth profile, the profile shift coefficient of cutter should be taken into consideration.

(2) Interference In Internal Gears

Three different types of interference can occur with internal gears: (a) Involute Interference, (b) Trochoid Interference, and (c) Trimming Interference.

(a) Involute Interference

This occurs between the dedendum of the external gear and the addendum of the internal gear. It is prevalent when the number of teeth of the external gear is small. Involute interference can be avoided by the conditions cited below:

$$\frac{z_1}{z_2} \geqq 1-\frac{\tan\alpha_{a2}}{\tan\alpha_w} \qquad (4.4)$$

Where α_{a2} is the pressure angle at a tip of the internal gear tooth.

$$\alpha_{a2}=\cos^{-1}\left(\frac{d_{b2}}{d_{a2}}\right) \qquad (4.5)$$

α_w : working pressure angle

$$\alpha_w=\cos^{-1}\left\{\frac{(z_2-z_1)\,m\cos\alpha}{2a}\right\} \qquad (4.6)$$

Equation (4.5) is true only if the tip diameter of the internal gear is bigger than the base circle:

$$d_{a2} \geqq d_{b2} \qquad (4.7)$$

For a standard internal gear, where α = 20°, Equation (4.7) is valid only if the number of teeth is z_2 > 34.

(b) Trochoid Interference

This refers to an interference occurring at the addendum of the external gear and the dedendum of the internal gear during recess tooth action. It tends to happen when the difference between the numbers of teeth of the two gears is small. Equation (4.8) presents the condition for avoiding trochoidal interference.

$$\theta_1\frac{z_1}{z_2}\ \text{inv}\,\alpha_w-\text{inv}\,\alpha_{a2} \geqq \theta_2 \qquad (4.8)$$

Here

$$\left.\begin{aligned}\theta_1&=\cos^{-1}\left(\frac{r_{a2}^2-r_{a1}^2-a^2}{2ar_{a1}}\right)+\text{inv}\,\alpha_{a1}-\text{inv}\,\alpha_w\\ \theta_2&=\cos^{-1}\left(\frac{a^2+r_{a2}^2-r_{a1}^2}{2ar_{a2}}\right)\end{aligned}\right\} \qquad (4.9)$$

where α_{a1} is the pressure angle of the spur gear tooth tip:

$$\alpha_{a1}=\cos^{-1}\left(\frac{d_{b1}}{d_{a1}}\right) \qquad (4.10)$$

In the meshing of an external gear and a standard internal gear α = 20°, trochoid interference is avoided if the difference of the number of teeth, z_2-z_1, is larger than 9.

(c) Trimming Interference

This occurs in the radial direction in that it prevents pulling the gears apart. Thus, the mesh must be assembled by sliding the gears together with an axial motion. It tends to happen when the numbers of teeth of the two gears are very close. Equation (4.11) indicates how to prevent this type of interference.

$$\theta_1 + \operatorname{inv}\alpha_{a1} - \operatorname{inv}\alpha_w \geqq \frac{z_2}{z_1}(\theta_2 + \operatorname{inv}\alpha_{a2} - \operatorname{inv}\alpha_w) \tag{4.11}$$

Here

$$\left.\begin{aligned}\theta_1 &= \sin^{-1}\sqrt{\frac{1-(\cos\alpha_{a1}/\cos\alpha_{a2})^2}{1-(z_1/z_2)^2}}\\ \theta_2 &= \sin^{-1}\sqrt{\frac{(\cos\alpha_{a2}/\cos\alpha_{a1})^2-1}{(z_2/z_1)^2-1}}\end{aligned}\right\} \tag{4.12}$$

This type of interference can occur in the process of cutting an internal gear with a pinion cutter. Should that happen, there is danger of breaking the tooling.

Table 4.8 (1) shows the limit for the pinion cutter to prevent trimming interference when cutting a standard internal gear, with pressure angle $\alpha_0 = 20°$, and no profile shift, i.e., $x_0 = 0$.

Table 4.8(1) The limit to prevent an internal gear from trimming interference

$\alpha_0 = 20°, x_0 = x_2 = 0$

z_0	15	16	17	18	19	20	21	22	24	25	27
z_2	34	34	35	36	37	38	39	40	42	43	45
z_0	28	30	31	32	33	34	35	38	40	42	
z_2	46	48	49	50	51	52	53	56	58	60	
z_0	44	48	50	56	60	64	66	80	96	100	
z_2	62	66	68	74	78	82	84	98	114	118	

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 22 (z_0 = 15 to 22) .

Table 4.8(2) shows the limit for a profile shifted pinion cutter to prevent trimming interference while cutting a standard internal gear. The correction (x_0) is the magnitude of shift which was assumed to be: $x_0 = 0.0075z_0 + 0.05$.

Table 4.8(2) The limit to prevent an internal gear from trimming interference

$\alpha_0 = 20°, x_2 = 0$

z_0	15	16	17	18	19	20	21	22	24	25	27
x_0	0.1625	0.17	0.1775	0.185	0.1925	0.2	0.2075	0.215	0.23	0.2375	0.2525
z_2	36	38	39	40	41	42	43	45	47	48	50
z_0	28	30	31	32	33	34	35	38	40	42	
x_0	0.26	0.275	0.2825	0.29	0.2975	0.305	0.3125	0.335	0.35	0.365	
z_2	52	54	55	56	58	59	60	64	66	68	
z_0	44	48	50	56	60	64	66	80	96	100	
x_0	0.38	0.41	0.425	0.47	0.5	0.53	0.545	0.65	0.77	0.8	
z_2	71	76	78	86	90	95	98	115	136	141	

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 19 (z_0 = 15 to 19) .

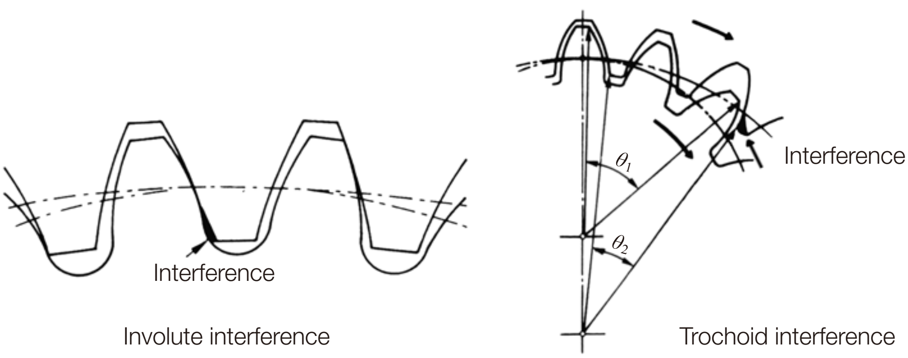


Fig.4.5 Involute interference and trochoid interference

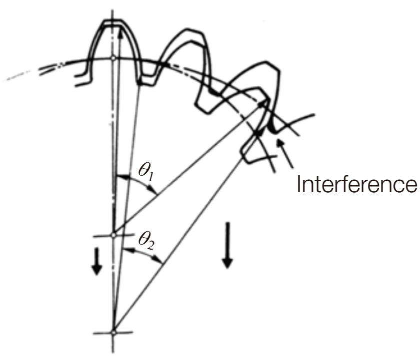


Fig.4.6 Trimming interference

4.3 Helical Gears

A helical gear such as shown in Figure 4.7 is a cylindrical gear in which the teeth flank are helicoid. The helix angle in reference cylinder is β, and the displacement of one rotation is the lead, p_z.

The tooth profile of a helical gear is an involute curve from an axial view, or in the plane perpendicular to the axis. The helical gear has two kinds of tooth profiles – one is based on a normal system, the other is based on a transverse system.

Pitch measured perpendicular to teeth is called normal pitch, p_n. And p_n divided by π is then a normal module, m_n.

$$m_n = \frac{p_n}{\pi} \tag{4.13}$$

The tooth profile of a helical gear with applied normal module, m_n, and normal pressure angle α_n belongs to a normal system.

In the axial view, the pitch on the reference is called the transverse pitch, p_t. And p_t divided by π is the transverse module, m_t.

$$m_t = \frac{p_t}{\pi} \tag{4.14}$$

These transverse module m_t and transverse pressure angle α_t at are the basic configuration of transverse system helical gear.
In the normal system, helical gears can be cut by the same gear hob if module m_n and pressure angle at are constant, no matter what the value of helix angle β.

It is not that simple in the transverse system. The gear hob design must be altered in accordance with the changing of helix angle β, even when the module m_t and the pressure angle at are the same.
Obviously, the manufacturing of helical gears is easier with the normal system than with the transverse system in the plane perpendicular to the axis.

When meshing helical gears, they must have the same helix angle but with opposite hands.

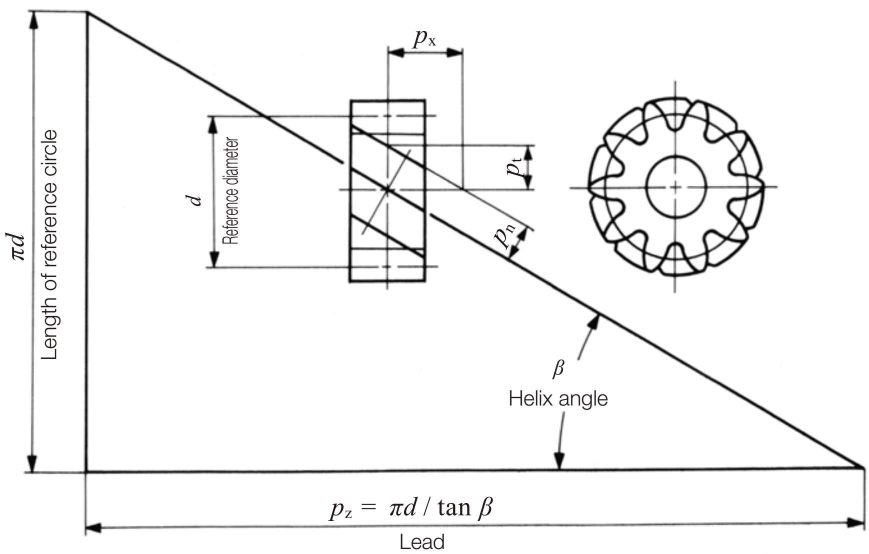


Fig.4.7 Fundamental relationship of a helical gear (Right-hand)

(1) Normal System Helical Gear

In the normal system, the calculation of a profile shifted helical gear, the working pitch diameter d_w and transverse working pressure angle α_{wt} is done per Equations (4.15). That is because meshing of the helical gears in the transverse plane is just like spur gears and the calculation is similar.

$$\left.\begin{aligned} d_{w1} &= 2a\frac{z_1}{z_1+z_2} \\ d_{w2} &= 2a\frac{z_2}{z_1+z_2} \\ \alpha_{wt} &= \cos^{-1}\left(\frac{d_{b1}+d_{b2}}{2a}\right) \end{aligned}\right\} \qquad (4.15)$$

Table 4.9 shows the calculation of profile shifted helical gears in the normal system. If normal profile shift coefficients x_{n1}, x_{n2} are zero, they become standard gears.

Table 4.9 The calculations of a profile shifted helical gear in the normal system (1)

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Normal module	m_n	Set Value	3	
2	Normal pressure angle	α_n		20°	
3	Reference cylinder helix angle	β		30°	
4	Number of teeth & helical hand	z		12 (L)	60 (R)
5	Normal coefficient of profile shift	x_n		+ 0.09809	0
6	Transverse pressure angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	22.79588°	
7	Involute function α_{wt}	$\mathrm{inv}\,\alpha_{wt}$	$2\tan\alpha_n\left(\frac{x_{n1}+x_{n2}}{z_1+z_2}\right)+\mathrm{inv}\,\alpha_t$	0.023405	
8	Transverse working pressure angle	α_{wt}	Find from involute Function Table	23.1126°	
9	Center distance modification coefficient	y	$\frac{z_1+z_2}{2\cos\beta}\left(\frac{\cos\alpha_t}{\cos\alpha_{wt}}-1\right)$	0.09744	
10	Center distance	a	$\left(\frac{z_1+z_2}{2\cos\beta}+y\right)m_n$	125.000	
11	Reference diameter	d	$\frac{zm_n}{\cos\beta}$	41.569	207.846
12	Base diameter	d_b	$d\cos\alpha_t$	38.322	191.611
13	Working pitch diameter	d_w	$\frac{d_b}{\cos\alpha_{wt}}$	41.667	208.333
14	Addendum	h_{a1} h_{a2}	$(1+y-x_{n2})m_n$ $(1+y-x_{n1})m_n$	3.292	2.998
15	Tooth depth	h	$\{2.25+y-(x_{n1}+x_{n2})\}m_n$	6.748	
16	Tip diameter	d_a	$d+2h_a$	48.153	213.842
17	Root diameter	d_f	d_a-2h	34.657	200.346

If center distance, α, is given, the normal profile shift coefficients x_{n1} and x_{n2} can be calculated from Table 4.10. These are the inverse equations from items 5 to 10 of Table 4.9.

Table 4.10 The calculations for a profile shifted helical gear in the normal system (2)

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Center distance	a	Set Value	125	
2	Center distance modification coefficient	y	$\dfrac{a}{m_n} - \dfrac{z_1 + z_2}{2\cos\beta}$	0.097447	
3	Transverse working pressure angle	α_{wt}	$\cos^{-1}\left(\dfrac{\cos\alpha_t}{\dfrac{2y\cos\beta}{z_1+z_2}+1}\right)$	23.1126°	
4	Sum of profile shift coefficient	$x_{n1}+x_{n2}$	$\dfrac{(z_1+z_2)(\mathrm{inv}\,\alpha_{wt} - \mathrm{inv}\,\alpha_t)}{2\tan\alpha_n}$	0.09809	
5	Normal profile shift coefficient	x_n	—	0.09809	0

The transformation from a normal system to a transverse system is accomplished by the following equations:

$$\left.\begin{aligned} x_t &= x_n \cos\beta \\ m_t &= \frac{m_n}{\cos\beta} \\ \alpha_t &= \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right) \end{aligned}\right\} \qquad (4.16)$$

(2) Transverse System Helical Gear

Table 4.11 shows the calculation of profile shifted helical gears in a transverse system. They become standard if $x_{t1} = x_{t2} = 0$.

Table 4.11 The calculations for a profile shifted helical gear in the transverse system (1)

No.	Item	Symbol	Formula	Example Pinion (1)	Example Gear (2)
1	Transverse module	m_t	Set Value	3	
2	Transverse pressure angle	α_t		20°	
3	Reference cylinder helix angle	β		30°	
4	Number of teeth & helical hand	z		12 (L)	60 (R)
5	Transverse profile shift coefficient	x_t		0.34462	0
6	Involute function α_{wt}	inv α_{wt}	$2\tan\alpha_t\left(\dfrac{x_{t1}+x_{t2}}{z_1+z_2}\right)+\mathrm{inv}\,\alpha_t$	0.0183886	
7	Transverse working pressure angle	α_{wt}	Find from Involute Function Table	21.3975°	
8	Center distance modification coefficient	y	$\dfrac{z_1+z_2}{2}\left(\dfrac{\cos\alpha_t}{\cos\alpha_{wt}}-1\right)$	0.33333	
9	Center distance	a	$\left(\dfrac{z_1+z_2}{2}+y\right)m_t$	109.0000	
10	Reference diameter	d	zm_t	36.000	180.000
11	Base diameter	d_b	$d\cos\alpha_t$	33.8289	169.1447
12	Working pitch diameter	d_w	$\dfrac{d_b}{\cos\alpha_{wt}}$	36.3333	181.6667
13	Addendum	h_{a1} h_{a2}	$(1+y-x_{t2})m_t$ $(1+y-x_{t1})m_t$	4.000	2.966
14	Tooth depth	h	$\{2.25+y-(x_{t1}+x_{t2})\}\ m_t$	6.716	
15	Tip diameter	d_a	$d+2h_a$	44.000	185.932
16	Root diameter	d_f	d_a-2h	30.568	172.500

Table 4.12 presents the inverse calculation of items 5 to 9 of Table 4.11.

Table 4.12 The calculations for a profile shifted helical gear in the transverse system (2)

No.	Item	Symbol	Formula	Example Pinion (1)	Example Gear (2)
1	Center distance	a	Set Value	109	
2	Center distance modification coefficient	y	$\dfrac{a}{m_t}-\dfrac{z_1+z_2}{2}$	0.33333	
3	Transverse working pressure angle	α_{wt}	$\cos^{-1}\left(\dfrac{\cos\alpha_t}{\dfrac{2y}{z_1+z_2}+1}\right)$	21.39752°	
4	Sum of profile shift coefficient	$x_{t1}+x_{t2}$	$\dfrac{(z_1+z_2)(\mathrm{inv}\,\alpha_{wt}-\mathrm{inv}\,\alpha_t)}{2\tan\alpha_t}$	0.34462	
5	Transverse profile shift coefficient	x_t	—	0.34462	0

The transformation from a transverse to a normal system is described by the following equations:

$$\left.\begin{aligned} x_n &= \frac{x_t}{\cos\beta} \\ m_n &= m_t\cos\beta \\ \alpha_n &= \tan^{-1}(\tan\alpha_t\cos\beta) \end{aligned}\right\} \quad (4.17)$$

(3) Helical Rack

Viewed in the transverse plane, the meshing of a helical rack and gear is the same as a spur gear and rack. Table 4.13 presents the calculation examples for a mated helical rack with normal module and normal pressure angle. Similarily, Table 4.14 presents examples for a helical rack in the transverse system (i.e., perpendicular to gear axis).

Table 4.13 The calculations for a helical rack in the normal system

No.	Item	Symbol	Formula	Example	
				Pinion	Rack
1	Normal module	m_n	Set Value	2.5	
2	Normal pressure angle	α_n		20°	
3	Reference cylinder helix angle	β		10° 57'49"	
4	Number of teeth & helical hand	z		20 (R)	− (L)
5	Normal profile shift coefficient	x_n		0	−
6	Pitch line height	H		−	27.5
7	Transverse pressure angle	α_t	$\tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\beta}\right)$	20.34160°	
8	Mounting distance	a	$\dfrac{zm_n}{2\cos\beta}+H+x_n m_n$	52.965	
9	Reference diameter	d	$\dfrac{zm_n}{\cos\beta}$	50.92956	−
10	Base diameter	d_b	$d\cos\alpha_t$	47.75343	
11	Addendum	h_a	$m_n(1+x_n)$	2.500	2.500
12	Tooth depth	h	$2.25m_n$	5.625	
13	Tip diameter	d_a	$d+2h_a$	55.929	−
14	Root diameter	d_f	d_a-2h	44.679	

The formulas of a standard helical rack are similar to those of Table 4.14 with only the normal profile shift coefficient $x_n = 0$. To mesh a helical gear to a helical rack, they must have the same helix angle but with opposite hands.

The displacement of the helical rack, l, for one rotation of the mating gear is the product of the transverse pitch and number of teeth.

$$l = \frac{\pi m_n}{\cos\beta}z \qquad (4.18)$$

According to the equations of Table 4.13, let transverse pitch p_t = 8 mm and displacement l = 160 mm. The transverse pitch and the displacement could be resolved into integers, if the helix angle were chosen properly.

Table 4.14 The calculations for a helical rack in the transverse system

No.	Item	Symbol	Formula	Example	
				Pinion	Rack
1	Transverse module	m_t	Set Value	2.5	
2	Transverse pressure angle	α_t		20°	
3	Reference cylinder helix angle	β		10° 57'49"	
4	Number of teeth & helical hand	z		20 (R)	− (L)
5	Transverse profile shift coefficient	x_t		0	−
6	Pitch line height	H		−	27.5
7	Mounting distance	a	$\frac{zm_t}{2} + H + x_t m_t$	52.500	
8	Reference diameter	d	zm_t	50.000	−
9	Base diameter	d_b	$d \cos \alpha_t$	46.98463	
10	Addendum	h_a	$m_t (1 + x_t)$	2.500	2.500
11	Tooth depth	h	$2.25m_t$	5.625	
12	Tip diameter	d_a	$d + 2h_a$	55.000	−
13	Root diameter	d_f	$d_a - 2h$	43.750	

In the meshing of transverse system helical rack and helical gear, the movement, l, for one turn of the helical gear is the transverse pitch multiplied by the number of teeth.

$$l = \pi m_t z \tag{4.19}$$

4.4 Bevel Gears

Bevel gears, whose pitch surfaces are cones, are used to drive intersecting axes. Bevel gears are classified according to their type of the tooth forms into Straight Bevel Gear, Spiral Bevel Gear, Zerol Bevel Gear, Skew Bevel Gear etc. The meshing of bevel gears means the pitch cone of two gears contact and roll with each other.

Let z_1 and z_2 be pinion and gear tooth numbers; shaft angle Σ ; and reference cone angles δ_1 and δ_2 ; then:

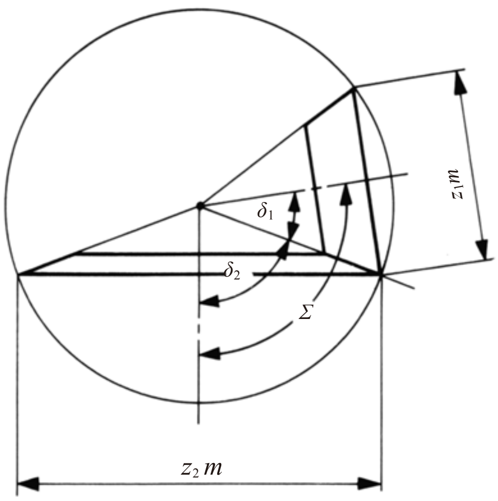


Fig. 4.8 The reference cone angle of bevel gear

$$\left.\begin{aligned}\tan\delta_1 &= \frac{\sin\Sigma}{\dfrac{z_2}{z_1}+\cos\Sigma}\\ \tan\delta_2 &= \frac{\sin\Sigma}{\dfrac{z_1}{z_2}+\cos\Sigma}\end{aligned}\right\} \qquad (4.20)$$

Generally, a shaft angle Σ = 90°is most used. Other angles (Figure 4.8) are sometimes used. Then, it is called “bevel gear in nonright angle drive”. The 90° case is called “bevel gear in right angle drive”.

When Σ = 90°, Equation (4.20) becomes:

$$\left.\begin{aligned}\delta_1 &= \tan^{-1}\left(\frac{z_1}{z_2}\right)\\ \delta_2 &= \tan^{-1}\left(\frac{z_2}{z_1}\right)\end{aligned}\right\} \qquad (4.21)$$

Miter gears are bevel gears with Σ = 90° and $z_1 = z_2$. Their transmission ratio $z_2/z_1 = 1$.

Figure 4.9 depicts the meshing of bevel gears.

The meshing must be considered in pairs. It is because the reference cone angles δ_1 and δ_2 are restricted by the gear ratio z_2/z_1. In the facial view, which is normal to the contact line of pitch cones, the meshing of bevel gears appears to be similar to the meshing of spur gears.

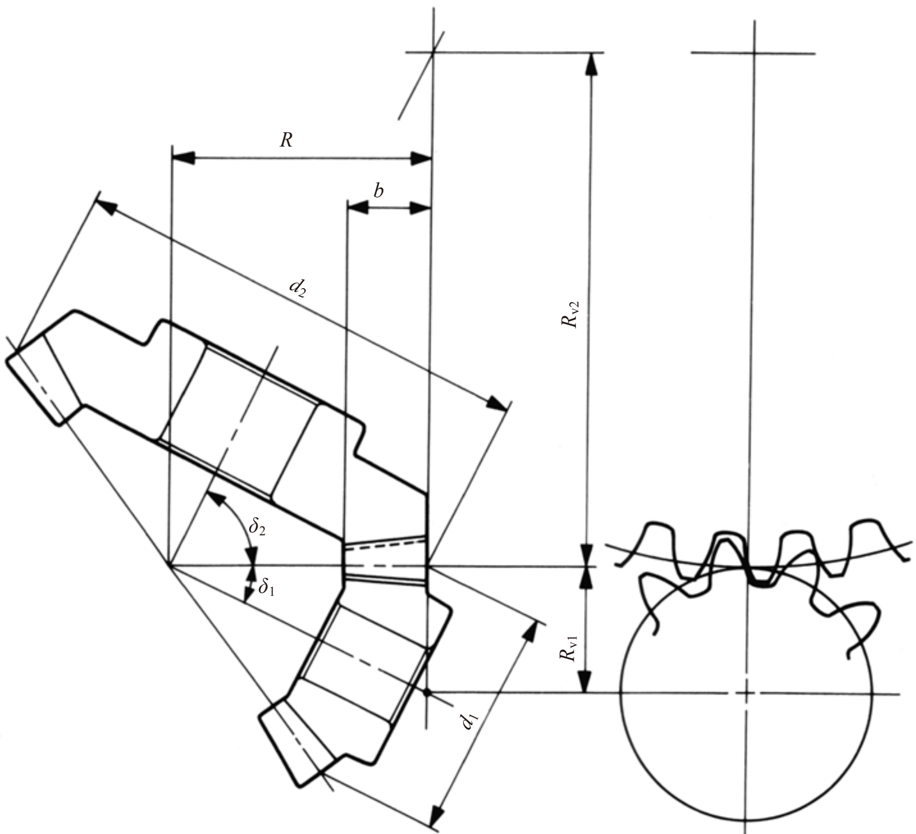


Fig. 4.9 The meshing of bevel gears

(1) Gleason Straight Bevel Gears

A straight bevel gear is a simple form of bevel gear having straight teeth which, if extended inward, would come together at the intersection of the shaft axes. Straight bevel gears can be grouped into the Gleason type and the standard type.

In this section, we discuss the Gleason straight bevel gear. The Gleason Company defines the tooth profile as: tooth depth h = 2.188m; tip and root clearance c = 0.188m; and working depth h_w = 2.000m.

The characteristics are:

○ Design specified profile shifted gears:

In the Gleason system, the pinion is positive shifted and the gear is negative shifted. The reason is to distribute the proper strength between the two gears. Miter gears, thus, do not need any shift.

○ The tip and root clearance is designed to be parallel:

The face cone of the blank is turned parallel to the root cone of the mate in order to eliminate possible fillet interference at the small end of the teeth.

Table 4.15 shows the minimum number of teeth to prevent undercut in the Gleason system at the shaft angle $\Sigma = 90°$

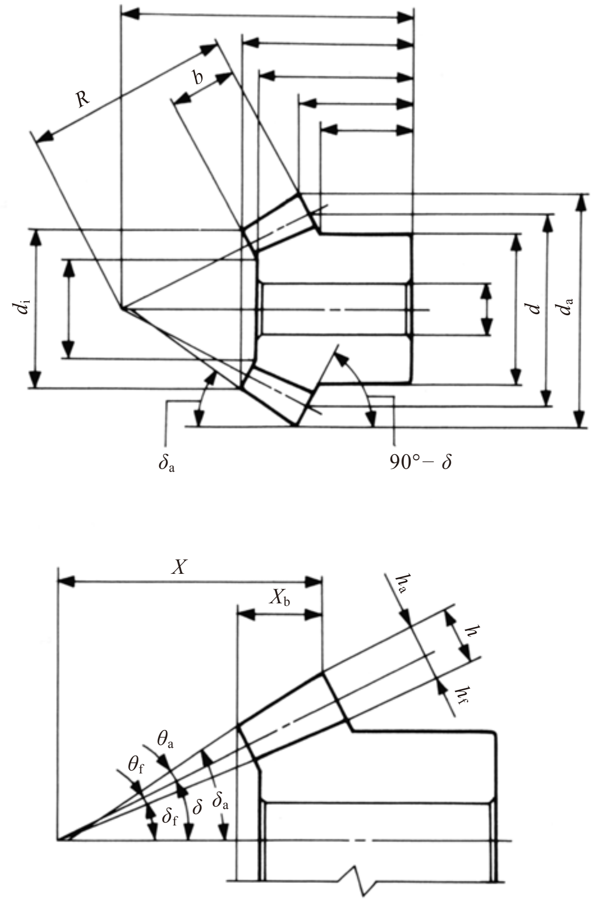


Fig. 4.10 Dimensions and angles of bevel gears

Table 4.15 The minimum numbers of teeth to prevent undercut

Pressure angle	Combination of number of teeth z_1/z_2					
(14.5°)	29/29 and higher	28/29 and higher	27/31 and higher	26/35 and higher	25/40 and higher	24/57 and higher
20°	16/16 and higher	15/17 and higher	14/20 and higher	13/30 and higher		
(25°)	13/13 and higher					

Table 4.16 presents equations for designing straight bevel gears in the Gleason system. The meanings of the dimensions and angles are shown in Figure 4.10 above. All the equations in Table 4.16 can also be applied to bevel gears with any shaft angle.

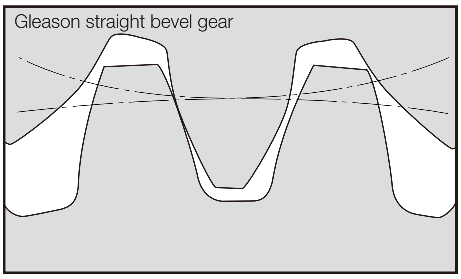
The straight bevel gear with crowning in the Gleason system is called a Coniflex gear. It is manufactured by a special Gleason "Coniflex" machine. It can successfully eliminate poor tooth contact due to improper mounting and assembly.

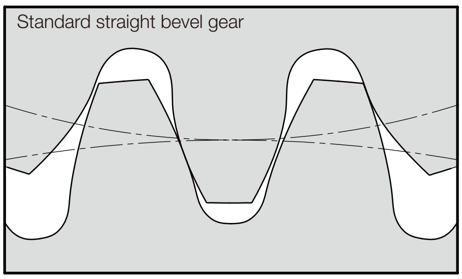
Tale 4.16 The calculations of straight bevel gears of the gleason system

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Shaft angle	Σ	Set Value	90°	
2	Module	m		3	
3	Reference pressure angle	α		20°	
4	Number of teeth	z		20	40
5	Reference diameter	d	zm	60	120
6	Reference cone angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\dfrac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
7	Cone distance	R	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
8	Facewidth	b	It should not exceed $R/3$	22	
9	Addendum	h_{a1} h_{a2}	$2.000m-h_{a2}$ $0.540m+\dfrac{0.460m}{\left(\dfrac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	4.035	1.965
10	Dedendum	h_f	$2.188m-h_a$	2.529	4.599
11	Dedendum angle	θ_f	$\tan^{-1}(h_f/R)$	2.15903°	3.92194°
12	Addendum angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.92194°	2.15903°
13	Tip angle	δ_a	$\delta+\theta_a$	30.48699°	65.59398°
14	Root angle	δ_f	$\delta-\theta_f$	24.40602°	59.51301°
15	Tip diameter	d_a	$d+2h_a\cos\delta$	67.2180	121.7575
16	Pitch apex to crown	X	$R\cos\delta-h_a\sin\delta$	58.1955	28.2425
17	Axial facewidth	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	19.0029	9.0969
18	Inner tip diameter	d_i	$d_a-\dfrac{2b\sin\delta_a}{\cos\theta_a}$	44.8425	81.6609

The first characteristic of a Gleason Straight Bevel Gear that it is a profile shifted tooth. From Figure 4.11, we can see the tooth profile of Gleason Straight Bevel Gear and the same of Standard Straight Bevel Gear.

Fig. 4.11 The tooth profile of straight bevel gears





(2) Standard Straight Bevel Gears

A bevel gear with no profile shifted tooth is a standard straight bevel gear. The are also referred to as Klingelnberg bevel gears. The applicable equations are in Table 4.17.

Table 4.17 The calculations for a standard straight bevel gears

No.	Item	Symbol	Formula	Example Pinion (1)	Example Gear (2)
1	Shaft angle	Σ	Set Value	90°	
2	Module	m		3	
3	Reference pressure angle	α		20°	
4	Number of teeth	z		20	40
5	Reference diameter	d	zm	60	120
6	Reference cone angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\dfrac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
7	Cone distance	R	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
8	Facewidth	b	It should not exceed $R/3$	22	
9	Addendum	h_a	$1.00m$	3.00	
10	Dedendum	h_f	$1.25m$	3.75	
11	Dedendum angle	θ_f	$\tan^{-1}(h_f/R)$	3.19960°	
12	Addendum angle	θ_a	$\tan^{-1}(h_a/R)$	2.56064°	
13	Tip angle	δ_a	$\delta+\theta_a$	29.12569°	65.99559°
14	Root angle	δ_f	$\delta-\theta_f$	23.36545°	60.23535°
15	Tip diameter	d_a	$d+2h_a\cos\delta$	65.3666	122.6833
16	Pitch apex to crown	X	$R\cos\delta-h_a\sin\delta$	58.6584	27.3167
17	Axial facewidth	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	19.2374	8.9587
18	Inner tip diameter	d_i	$d_a-\dfrac{2b\sin\delta_a}{\cos\theta_a}$	43.9292	82.4485

These equations can also be applied to bevel gear sets with other than 90° shaft angles.

(3) Gleason Spiral Bevel Gears

A spiral bevel gear is one with a spiral tooth flank as in Figure 4.12. The spiral is generally consistent with the curve of a cutter with the diameter d_c. The spiral angle β is the angle between a generatrix element of the pitch cone and the tooth flank. The spiral angle just at the tooth flank center is called the mean spiral angle β_m. In practice, the term spiral angle refers to the mean spiral angle.

All equations in Table 4.20 are specific to the manufacturing method of Spread Blade or of Single Side from Gleason. If a gear is not cut per the Gleason system, the equations will be different from these.

The tooth profile of a Gleason spiral bevel gear shown here has the tooth depth h = 1.888m; tip and root clearance c = 0.188m; and working depth h_w = 1.700m. These Gleason spiral bevel gears belong to a stub gear system. This is applicable to gears with modules m > 2.1.

Table 4.18 shows the minimum number of teeth to avoid undercut in the Gleason system with shaft angle Σ = 90° and pressure angle α_n = 20°.

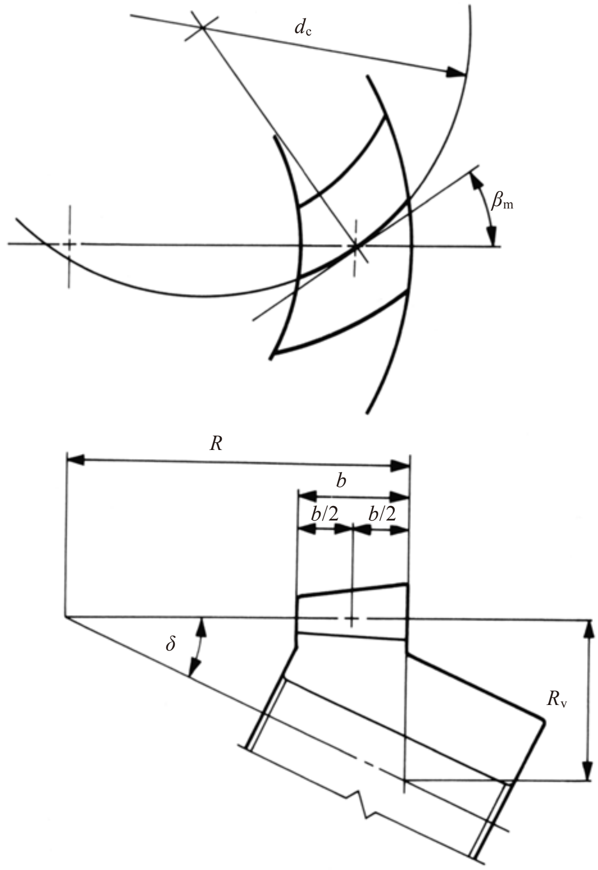


Fig.4.12 Spiral Bevel Gear (Left-hand)

Table 4.18 The minimum numbers of teeth to prevent undercut $\beta = 35°$

Pressure angle	Combination of numbers of teeth z_1/z_2					
20°	17/17 and higher	16/18 and higher	15/19 and higher	14/20 and higher	13/22 and higher	12/26 and higher

If the number of teeth is less than 12, Table 4.19 is used to determine the gear sizes.

Table 4.19 Dimentions for pinions with number of teeth less than 12

Number of teeth in pinion	z_1	6	7	8	9	10	11
Number of teeth in gear	z_2	34 and higher	33 and higher	32 and higher	31 and higher	30 and higher	29 and higher
Working depth	h_w	1.500	1.560	1.610	1.650	1.680	1.695
Tooth depth	h	1.666	1.733	1.788	1.832	1.865	1.882
Gear addendum	h_{a2}	0.215	0.270	0.325	0.380	0.435	0.490
Pinion addendum	h_{a1}	1.285	1.290	1.285	1.270	1.245	1.205
Tooth thickness of gear s_2	30	0.911	0.957	0.975	0.997	1.023	1.053
	40	0.803	0.818	0.837	0.860	0.888	0.948
	50	—	0.757	0.777	0.828	0.884	0.946
	60	—	—	0.777	0.828	0.883	0.945
Normal pressure angle	α_n	20°					
Spiral angle	β	35° ～ 40°					
Shaft angle	Σ	90°					

NOTE: All values in the table are based on m = 1.

Table 4.20 shows the calculations for spiral bevel gears in the Gleason system

Table 4.20 The calculations for spiral bevel gears in the Gleason system

No.	Item	Symbol	Formula	Example Pinion (1)	Example Gear (2)
1	Shaft angle	Σ	Set Value	90°	
2	Module	m		3	
3	Normal pressure angle	α_n		20°	
4	Mean spiral angle	β_m		35°	
5	Number of teeth and spiral hand	z		20 (L)	40 (R)
6	Transverse pressure angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta_m}\right)$	23.95680	
7	Reference diameter	d	zm	60	120
8	Reference cone angle	δ_1 δ_2	$\tan^{-1}\left(\frac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
9	Cone distance	R	$\frac{d_2}{2\sin\delta_2}$	67.08204	
10	Facewidth	b	It should be less than $0.3R$ or $10m$	20	
11	Addendum	h_{a1} h_{a2}	$1.700m-h_{a2}$ $0.460m+\frac{0.390m}{\left(\frac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	3.4275	1.6725
12	Dedendum	h_f	$1.888m-h_a$	2.2365	3.9915
13	Dedendum angle	θ_f	$\tan^{-1}(h_f/R)$	1.90952°	3.40519°
14	Addendum angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.40519°	1.90952°
15	Tip angle	δ_a	$\delta+\theta_a$	29.97024°	65.34447°
16	Root angle	δ_f	$\delta-\theta_f$	24.65553°	60.02976°
17	Tip diameter	d_a	$d+2h_a\cos\delta$	66.1313	121.4959
18	Pitch apex to crown	X	$R\cos\delta-h_a\sin\delta$	58.4672	28.5041
19	Axial facewidth	X_b	$\frac{b\cos\delta_a}{\cos\theta_a}$	17.3563	8.3479
20	Inner tip diameter	d_i	$d_a-\frac{2b\sin\delta_a}{\cos\theta_a}$	46.1140	85.1224

All equations in Table 4.20 are also applicable to Gleason bevel gears with any shaft angle. A spiral bevel gear set requires matching of hands; left-hand and right-hand as a pair.

(4) Gleason Zerol Bevel Gears

When the spiral angle bm = 0, the bevel gear is called a Zerol bevel gear. The calculation equations of Table 4.16 for Gleason straight bevel gears are applicable. They also should take care again of the rule of hands; left and right of a pair must be matched. Figure 4.13 is a left-hand Zerol bevel gear.

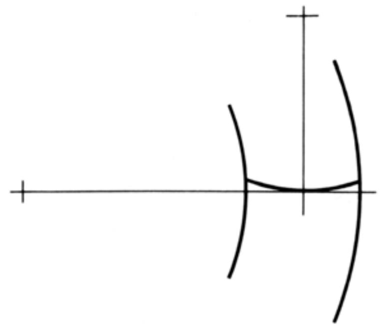


Fig. 4.13 Left-hand zerol bevel gear

4.5 Screw Gears

Screw gearing includes various types of gears used to drive nonparallel and nonintersecting shafts where the teeth of one or both members of the pair are of screw form. Figure 4.14 shows the meshing of screw gears.
Two screw gears can only mesh together under the conditions that normal modules (m_{n1}) and (m_{n2}) and normal pressure angles (α_{n1}, α_{n2}) are the same.

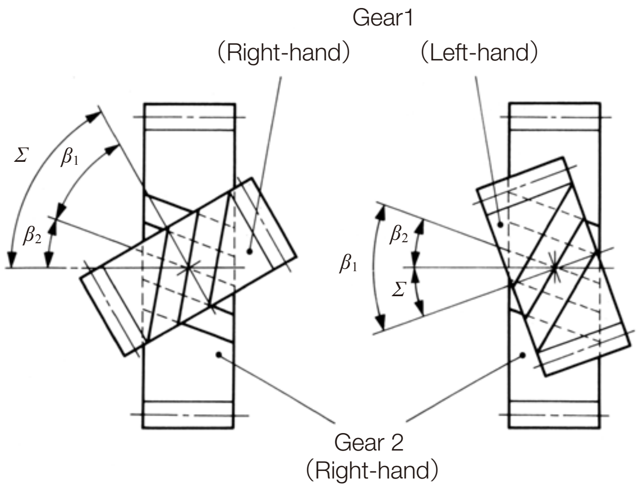


Fig.4.14 Screw gears of nonparallel and nonintersecting axes

Let a pair of screw gears have the shaft angle Σ and helix angles β_1 and β_2:

$$\left.\begin{array}{l}\text{If they have the same hands, then:}\\ \quad \Sigma = \beta_1 + \beta_2 \\ \text{If they have the opposite hands, then:}\\ \quad \Sigma = \beta_1 - \beta_2 \text{ or } \Sigma = \beta_2 - \beta_1\end{array}\right\} \tag{4.22}$$

If the screw gears were profile shifted, the meshing would become a little more complex. Let β_{w1}, β_{w2} represent the working pitch cylinder;

$$\left.\begin{array}{l}\text{If they have the same hands, then:}\\ \quad \Sigma = \beta_{w1} + \beta_{w2} \\ \text{If they have the opposite hands, then:}\\ \quad \Sigma = \beta_{w1} - \beta_{w2} \text{ or } \Sigma = \beta_{w2} - \beta_{w1}\end{array}\right\} \tag{4.23}$$

Table 4.21 presents equations for a profile shifted screw gear pair. When the normal profile shift coefficients $x_{n1} = x_{n2} = 0$, the equations and calculations are the same as for standard gears.

Table 4.21 The equations for a screw gear pair on nonparallel and Nonintersecting axes in the normal system

No.	Item	Symbol	Formula	Example Pinion (1)	Example Gear (2)
1	Normal module	m_n	Set Value	3	
2	Normal pressure angle	α_n	Set Value	20°	
3	Reference cylinder helix angle	β	Set Value	20°	30°
4	Number of teeth & helical hand	z	Set Value	15 (R)	24 (R)
5	Normal profile shift coefficient	x_n	Set Value	0.4	0.2
6	Number of teeth of an Equivalent spur gear	z_v	$\dfrac{z}{\cos^3\beta}$	18.0773	36.9504
7	Transverse pressure angle	α_t	$\tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\beta}\right)$	21.1728°	22.7959°
8	Involute function α_{wn}	$\mathrm{inv}\alpha_{wn}$	$2\tan\alpha_n\left(\dfrac{x_{n1}+x_{n2}}{z_{v1}+z_{v2}}\right)+\mathrm{inv}\,\alpha_n$	0.0228415	
9	Normal working pressure angle	α_{wn}	Find from involute function table	22.9338°	
10	Transverse working pressure angle	α_{wt}	$\tan^{-1}\left(\dfrac{\tan\alpha_{wn}}{\cos\beta}\right)$	24.2404°	26.0386°
11	Center distance modification coefficient	y	$\dfrac{1}{2}(z_{v1}+z_{v2})\left(\dfrac{\cos\alpha_n}{\cos\alpha_{wn}}-1\right)$	0.55977	
12	Center distance	a	$\left(\dfrac{z_1}{2\cos\beta_1}+\dfrac{z_2}{2\cos\beta_2}+y\right)m_n$	67.1925	
13	Reference diameter	d	$\dfrac{zm_n}{\cos\beta}$	47.8880	83.1384
14	Base diameter	d_b	$d\cos\alpha_t$	44.6553	76.6445
15	Working pitch diameter	d_{w1} d_{w2}	$2a\dfrac{d_1}{d_1+d_2}$ $2a\dfrac{d_2}{d_1+d_2}$	49.1155	85.2695
16	Working helix angle	β_w	$\tan^{-1}\left(\dfrac{d_w}{d}\tan\beta\right)$	20.4706°	30.6319°
17	Shaft angle	Σ	$\beta_{w1}+\beta_{w2}$ or $\beta_{w1}-\beta_{w2}$	51.1025°	
18	Addendum	h_{a1} h_{a2}	$(1+y-x_{n2})m_n$ $(1+y-x_{n1})m_n$	4.0793	3.4793
19	Tooth depth	h	$\{2.25+y-(x_{n1}+x_{n2})\}m_n$	6.6293	
20	Tip diameter	d_a	$d+2h_a$	56.0466	90.0970
21	Root diameter	d_f	d_a-2h	42.7880	76.8384

Standard screw gears have relations as follows:

$$\left.\begin{array}{ll} d_{w1}=d_1 & d_{w2}=d_2 \\ \beta_{w1}=\beta_1 & \beta_{w2}=\beta_2 \end{array}\right\} \qquad (4.24)$$

4.6 Cylindrical Worm Gear Pair

Cylindrical worms may be considered cylindrical type gears with screw threads. Generally, the mesh has a 90° shaft angle. The number of threads in the worm is equivalent to the number of teeth in a gear of a screw type gear mesh. Thus, a one-thread worm is equivalent to a one-tooth gear; and two-threads equivalent to two-teeth, etc. Referring to Figure 4.15, for a reference cylinder lead angle γ, measured on the pitch cylinder, each rotation of the worm makes the thread advance one lead p_z

There are four worm tooth profiles in JIS B 1723-1977, as defined below.

Type I ：The tooth profile is trapezoidal on the axial plane.

Type II：The tooth profile is trapezoid on the plane normal to the space.

Type Ⅲ： The tooth profile which is obtained by inclining the axis of the milling or grinding, of which cutter shape is trapezoidal on the cutter axis, by the lead angle to the worm axis.）

Type IV：The tooth profile is of involute curve on the plane of rotation.

KHK stock worm gear products are all Type III. Worm profiles (Fig 4.15). The cutting tool used to process worm gears is called a single-cutter that has a single-edged blade. The cutting of worm gears is done with worm cutting machine.

Because the worm mesh couples nonparallel and nonintersecting axes, the axial plane of worm does not

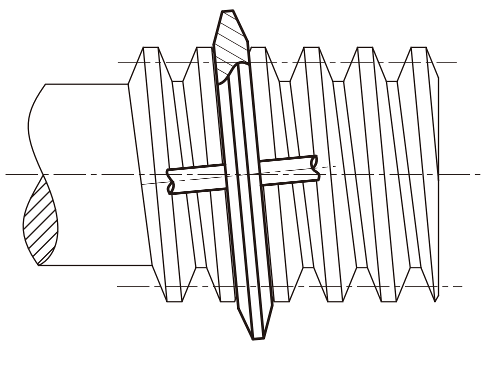


Fig. 4.15 Cutting / Grinding for Type III Worm

correspond with the axial plane of worm wheel. The axial plane of worm corresponds with the transverse plane of worm wheel. The transverse plane of worm corresponds with the axial plane of worm wheel. The common plane of the worm and worm wheel is the normal plane. Using the normal module, m_n, is most popular. Then, an ordinary hob can be used to cut the worm wheel.

Table 4.22 presents the relationships among worm and worm wheel with regard to axial plane, transverse plane, normal plane, module, pressure angle, pitch and lead.

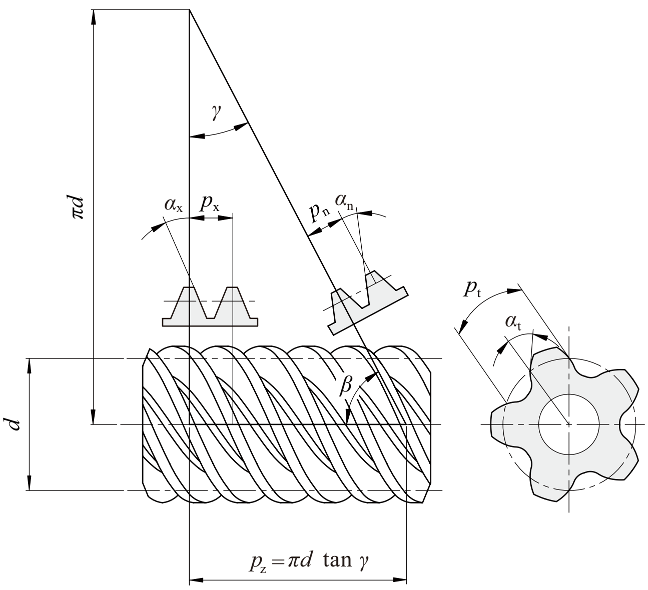


Fig. 4.16 Cylindrical worm (Right-hand)

Table 4.22 The relations of cross sections of worm gear pairs

Worm		
Axial plane	Normal plane	Transverse plane
$m_x = \dfrac{m_n}{\cos\gamma}$	m_n	$m_t = \dfrac{m_n}{\sin\gamma}$
$\alpha_x = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\gamma}\right)$	α_n	$\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$
$p_x = \pi m_x$	$p_n = \pi m_n$	$p_t = \pi m_t$
$p_z = \pi m_x z$	$p_z = \dfrac{\pi m_n z}{\cos\gamma}$	$p_z = \pi m_t z \tan\gamma$
Transverse plane	Normal plane	Axial plane
Worm wheel		

Reference to Figure 4.16 can help the understanding of the relationships in Table 4.22. They are similar to the relations in Formulas (4.16) and (4.17) in that the helix angle β be substituted by $(90° - \gamma)$. We can consider that a worm with lead angle γ is almost the same as a helical gear with helix angle $(90° - \gamma)$.

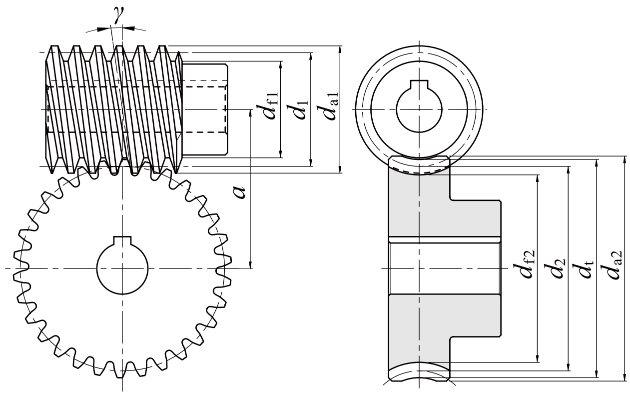


Fig. 4.17 Dimentions of cylindrical worm gear pair

(1) Axial Module Worm Gear Pair

Table 4.23 presents the equations, for dimensions shown in Figure 4.16, for worm gears with axial module, m_x, and normal pressure angle $\alpha_n = 20°$.

Table 4.23 The calculations for an axial module system worm gear pair

No.	Item	Symbol	Formula	Example	
				Worm (1)	Wheel (2)
1	Axial module	m_x	Set Value	3	
2	Normal pressure angle	(α_n)		(20°)	
3	No. of threads, no. of teeth	z		Double Thread (R)	30 (R)
4	Coefficient of Profile shift	x_{t2}		—	0
5	Reference diameter	d_1 d_2	(Qm_x) NOTE 1 $z_2 m_x$	44.000	90.000
6	Reference cylinder lead angle	γ	$\tan^{-1}\left(\frac{m_x z_1}{d_1}\right)$	7.76517°	
7	Center distance	a	$\frac{d_1 + d_2}{2} + x_{t2} m_x$	67.000	
8	Addendum	h_{a1} h_{a2}	$1.00\, m_x$ $(1.00 + x_{t2})\, m_x$	3.000	3.000
9	Tooth depth	h	$2.25\, m_x$	6.750	
10	Tip diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_x$ NOTE 2	50.000	99.000
11	Throat diameter	d_t	$d_2 + 2h_{a2}$	—	96.000
12	Throat surface radius	r_i	$\frac{d_1}{2} - h_{a1}$	—	19.000
13	Root diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_t - 2h$	36.500	82.500

NOTE 1. Diameter factor, Q, means reference diameter of worm, d_1, over axial module, m_x.

$$Q = \frac{d_1}{m_x}$$

NOTE 2. There are several calculation methods of worm wheel tip diameter d_{a2} besides those in Table 4.25.

NOTE 3. The facewidth of worm, b_1, would be sufficient if: $b_1 = \pi m_x (4.5 + 0.02 z_2)$

NOTE 4. Effective facewidth of worm wheel $b_w = 2m_x\sqrt{Q+1}$. So the actual facewidth of $b_2 \geqq b_w + 1.5m_x$ would be enough.

(2) Normal Module System Worm Gear Pair

The equations for normal module system worm gears are based on a normal module, m_n, and normal pressure angle, $\alpha_n = 20°$. See Table 4.24.

Table 4.24 The calculations for a normal module system worm gear pair

No.	Item	Symbol	Formula	Example: Worm (1)	Example: Wheel (2)
1	Normal module	m_n	Set Value	3	
2	Normal pressure angle	α_n		(20°)	
3	No. of threads, No. of teeth	z		Double (R)	30 (R)
4	Reference diameter of worm	d_1		44.000	—
5	Normal profile shift coefficient	x_{n2}		—	−0.1414
6	Reference cylinder lead angle	γ	$\sin^{-1}\left(\frac{m_n z_1}{d_1}\right)$	7.83748°	
7	Reference diameter of worm wheel	d_2	$\frac{z_2 m_n}{\cos\gamma}$	—	90.8486
8	Center distance	a	$\frac{d_1 + d_2}{2} + x_{n2} m_n$	67.000	
9	Addendum	h_{a1} h_{a2}	$1.00\, m_n$ $(1.00 + x_{n2})\, m_n$	3.000	2.5758
10	Tooth depth	h	$2.25\, m_n$	6.75	
11	Tip diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_n$	50.000	99.000
12	Throat diameter	d_t	$d_2 + 2h_{a2}$	—	96.000
13	Throat surface radius	r_i	$\frac{d_1}{2} - h_{a1}$	—	19.000
14	Root diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_t - 2h$	36.500	82.500

NOTE: All notes are the same as those of Table 4.23.

(3) Crowning of the Tooth

Crowning is critically important to worm gears. Not only can it eliminate abnormal tooth contact due to incorrect assembly, but it also provides for the forming of an oil film, which enhances the lubrication effect of the mesh. This can favorably impact endurance and transmission efficiency of the worm mesh. There are four methods of crowning worm gear pair:

(a) Cut Worm Wheel with a Hob Cutter of Greater Reference Diameter than the Worm.

A crownless worm wheel results when it is made by using a hob that has an identical pitch diameter as that of the worm. This crownless worm wheel is very difficult to assemble correctly. Proper tooth contact and a complete oil film are usually not possible.

However, it is relatively easy to obtain a crowned worm wheel by cutting it with a hob whose reference diameter is slightly larger than that of the worm.

This is shown in Figure 4.18. This creates teeth contact in the center region with space for oil film formation.

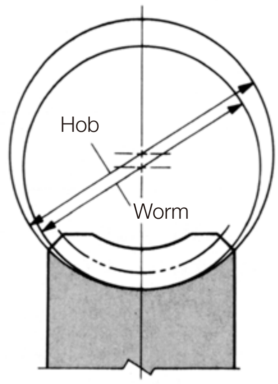


Fig.4.18 The method of using a greater diameter hob

(b) Recut With Hob Center Position Adjustment.

The first step is to cut the worm wheel at standard center distance. This results in no crowning. Then the worm wheel is finished with the same hob by recutting with the hob axis shifted parallel to the worm wheel axis by $\pm\Delta h$. This results in a crowning effect, shown in Figure 4.19.

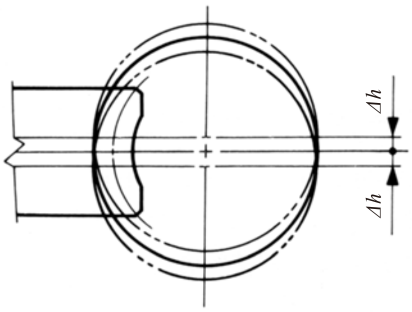


Fig.4.19 Offsetting up or down

(c) Hob Axis Inclining $\Delta\theta$ From Standard Position.

In standard cutting, the hob axis is oriented at the proper angle to the worm wheel axis. After that, the hob axis is shifted slightly left and then right, $\Delta\theta$, in a plane parallel to the worm wheel axis, to cut a crown effect on the worm wheel tooth.

This is shown in Figure 4.20. Only method (a) is popular. Methods (b) and (c) are seldom used.

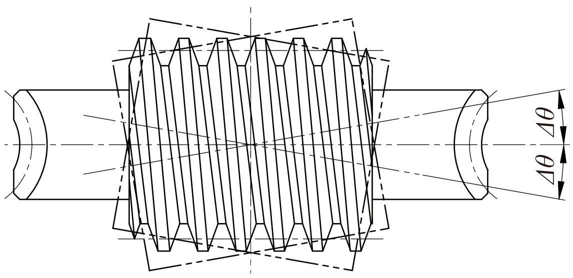


Fig. 4.20 Inclining right or left

(d) Use a Worm with a Larger Pressure Angle than the Worm Wheel.

This is a very complex method, both theoretically and practically. Usually, the crowning is done to the worm wheel, but in this method the modification is on the worm. That is, to change the pressure angle and pitch of the worm without changing base pitch, in accordance with the relationships shown in Equations 4.25:

$$p_x \cos\alpha_x = p_{wx} \cos\alpha_{wx} \tag{4.25}$$

In order to raise the pressure angle from before change, α_{wx}, to after change, α_x, it is necessary to increase the axial pitch, p_{wx}, to a new value, p_x, per Equation (4.25). The amount of crowning is represented as the space between the worm and worm wheel at the meshing point A in Figure 4.22. This amount may be approximated by the following equation:

$$\text{Amount of crowning} \fallingdotseq k\,\frac{p_x - p_{wx}}{p_{wx}}\,\frac{d_1}{2} \tag{4.26}$$

Where d_1 : Reference diameter of worm

k : Factor from Table 4.25 and Figure 4.21

Table 4.25 The value of factor k

α_x	14.5°	17.5°	20°	22.5°
k	0.55	0.46	0.41	0.375

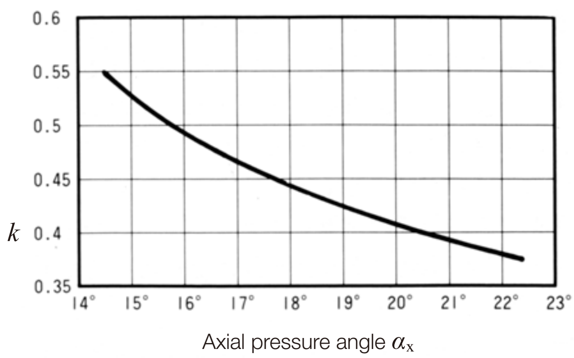


Fig. 4.21 The value of factor (k)

Table 4.26 shows an example of calculating worm crowning.

Table 4.26 The calculations for worm crowning

No.	Item	Symbol	Formula	Example
1	Axial module	m_{wx}	NOTE: This is the data before crowning.	3
2	Normal pressure angle	α_{wn}		20°
3	Number of threads of worm	z_1		2
4	Reference diameter of worm	d_1		44.000
5	Reference cylinder lead angle	γ_w	$\tan^{-1}\left(\dfrac{m_{wx} z_1}{d_1}\right)$	7.765166°
6	Axial pressure angle	α_{wx}	$\tan^{-1}\left(\dfrac{\tan \alpha_{wn}}{\cos \gamma_w}\right)$	20.170236°
7	Axial pitch	p_{wx}	πm_{wx}	9.424778
8	Lead	p_{wz}	$\pi m_{wx}\, z_1$	18.849556
9	Amount of crowning	C_R	It should be determined by considering the size of tooth contact .	0.04
10	Factor	k	From Table 4.26	0.41
※ After crowning				
11	Axial pitch	p_x	$p_{wx}\left(\dfrac{2C_R}{kd_1}+1\right)$	9.466573
12	Axial pressure angle	α_x	$\cos^{-1}\left(\dfrac{p_{wx}}{p_x}\cos\alpha_{wx}\right)$	20.847973°
13	Axial module	m_x	$\dfrac{p_x}{\pi}$	3.013304
14	Reference cylinder lead angle	γ	$\tan^{-1}\left(\dfrac{m_x z_1}{d_1}\right)$	7.799179°
15	Normal pressure angle	α_n	$\tan^{-1}(\tan\alpha_x \cos\gamma)$	20.671494°
16	Lead	p_z	$\pi m_x\, z_1$	18.933146

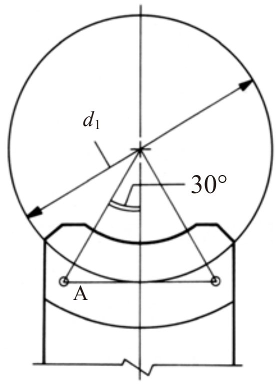


Fig.4.22 Position A is the point of determining crowning amount

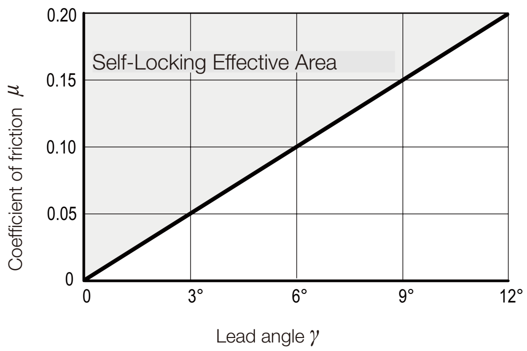


Fig. 4.23 The critical limit of self-locking of lead angle g and coefficient of friction m

(4) Self-Locking Of Worm Gear Pairs

Self-locking is a unique characteristic of worm meshes that can be put to advantage. It is the feature that a worm cannot be driven by the worm wheel. It is very useful in the design of some equipment, such as lifting, in that the drive can stop at any position without concern that it can slip in reverse. However, in some situations it can be detrimental if the system requires reverse sensitivity, such as a servomechanism.

Self-locking does not occur in all worm meshes, since it requires special conditions as outlined here. In this analysis, only the driving force acting upon the tooth surfaces is considered without any regard to losses due to bearing friction, lubricant agitation, etc. The governing conditions are as follows:

Let F_{t1} = tangential driving force of worm

Then,

$$F_{t1} = F_n\left(\cos\alpha_n \sin\gamma - \mu\cos\gamma\right) \qquad (4.27)$$

If $F_{t1} > 0$ then there is no self-locking effect at all. Therefore, $F_{t1} \le 0$ is the critical limit of self-locking.

Let α_n in Equation (4.27) be 20°, then the condition:

$F_{t1} \le 0$ will become:

(cos 20° sing – mcosg) ≤ 0

Figure 4.22 shows the critical limit of self-locking for lead angle g and coefficient of friction m. Practically, it is very hard to assess the exact value of coefficient of friction μ. Further, the bearing loss, lubricant agitation loss, etc. can add many side effects. Therefore, it is not easy to establish precise self-locking conditions.

However, it is true that the smaller the lead angle γ, the more likely the self-locking condition will occur.

5 Tooth Thickness

There are direct and indirect methods for measuring tooth thickness. In general, there are three methods:

- Chordal tooth thickness measurement
- Span measurement
- Over pin or ball measurement

5.1 Chordal Tooth Thickness Measurement

This method employs a tooth caliper that is referenced from the gear's tip diameter. Thickness is measured at the reference circle. See Figure 5.1.

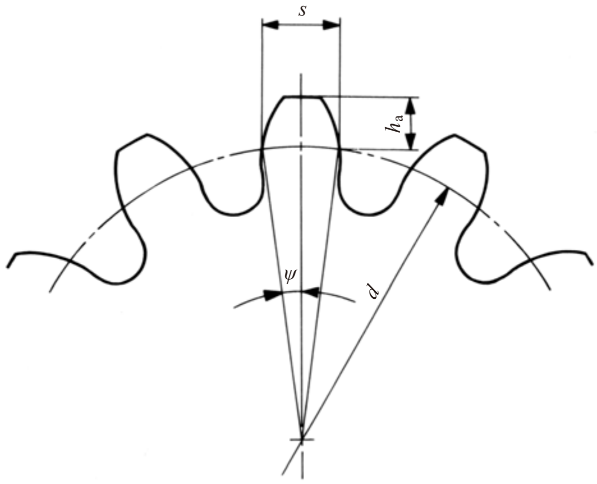


Fig.5.1 Chordal tooth thickness method

(1) Spur Gears

Table 5.1 presents equations for each chordal tooth thickness measurement.

Table 5.1 Equations for spur gear chordal tooth thickness

No.	Item	Symbol	Formula	Example
1	Tooth thickness	s	$\left(\frac{\pi}{2} + 2x \tan\alpha\right) m$	$m = 10$ $\alpha = 20°$ $z = 12$ $x = +0.3$ $h_a = 13.000$ $s = 17.8918$ $\psi = 8.54270°$ $\bar{s} = 17.8256$ $\bar{h}_a = 13.6657$
2	Tooth thickness half angle	ψ	$\frac{90}{z} + \frac{360x \tan\alpha}{\pi z}$	
3	Chordal tooth thickness	$\bar{s}$	$zm \sin\psi$	
4	Chordal height	$\bar{h}_a$	$\frac{zm}{2}(1 - \cos\psi) + h_a$	

(2) Spur Racks and Helical Racks

The governing equations become simple since the rack tooth profile is trapezoid, as shown in Table 5.2.

Table 5.2 Chordal tooth thickness of racks

No.	Item	Symbol	Formula	Example
1	Chordal tooth thickness	$\bar{s}$	$\frac{\pi m}{2}$ or $\frac{\pi m_n}{2}$	$m = 3$ $\alpha = 20°$ $\bar{s} = 4.7124$ $\bar{h}_a = 3.0000$
2	Chordal height	$\bar{h}_a$	h_a	

NOTE: These equations are also applicable to helical racks.

(3) Helical Gears

The chordal tooth thickness of helical gears should be measured on the normal plane basis as shown in Table 5.3. Table 5.4 presents the equations for chordal tooth thickness of helical gears in the transverse system.

Table 5.3 Equations for chordal tooth thickness of helical gears in the normal system

No.	Item	Symbol	Formula	Example
1	Normal tooth thickness	s_n	$\left(\frac{\pi}{2}+2x_n\tan\alpha_n\right)m_n$	$m_n = 5$ $\alpha_n = 20°$ $\beta = 25°\,00'\,00''$ $z = 16$ $x_n = +0.2$ $h_a = 6.0000$ $s_n = 8.5819$ $z_v = 21.4928$ $\psi_v = 4.57556°$ $\bar{s} = 8.5728$ $\bar{h}_a = 6.1712$
2	Number of teeth of an equivalent spur gear	z_v	$\frac{z}{\cos^3\beta}$	
3	Tooth thickness half angle	ψ_v	$\frac{90}{z_v}+\frac{360x_n\tan\alpha_n}{\pi z_v}$	
4	Chordal tooth thickness	$\bar{s}$	$z_v m_n \sin\psi_v$	
5	Chordal height	$\bar{h}_a$	$\frac{z_v m_n}{2}(1-\cos\psi_v)+h_a$	

Table 5.4 Equations for chordal tooth thickness of helical gears in the transverse system

No.	Item	Symbol	Formula	Example
1	Normal tooth thickness	s_n	$\left(\frac{\pi}{2}+2x_t\tan\alpha_t\right)m_t\cos\beta$	$m_t = 2.5$ $\alpha_t = 20°$ $\beta = 21°\,30'\,00''$ $z = 20$ $x_t = 0$ $h_a = 2.5$ $s_n = 3.6537$ $z_v = 24.8311$ $\psi_v = 3.62448°$ $\bar{s} = 3.6513$ $\bar{h}_a = 2.5578$
2	Number of teeth in an equivalent spur gear	z_v	$\frac{z}{\cos^3\beta}$	
3	Tooth thickness half angle	ψ_v	$\frac{90}{z_v}+\frac{360x_t\tan\alpha_t}{\pi z_v}$	
4	Chordal tooth thickness	$\bar{s}$	$z_v m_t\cos\beta\sin\psi_v$	
5	Chordal height	$\bar{h}_a$	$\frac{z_v m_t\cos\beta}{2}(1-\cos\psi_v)+h_a$	

(4) Bevel Gears

Table 5.5 shows the equations for chordal tooth thickness of a Gleason straight bevel gear. Table 5.6 shows the same of a standard straight bevel gear. Table 5.7 the same of a Gleason spiral bevel gear.

Table 5.5 Equations for chordal tooth thickness of Gleason straight bevel gears

No.	Item	Symbol	Formula	Example
1	Tooth thickness factor (Coefficient of horizontal profile shift)	K	Obtain from Figure 5.2	$m = 4$ $\alpha = 20°$ $\Sigma = 90°$ $z_1 = 16$ $z_2 = 40$ $z_1/z_2 = 0.4$ $K = 0.0259$ $h_{a1} = 5.5456$ $h_{a2} = 2.4544$ $\delta_1 = 21.8014°$ $\delta_2 = 68.1986°$ $s_1 = 7.5119$ $s_2 = 5.0545$ $\bar{s}_1 = 7.4946$ $\bar{s}_2 = 5.0536$ $\bar{h}_{a1} = 5.7502$ $\bar{h}_{a2} = 2.4692$
2	Tooth thickness	s_1 s_2	$\pi m - s_2$ $\frac{\pi m}{2}-(h_{a1}-h_{a2})\tan\alpha - Km$	
3	Chordal tooth thickness	$\bar{s}$	$s-\frac{s^3}{6d^2}$	
4	Chordal height	$\bar{h}_a$	$h_a+\frac{s^2\cos\delta}{4d}$	

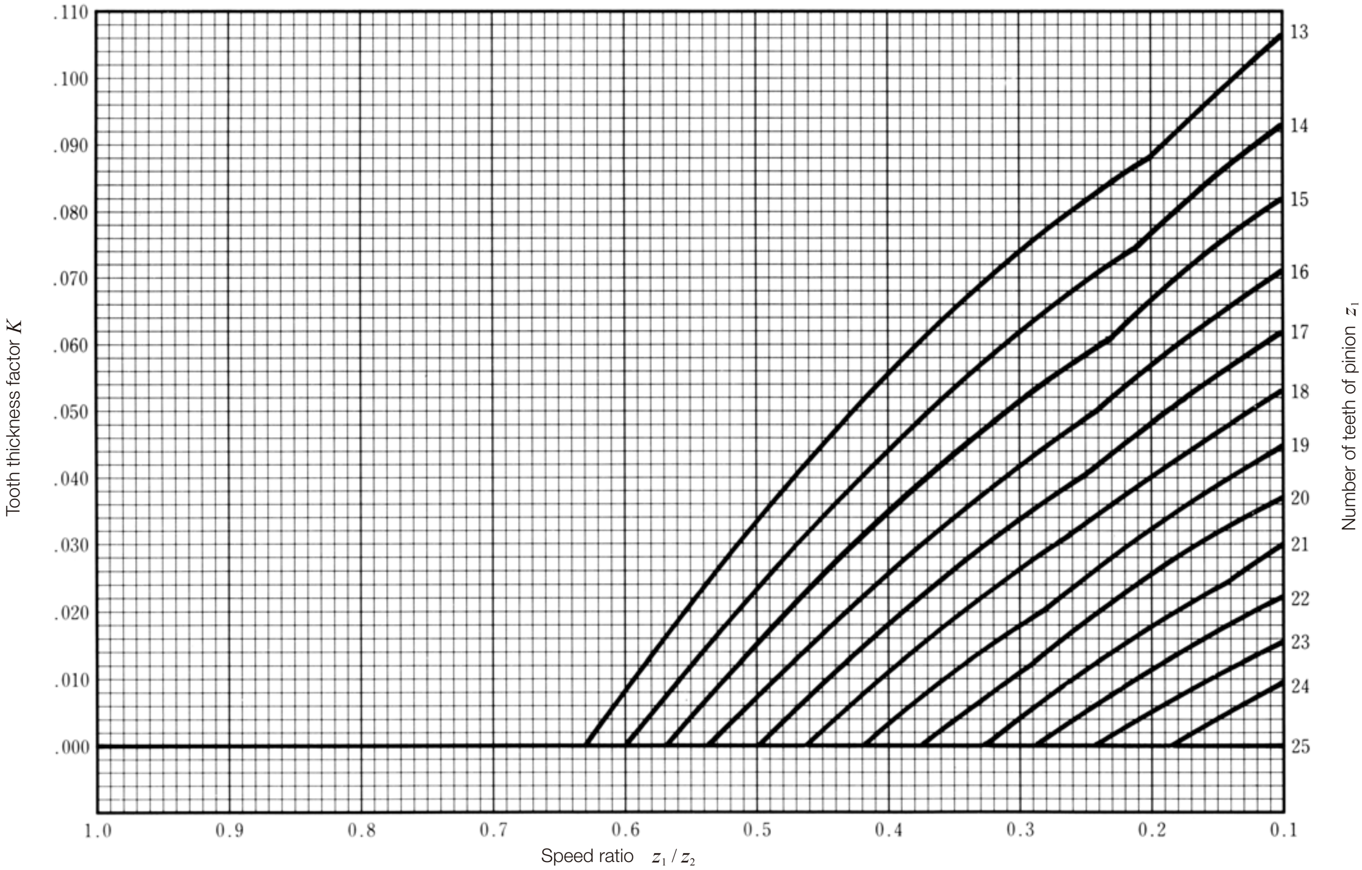


Fig.5.2 Chart to determine the tooth thickness factor k for Gleason straight bevel gear

Table 5.6 Equations for chordal tooth thickness of standard straight bevel gears

No.	Item	Symbol	Formula	Example
1	Tooth thickness	s	$\dfrac{\pi m}{2}$	$m = 4$ $\alpha = 20°$ $\Sigma = 90°$ $z_1 = 16$ $z_2 = 40$ $d_1 = 64$ $d_2 = 160$ $h_a = 4.0000$ $\delta_1 = 21.8014°$ $\delta_2 = 68.1986°$ $s = 6.2832$ $z_{v1} = 17.2325$ $z_{v2} = 107.7033$ $R_{v1} = 34.4650$ $R_{v2} = 215.4066$ $\psi_{v1} = 5.2227°$ $\psi_{v2} = 0.83563°$ $\bar{s}_1 = 6.2745$ $\bar{s}_2 = 6.2830$ $\bar{h}_{a1} = 4.1431$ $\bar{h}_{a2} = 4.0229$
2	Number of teeth of an equivalent spur gear	z_v	$\dfrac{z}{\cos\delta}$	
3	Back cone distance	R_v	$\dfrac{d}{2\cos\delta}$	
4	Tooth thickness half angle	ψ_v	$\dfrac{90}{z_v}$	
5	Chordal tooth thickness	$\bar{s}$	$z_v m \sin\psi_v$	
6	Chordal height	$\bar{h}_a$	$h_a + R_v(1 - \cos\psi_v)$	

If a straight bevel gear is cut by a Gleason straight bevel cutter, the tooth angle should be adjusted according to:

Tooth angle (°) =

$$\frac{180}{\pi R}\left(\frac{s}{2} + h_f \tan\alpha\right) \qquad (5.1)$$

This angle is used as a reference in determining the tooth thickness, s, when setting up the gear cutting machine.

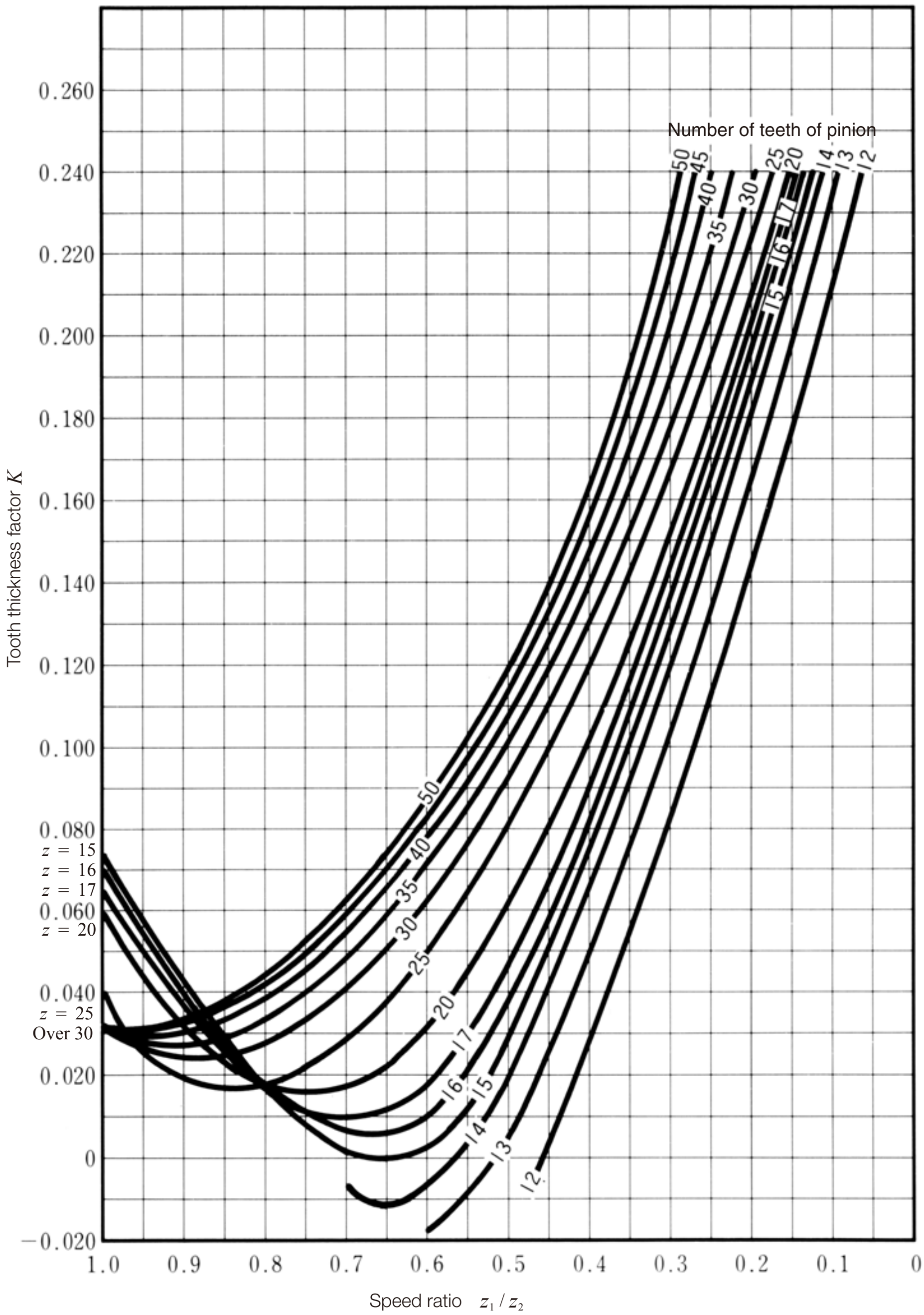


Fig.5.3 Chart to determine the tooth thickness factor k for Gleason spiral bevel gears

Table 5.7 Equations for chordal tooth thickness of Gleason spiral bevel gears

No.	Item	Symbol	Formula	Example
1	Tooth thickness factor	K	Obtain from Figure 5.3	$\Sigma = 90°$ $m = 3$ $\alpha_n = 20°$ $z_1 = 20$ $z_2 = 40$ $\beta_m = 35°$ $h_{a1} = 3.4275$ $h_{a2} = 1.6725$ $K = 0.060$ $p = 9.4248$ $s_1 = 5.6722$ $s_2 = 3.7526$
2	Tooth thickness	s_1	$p - s_2$	
		s_2	$\dfrac{p}{2} - (h_{a1} - h_{a2}) \dfrac{\tan \alpha_n}{\cos \beta_m} Km$	

The calculations of chordal tooth thickness of a Gleason spiral bevel gear are so complicated that we do not intend to go further in this presentation.

(5) Worm Gear Pair

Table 5.8 presents equations for chordal tooth thickness of axial module worm gear pairs. Table 5.9 presents the same of normal module worm gear pairs.

Table 5.8 Equations for chordal tooth thickness of an axial module worm gear pair

No.	Item	Symbol	Formula	Example
1	Axial tooth thickness of worm	s_{x1}	$\dfrac{\pi m_x}{2}$	$m_x = 3$ $m_t = 3$ $\alpha_n = 20°$ $z_1 = 2$ $z_2 = 30$ $d_1 = 38$ $d_2 = 90$ $a = 65$ $x_{t2} = +0.33333$ $h_{a1} = 3.0000$ $h_{a2} = 4.0000$ $\gamma = 8.97263°$ $\alpha_t = 20.22780°$ $s_{x1} = 4.71239$ $s_{t2} = 5.44934$ $z_{v2} = 31.12885$ $\psi_{v2} = 3.34335°$ $\bar{s}_1 = 4.6547$ $\bar{s}_2 = 5.3796$ $\bar{h}_{a1} = 3.0035$ $\bar{h}_{a2} = 4.0785$
	Transverse tooth thickness of worm wheel	s_{t2}	$\left(\dfrac{\pi}{2} + 2x_{t2}\tan\alpha_t\right)m_t$	
2	No. of teeth in an equivalent spur gear (Worm wheel)	z_{v2}	$\dfrac{z_2}{\cos^3\gamma}$	
3	Tooth thickness half angle (Worm wheel)	ψ_{v2}	$\dfrac{90}{z_{v2}} + \dfrac{360\,x_{t2}\tan\alpha_t}{\pi z_{v2}}$	
4	Chordal tooth thickness	$\bar{s}_1$ $\bar{s}_2$	$s_{x1}\cos\gamma$ $z_{v2}\,m_t\cos\gamma\sin\psi_{v2}$	
5	Chordal height	$\bar{h}_{a1}$	$h_{a1} + \dfrac{(s_{x1}\sin\gamma\cos\gamma)^2}{4d_1}$	
		$\bar{h}_{a2}$	$h_{a2} + \dfrac{z_{v2}\,m_t\cos\gamma}{2}(1-\cos\psi_{v2})$	

Table 5.9 Equations for chordal tooth thickness of a normal module worm gear pair

No.	Item	Symbol	Formula	Example
1	Normal tooth thickness of worm	s_{n1}	$\dfrac{\pi m_n}{2}$	$m_n = 3$ $\alpha_n = 20°$ $z_1 = 2$ $z_2 = 30$ $d_1 = 38$ $d_2 = 91.1433$ $a = 65$ $x_{n2} = 0.14278$ $h_{a1} = 3.0000$ $h_{a2} = 3.42835$ $\gamma = 9.08472°$ $s_{n1} = 4.71239$ $s_{n2} = 5.02419$ $z_{v2} = 31.15789$ $\psi_{v2} = 3.07964°$ $\bar{s}_1 = 4.7124$ $\bar{s}_2 = 5.0218$ $\bar{h}_{a1} = 3.0036$ $\bar{h}_{a2} = 3.4958$
	Transverse tooth thickness of worm wheel	s_{n2}	$\left(\dfrac{\pi}{2} + 2x_{n2}\tan\alpha_n\right)m_n$	
2	No. of teeth in an equivalent spur gear (Worm wheel)	z_{v2}	$\dfrac{z_2}{\cos^3\gamma}$	
3	Tooth thickness half angle (Worm wheel)	ψ_{v2}	$\dfrac{90}{z_{v2}} + \dfrac{360\,x_{n2}\tan\alpha_n}{\pi z_{v2}}$	
4	Chordal tooth thickness	$\bar{s}_1$ $\bar{s}_2$	s_{n1} $z_{v2}\,m_n\sin\psi_{v2}$	
5	Chordal height	$\bar{h}_{a1}$	$h_{a1} + \dfrac{(s_{n1}\sin\gamma)^2}{4d_1}$	
		$\bar{h}_{a2}$	$h_{a2} + \dfrac{z_{v2}\,m_n}{2}(1-\cos\psi_{v2})$	

5.2 Span Measurement of Teeth

Span measurement of teeth, W, is a measure over a number of teeth, k, made by means of a special tooth thickness micrometer. The value measured is the sum of normal tooth thickness on the base circle, s_{bn}, and normal pitch, $p_{bn}(k-1)$. See Figure 5.4.

(1) Spur and Internal Gears

The applicable equations are presented in Table 5.10.

Table 5.10 Span measurement calculations for spur and internal gear teeth

No.	Item	Symbol	Formula	Example
1	Span number of teeth	k	$k_{th} = zK(f) + 0.5$ NOTE 1 Select the nearest natural number of k_{th} as k	$m = 3$ $\alpha = 20°$ $z = 24$ $x = +0.4$ $k_{th} = 3.78787$ $k = 4$ $W = 32.8266$
2	Span measurement over k teeth	W	$m \cos\alpha \{\pi(k-0.5) + z \operatorname{inv}\alpha\} + 2xm \sin\alpha$	

NOTE :

$$K(f) = \frac{1}{\pi}\{\sec\alpha\sqrt{(1+2f)^2-\cos^2\alpha} - \operatorname{inv}\alpha - 2f\tan\alpha\} \quad (5.2)$$

Where $f = \dfrac{x}{z}$

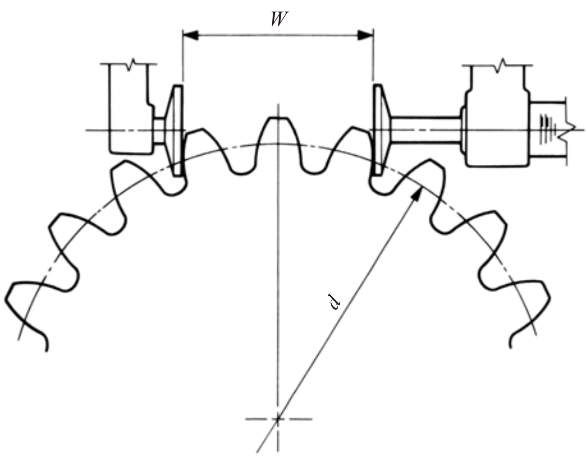


Fig.5.4 Span measurement over k teeth (spur gear)

Figure 5.4 shows the span measurement of a spur gear. This measurement is on the outside of the teeth.

For internal gears the tooth profile is opposite to that of the external spur gear. Therefore, the measurement is between the inside of the tooth profiles.

(2) Helical Gears

Tables 5.11 and 5.12 present equations for span measurement of the normal and the transverse systems, respectively, of helical gears.

Table 5.11 Equations for the span measurement of normal system helical gears

No.	Item	Symbol	Formula	Example
1	Span number of teeth	k	$k_{th} = zK(f,\beta) + 0.5$ NOTE 1 Select the nearest natural number of k_{th} as k	$m_n = 3$, $\alpha_n = 20°$, $z = 24$ $\beta = 25°00'00''$ $x_n = +0.4$ $\alpha_t = 21.88023°$ $k_{th} = 4.63009$ $k = 5$ $W = 42.0085$
2	Span measurement over k teeth	W	$m_n \cos\alpha_n \{\pi(k - 0.5) + z \operatorname{inv}\alpha_t\} + 2x_n m_n \sin\alpha_n$	

NOTE :

$$K(f,\beta) = \frac{1}{\pi}\left\{\left(1+\frac{\sin^2\beta}{\cos^2\beta+\tan^2\alpha_n}\right)\sqrt{(\cos^2\beta+\tan^2\alpha_n)(\sec\beta+2f)^2-1} - \operatorname{inv}\alpha_t - 2f\tan\alpha_n\right\} \quad (5.3)$$

Where $f = \dfrac{x_n}{z}$

See page 655 to find the figures showing how to determine the number of span number of teeth of a profile shifted spur and helical gears.

Table 5.12 Equations for span measurement of transverse system helical gears

No.	Item	Symbol	Formula	Example
1	Span number of teeth	k	$k_{th} = zK\ (f,\beta) + 0.5$ NOTE 1 Select the nearest natural number of k_{th} as k	$m_t = 3$, $\alpha_t = 20°$, $z = 24$ $\beta = 22°30'00"$ $x_t = +0.4$ $\alpha_n = 18.58597°$ $k_{th} = 4.31728$ $k = 4$ $W = 30.5910$
2	Span measurement over k teeth	W	$m_t \cos\beta \cos\alpha_n \{\pi(k - 0.5) + z \operatorname{inv}\alpha_t\} + 2x_t m_t \sin\alpha_n$	

NOTE :

$$K(f,\beta) = \frac{1}{\pi}\left\{\left(1 + \frac{\sin^2\beta}{\cos^2\beta + \tan^2\alpha_n}\right)\sqrt{(\cos^2\beta + \tan^2\alpha_n)(\sec\beta + 2f)^2 - 1} - \operatorname{inv}\alpha_t - 2f\tan\alpha_n\right\} \quad (5.4)$$

where $f = \dfrac{x_t}{z\cos\beta}$

There is a requirement of a minimum facewidth to make a helical gear span measurement. Let b_{min} be the minimum value for facewidth. See Fig. 5.5.

Then $b_{min} = W\sin\beta_b + \Delta b$ (5.5)

Where β_b is the helix angle at the base cylinder,

$$\left.\begin{aligned}\beta_b &= \tan^{-1}(\tan\beta\cos\alpha_t)\\ &= \sin^{-1}(\sin\beta\cos\alpha_n)\end{aligned}\right\} \quad (5.6)$$

From the above, we can determine $\Delta b > 3$ mm to make a stable measurement of W.

Refer to page 752 to 755 to review the data sheet "Span Measurement Over k Teeth of Standard Spur Gears" (Pressure Angle: 20℃ , 14.5℃).

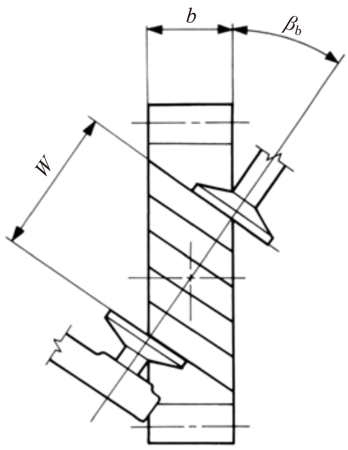


Fig.5.5 Facewidth of helical gear

5.3 Measurement Over Rollers (or generally called over pin/ball measurement)

As shown in Figure 5.6, measurement is made over the outside of two pins that are inserted in diametrically opposite tooth spaces, for even tooth number gears, and as close as possible for odd tooth number gears. The procedure for measuring a rack with a pin or a ball is as shown in Figure 5.8 by putting pin or ball in the tooth space and using a micrometer between it and a reference surface.

Internal gears are similarly measured, except that the measurement is between the pins. See Figure 3.9. Helical gears can only be measured with balls. In the case of a worm, three pins are used, as shown in Figure 5.10. This is similar to the procedure of measuring a screw thread.

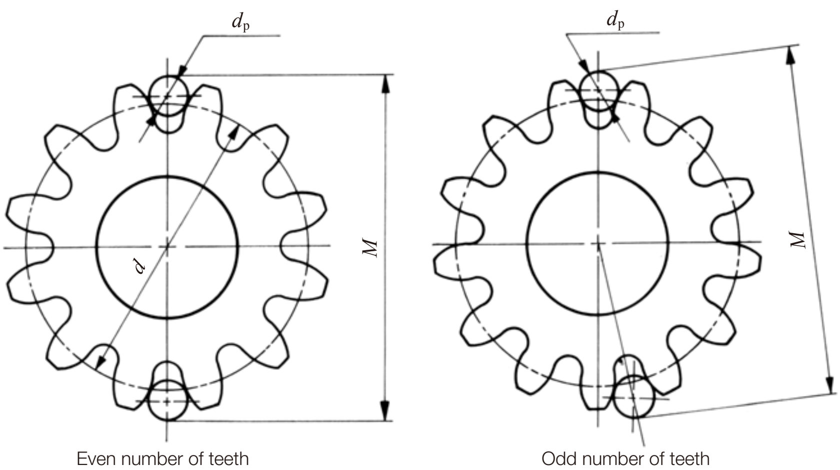


Fig. 5.6 Over pin (ball) measurement

(1) Spur Gears

In measuring a standard gear, the size of the pin must meet the condition that its surface should have a tangent point at the standard pitch circle. When measuring a shifted gear, the surface of the pin should have a tangent point at the $d + 2xm$ circle. Under the condition mentioned above, Table 5.13 indicates formulas to determine the diameter of the pin (ball) for the spur gear in Figure 5.7.

Table 5.13 Equations for calculating ideal pin diameters

No.	Item	Symbol	Formula	Example
1	Spacewidth half angle	η	$\left(\frac{\pi}{2z} - \mathrm{inv}\,\alpha\right) - \frac{2x\tan\alpha}{z}$	$m = 1$ $\alpha = 20°$ $z = 20$ $x = 0$ $\eta = 0.0636354$ $\alpha' = 20°$ $\varphi = 0.4276057$ $d'_p = 1.7245$
2	Pressure angle at the point pin is tangent to tooth surface	α'	$\cos^{-1}\left\{\frac{zm\cos\alpha}{(z+2x)m}\right\}$	
3	Pressure angle at pin center	φ	$\tan\alpha' + \eta$	
4	Ideal pin diameter	d'_p	$zm\cos\alpha\ (\mathrm{inv}\,\varphi + \eta)$	

NOTE: The units of angles η and φ are radians.

The ideal diameters of pins when calculated from the equations of Table 5.13 may not be practical. So, in practice, we select a standard pin diameter close to the ideal value. After the actual diameter of pin dp is determined, the over pin measurement M can be calculated from Table 5.14.

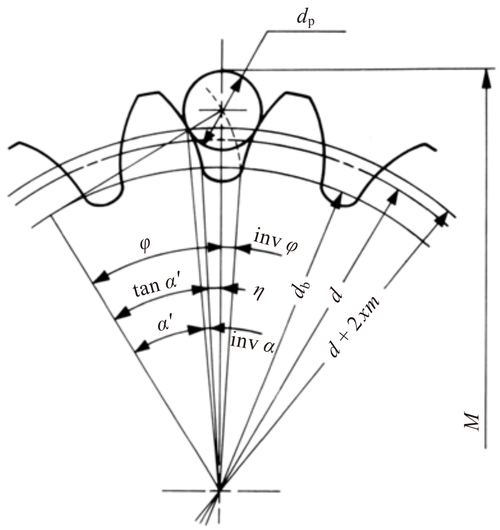


Fig.5.7 Over pins measurement of spur gear

Table 5.14 Equations for over pins measurement of spur gears

No.	Item	Symbol	Formula	Example
1	Pin diameter	d_p	NOTE 1	$d_p = 1.7$ $\mathrm{inv}\varphi = 0.0268197$ $\varphi = 24.1350°$ $M = 22.2941$
2	Involute function φ	$\mathrm{inv}\,\varphi$	$\frac{d_p}{zm\cos\alpha} - \frac{\pi}{2z} + \mathrm{inv}\,\alpha + \frac{2x\tan\alpha}{z}$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Measurement over pin (ball)	M	Even teeth $\frac{zm\cos\alpha}{\cos\varphi} + d_p$	
			Odd teeth $\frac{zm\cos\alpha}{\cos\varphi}\cos\frac{90°}{z} + d_p$	

NOTE: The value of the ideal pin diameter from Table 5.13, or its approximate value, is applied as the actual diameter of pin d_p here.

Table 5.15 is a dimensional table under the condition of module m = 1 and pressure angle α = 20° with which the pin has the tangent point at $d + 2xm$ circle.

Table 5.15 The size of pin which has the tangent point at $d + 2xm$ circle for spur gears

$m = 1$、$\alpha = 20°$

No. of teeth z	Profile shift coefficient x							
	− 0.4	− 0.2	0	0.2	0.4	0.6	0.8	1.0
10		1.6347	1.7886	1.9979	2.2687	2.6079	3.0248	3.5315
20	1.6231	1.6599	1.7244	1.8149	1.9306	2.0718	2.2389	2.4329
30	1.6418	1.6649	1.7057	1.7632	1.8369	1.9267	2.0324	2.1542
40	1.6500	1.6669	1.6967	1.7389	1.7930	1.8589	1.9365	2.0257
50	1.6547	1.6680	1.6915	1.7247	1.7675	1.8196	1.8810	1.9515
60	1.6577	1.6687	1.6881	1.7155	1.7509	1.7940	1.8448	1.9032
70	1.6598	1.6692	1.6857	1.7090	1.7391	1.7759	1.8193	1.8691
80	1.6613	1.6695	1.6839	1.7042	1.7304	1.7625	1.8003	1.8438
90	1.6625	1.6698	1.6825	1.7005	1.7237	1.7521	1.7857	1.8242
100	1.6635	1.6700	1.6814	1.6975	1.7184	1.7439	1.7740	1.8087
110	1.6642	1.6701	1.6805	1.6951	1.7140	1.7372	1.7645	1.7960
120	1.6649	1.6703	1.6797	1.6931	1.7104	1.7316	1.7567	1.7855
130	1.6654	1.6704	1.6791	1.6914	1.7074	1.7269	1.7500	1.7766
140	1.6659	1.6705	1.6785	1.6900	1.7048	1.7229	1.7443	1.7690
150	1.6663	1.6706	1.6781	1.6887	1.7025	1.7194	1.7394	1.7625
160	1.6666	1.6706	1.6777	1.6876	1.7006	1.7164	1.7351	1.7567
170	1.6669	1.6707	1.6773	1.6867	1.6988	1.7137	1.7314	1.7517
180	1.6672	1.6707	1.6770	1.6858	1.6973	1.7114	1.7280	1.7472
190	1.6674	1.6708	1.6767	1.6851	1.6959	1.7093	1.7250	1.7432
200	1.6676	1.6708	1.6764	1.6844	1.6947	1.7074	1.7223	1.7396

(2) Spur Racks and Helical Racks

In measuring a rack, the pin is ideally tangent with the tooth flank at the pitch line. The equations in Table 5.16A can, thus, be derived. In the case of a helical rack, module m, and pressure angle α, in Table 5.16A, can be substituted by normal module m_n , and normal pressure angle α_n , resulting in Table 5.16B.

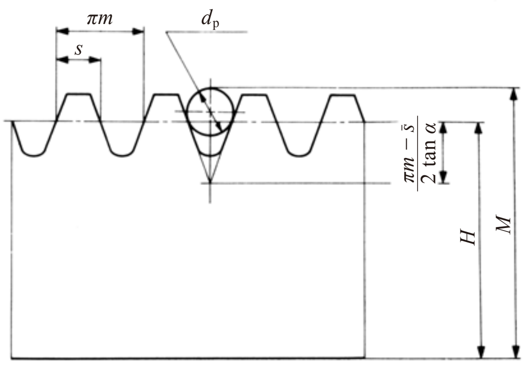


Fig. 5.8 Over pins measurement for a rack using a pin or a ball

Table 5.16A Equations for over pins measurement of spur racks

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\dfrac{\pi m - \bar{s}}{\cos \alpha}$	m = 1 α = 20° s = 1.5708 d'_p = 1.6716 d_p = 1.7 H = 14.0000 M = 15.1774
2	Measurement over pin (ball)	M	$H - \dfrac{\pi m - \bar{s}}{2 \tan \alpha} + \dfrac{d_p}{2}\left(1 + \dfrac{1}{\sin \alpha}\right)$	

Table 5.16B Equations for Over Pins Measurement of Helical Racks

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\dfrac{\pi m_n - \bar{s}}{\cos\alpha_n}$	m_n = 1 α_n = 20°、β = 15° s = 1.5708 d'_p = 1.6716 d_p = 1.7 H = 14.0000 M = 15.1774
2	Measurement over pin (ball)	M	$H - \dfrac{\pi m_n - \bar{s}}{2\tan\alpha_n} + \dfrac{d_p}{2}\left(1 + \dfrac{1}{\sin\alpha_n}\right)$	

(3) Internal Gears

As shown in Figure 5.9, measuring an internal gear needs a proper pin which has its tangent point at $d + 2xm$ circle. The equations are in Table 5.17 for obtaining the ideal pin diameter. The equations for calculating the between pin measurement, M, are given in Table 5.18.

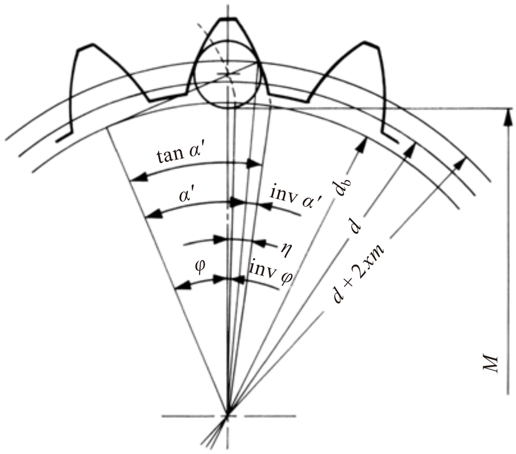


Fig. 5.9 Between pin dimension of internal gears

Table 5.17 Equations for calculating pin diameter for internal gears

No.	Item	Symbol	Formula	Example
1	Spacewidth half angle	η	$\left(\dfrac{\pi}{2z} + \mathrm{inv}\,\alpha\right) + \dfrac{2x\tan\alpha}{z}$	m = 1 α = 20° z = 40 x = 0 η = 0.054174 α' = 20° φ = 0.309796 d'_p = 1.6489
2	Pressure angle at the point pin is tangent to tooth surface	α'	$\cos^{-1}\left\{\dfrac{zm\cos\alpha}{(z+2x)m}\right\}$	
3	Pressure angle at pin center	φ	$\tan\alpha' - \eta$	
4	Ideal pin diameter	d'_p	$zm\cos\alpha\,(\eta - \mathrm{inv}\,\varphi)$	

NOTE: The units of angles η, φ are radians.

Tabl 5.18 Equations for between pins measurement of internal gears

No.	Item	Symbol	Formula		Example
1	Pin (ball) diameter	d_p	see NOTE 1		d_p = 1.7 invφ = 0.0089467 φ = 16.9521° M = 37.5951
2	Involute function φ	inv φ	$\left(\dfrac{\pi}{2z} + \mathrm{inv}\,\alpha\right) - \dfrac{d_p}{zm\cos\alpha} + \dfrac{2x\tan\alpha}{z}$		
3	Pressure angle at pin center	φ	Find from involute function table		
4	Between pins measurement	M	Even teeth	$\dfrac{zm\cos\alpha}{\cos\varphi} - d_p$	
			Odd teeth	$\dfrac{zm\cos\alpha}{\cos\varphi}\cos\dfrac{90°}{z} - d_p$	

NOTE: First, calculate the ideal pin diameter. Then, choose the nearest practical actual pin size.

Table 5.19 lists ideal pin diameters for standard and profile shifted gears under the condition of module $m = 1$ and pressure angle $\alpha = 20°$, which makes the pin tangent to the reference circle $d + 2xm$.

Table 5.19 The size of pin that is tangent at reference circle $d + 2xm$ for internal gears

$m = 1$、$\alpha = 20°$

No. of teeth z	Profile shift coefficient x							
	− 0.4	− 0.2	0	0.2	0.4	0.6	0.8	1.0
10	—	1.4789	1.5936	1.6758	1.7283	1.7519	1.7460	1.7092
20	1.4687	1.5604	1.6284	1.6759	1.7047	1.7154	1.7084	1.6837
30	1.5309	1.5942	1.6418	1.6751	1.6949	1.7016	1.6956	1.6771
40	1.5640	1.6123	1.6489	1.6745	1.6895	1.6944	1.6893	1.6744
50	1.5845	1.6236	1.6532	1.6740	1.6862	1.6900	1.6856	1.6732
60	1.5985	1.6312	1.6562	1.6737	1.6839	1.6870	1.6832	1.6725
70	1.6086	1.6368	1.6583	1.6734	1.6822	1.6849	1.6815	1.6721
80	1.6162	1.6410	1.6600	1.6732	1.6810	1.6833	1.6802	1.6718
90	1.6222	1.6443	1.6612	1.6731	1.6800	1.6820	1.6792	1.6717
100	1.6270	1.6470	1.6622	1.6729	1.6792	1.6810	1.6784	1.6715
110	1.6310	1.6492	1.6631	1.6728	1.6785	1.6801	1.6778	1.6715
120	1.6343	1.6510	1.6638	1.6727	1.6779	1.6794	1.6772	1.6714
130	1.6371	1.6525	1.6644	1.6727	1.6775	1.6788	1.6768	1.6714
140	1.6395	1.6539	1.6649	1.6726	1.6771	1.6783	1.6764	1.6714
150	1.6416	1.6550	1.6653	1.6725	1.6767	1.6779	1.6761	1.6713
160	1.6435	1.6561	1.6657	1.6725	1.6764	1.6775	1.6758	1.6713
170	1.6451	1.6570	1.6661	1.6724	1.6761	1.6771	1.6755	1.6713
180	1.6466	1.6578	1.6664	1.6724	1.6759	1.6768	1.6753	1.6713
190	1.6479	1.6585	1.6666	1.6723	1.6757	1.6766	1.6751	1.6713
200	1.6490	1.6591	1.6669	1.6723	1.6755	1.6763	1.6749	1.6713

(4) Helical Gears

The ideal pin that makes contact at the $d + 2x_n m_n$ reference circle of a helical gear can be obtained from the same above equations, but with the teeth number z substituted by the equivalent (virtual) teeth number z_v.

Table 5.20 presents equations for deriving over pin diameters. Table 5.21 presents equations for calculating over pin measurements for helical gears in the normal system.

Table 5.20 Equations for calculating pin diameter for helical gears in the normal system

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\dfrac{z}{\cos^3\beta}$	$m_n = 1$ $\alpha_n = 20°$ $z = 20$ $\beta = 15°00'00''$ $x_n = +0.4$ $z_v = 22.19211$ $\eta_v = 0.0427566$ $\alpha'_v = 24.90647°$ $\varphi_v = 0.507078$ $d'_p = 1.9020$
2	Spacewidth half angle	η_v	$\dfrac{\pi}{2z_v} - \mathrm{inv}\,\alpha_n - \dfrac{2x_n \tan\alpha_n}{z_v}$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\dfrac{z_v\cos\alpha_n}{z_v + 2x_n}\right)$	
4	Pressure angle at pin center	φ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_n \cos\alpha_n\ (\mathrm{inv}\,\varphi_v + \eta_v)$	

NOTE: The units of angles η_v and φ_v are radians.

Table 5.21 Equations for calculating over pins measurement for helical gears in the normal system

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE 1	$d_p = 2$ $\alpha_t = 20.646896°$ $\text{inv}\,\varphi = 0.058890$ $\varphi = 30.8534°$ $M = 24.5696$
2	Involute function φ	$\text{inv}\,\varphi$	$\frac{d_p}{m_n z \cos\alpha_n} - \frac{\pi}{2z} + \text{inv}\,\alpha_t + \frac{2x_n \tan\alpha_n}{z}$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Measurement over pin (ball)	M	Even Teeth $\frac{zm_n \cos\alpha_t}{\cos\beta \cos\varphi} + d_p$	
			Odd Teeth $\frac{zm_n \cos\alpha_t}{\cos\beta \cos\varphi} \cos\frac{90°}{z} + d_p$	

NOTE 1: The ideal pin diameter of Table 5.20, or its approximate value, is entered as the actual diameter of d_p.

Table 5.22 and Table 5.23 present equations for calculating pin measurements for helical gears in the transverse (perpendicular to axis) system.

Table 5.22 Equations for calculating pin diameter for helical gears in the transverse system

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\frac{z}{\cos^3\beta}$	$m_t = 3$ $\alpha_t = 20°$ $z = 36$ $\beta = 33°33'\,26.3''$ $\alpha_n = 16.87300°$ $x_t = +0.2$ $z_v = 62.20800$ $\eta_v = 0.014091$ $\alpha'_v = 18.26390$ $\varphi_v = 0.34411$ $\text{inv}\,\varphi_v = 0.014258$ $d'_p = 4.2190$
2	Spacewidth half angle	η_v	$\frac{\pi}{2z_v} - \text{inv}\,\alpha_n - \frac{2x_t \tan\alpha_t}{z_v}$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\frac{z_v \cos\alpha_n}{z_v + 2\frac{x_t}{\cos\beta}}\right)$	
4	Pressure angle at pin center	φ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_t \cos\beta \cos\alpha_n\ (\text{inv}\,\varphi_v + \eta_v)$	

NOTE: The units of angles η_v and φ_v are radians.

Table 5.23 Equations for calculating over pins measurement for helical gears in the transverse system

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE 1	$d_p = 4.5$ $\text{inv}\,\varphi = 0.027564$ $\varphi = 24.3453°$ $M = 115.892$
2	Involute function φ	$\text{inv}\,\varphi$	$\frac{d_p}{m_t z \cos\beta \cos\alpha_n} - \frac{\pi}{2z} + \text{inv}\,\alpha_t + \frac{2x_t \tan\alpha_t}{z}$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Measurement over pin (ball)	M	Even teeth $\frac{zm_t \cos\alpha_t}{\cos\varphi} + d_p$	
			Odd teeth $\frac{zm_t \cos\alpha_t}{\cos\varphi} \cos\frac{90°}{z} + d_p$	

NOTE: The ideal pin diameter of Table 5.22, or its approximate value is applied as the actual diameter of pin dp here.

(5) Three Wire Method of Worm Measurement

The tooth profile of type III worms which are most popular are cut by standard cutters with a pressure angle α_0 = 20°. This results in the normal pressure angle of the worm being a bit smaller than 20°. The equation below shows how to calculate a type III worm in an AGMA system.

$$\alpha_n = \alpha_0 - \frac{90}{z_1} \frac{r}{r_0 \cos^2 \gamma + r} \sin^3 \gamma \qquad (5.7)$$

Where
r : Worm reference radius
r_0 : Cutter radius
z_1 : Number of threads
γ : Lead angle of worm

The exact equation for a three wire method of type III worm is not only difficult to comprehend, but also hard to calculate precisely. We will introduce two approximate calculation methods here:

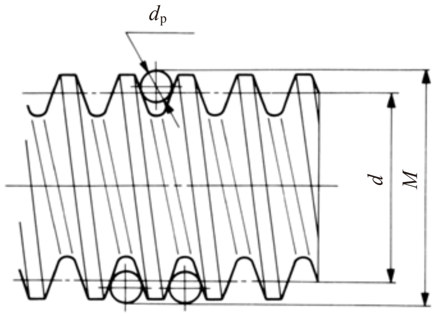


Fig. 5.10 Three wire method of a worm

(a) Regard the tooth profile of the worm as a straight tooth profile of a rack and apply its equations.

Using this system, the three wire method of a worm can be calculated by Table 5.24.

Table 5.24 Equations for three wire method of worm measurement, (a)-1

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\dfrac{\pi m_x}{2\cos\alpha_x}$	m_x = 2, α_n = 20°, z_1 = 1, d_1 = 31, γ = 3.691386°, α_x = 20.03827°, d'_p = 3.3440, d_p = 3.3, M = 35.3173
2	Three wire measurement	M	$d_1 - \dfrac{\pi m_x}{2\tan\alpha_x} + d_p\left(1 + \dfrac{1}{\sin\alpha_x}\right)$	

These equations presume the worm lead angle to be very small and can be neglected. Of course, as the lead angle gets larger, the equations' error gets correspondingly larger. If the lead angle is considered as a factor, the equations are as in Table 5.25.

Table 5.25 Equations for three wire method of worm measurement, (a)-2

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\dfrac{\pi m_n}{2\cos\alpha_n}$	m_x = 2, α_n = 20°, z_1 = 1, d_1 = 31, γ = 3.691386°, m_n = 1.99585, d'_p = 3.3363, d_p = 3.3, M = 35.3344
2	Three wire measurement	M	$d_1 - \dfrac{\pi m_n}{2\tan\alpha_n} + d_p\left(1 + \dfrac{1}{\sin\alpha_n}\right) - \dfrac{(d_p\cos\alpha_n\sin\gamma)^2}{2d_1}$	

(b) Consider a worm to be a helical gear.

This means applying the equations for calculating over pins measurement of helical gears to the case of three wire method of a worm. Because the tooth profile of Type III worm is not an involute curve, the method yields an approximation. However, the accuracy is adequate in practice.

Tables 5.26 and 5.27 contain equations based on the axial system. Tables 5.28 and 5.29 are based on the normal system.

Table 5.26 Equations for calculating pin diameter for worms in the axial system

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\dfrac{z_1}{\cos^3(90° - \gamma)}$	m_x = 2 α_n = 20° z_1 = 1 d_1 = 31 γ = 3.691386° z_v = 3747.1491 η_v = − 0.014485 α'_v = 20° φ_v = 0.349485 inv φ_v = 0.014960 d'_p = 3.3382
2	Spacewidth half angle	η_v	$\dfrac{\pi}{2z_v} - \mathrm{inv}\,\alpha_n$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\dfrac{z_v \cos\alpha_n}{z_v}\right)$	
4	Pressure angle at pin center	φ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_x \cos\gamma \cos\alpha_n (\mathrm{inv}\,\varphi_v + \eta_v)$	

NOTE: The units of angles η_v and φ_v are radians.

Table 5.27 Equations for three wire method for worms in the axial system

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE 1	d_p = 3.3 α_t = 76.96878° inv α_t = 4.257549 inv φ = 4.446297 φ = 80.2959° M = 35.3345
2	Involute function φ	inv φ	$\dfrac{d_p}{m_x z_1 \cos\gamma \cos\alpha_n} - \dfrac{\pi}{2z_1} + \mathrm{inv}\,\alpha_t$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Three wire measurement	M	$\dfrac{z_1 m_x \cos\alpha_t}{\tan\gamma\cos\varphi} + d_p$	

NOTE 1. The value of ideal pin diameter from Table 5.26, or its approximate value, is to be used as the actual pin diameter, d_p.

NOTE 2. $\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$

Tables 5.28 and 5.29 show the calculation of a worm in the normal module system. Basically, the normal module system and the axial module system have the same form of equations. Only the notations of module make them different.

Table 5.28 Equations for calculating pin diameter for worms in the normal system

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\dfrac{z_1}{\cos^3(90° - \gamma)}$	$m_n = 2.5$ $\alpha_n = 20°$ $z_1 = 1$ $d_1 = 37$ $\gamma = 3.874288°$ $z_v = 3241.792$ $\eta_v = -0.014420$ $\alpha'_v = 20°$ $\varphi_v = 0.349550$ $\mathrm{inv}\,\varphi_v = 0.0149687$ $d'_p = 4.1785$
2	Spacewidth half angle	η_v	$\dfrac{\pi}{2z_v} - \mathrm{inv}\,\alpha_n$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\dfrac{z_v \cos\alpha_n}{z_v}\right)$	
4	Pressure angle at pin center	φ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_n \cos\alpha_n (\mathrm{inv}\,\varphi_v + \eta_v)$	

NOTE: The units of angles η_v and φ_v are radians.

Table 5.29 Equations for three wire method for worms in the normal system

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE 1	$d_p = 4.2$ $\alpha_t = 79.48331°$ $\mathrm{inv}\,\alpha_t = 3.999514$ $\mathrm{inv}\,\varphi = 4.216536$ $\varphi = 79.8947°$ $M = 42.6897$
2	Involute function φ	$\mathrm{inv}\,\varphi$	$\dfrac{d_p}{m_n z_1 \cos\alpha_n} - \dfrac{\pi}{2z_1} + \mathrm{inv}\,\alpha_t$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Three wire measurement	M	$\dfrac{z_1 m_n \cos\alpha_t}{\sin\gamma\cos\varphi} + d_p$	

NOTE 1. The value of ideal pin diameter from Table 5.28, or its approximate value, is to be used as the actual pin diameter, d_p.

NOTE 2. $\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$

6 Backlash

For smooth rotation of meshed gears, backlash is necessary. Backlash is the amount by which a tooth space exceeds the thickness of a gear tooth engaged in mesh. Backlashes are classified in the following ways.

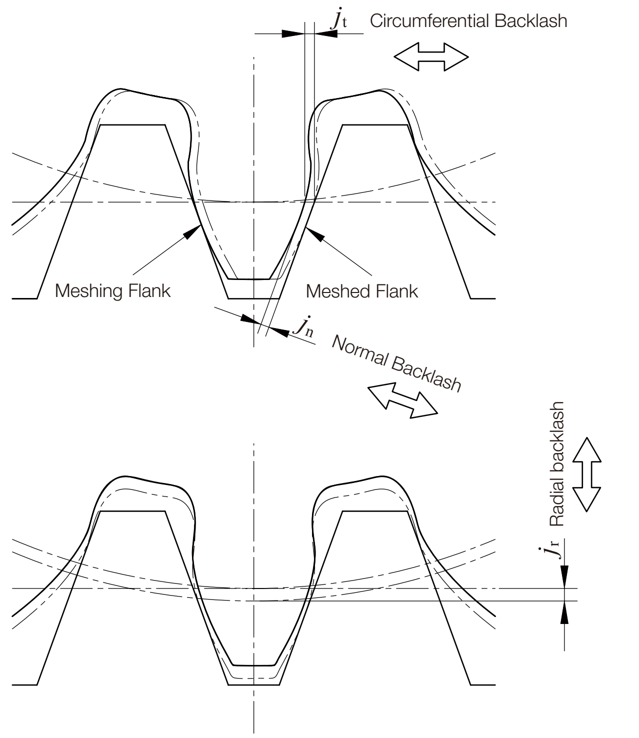


Fig. 6.1 Circumferential Backlash / Normal Backlash and Radial Backlash

6.1 Types of Backlashes

(1) Circumferential Backlash (j_t)

Circumferential Backlash is the length of arc on the pitch circle. The length is the distance the gear is rotated until the meshed tooth flank makes contacts while the other mating gear is held stationary.

(2) Normal Backlash (j_n)

The minimum distance between each meshed tooth flank in a pair of gears, when it is set so the tooth surfaces are in contact.

(3) Angular Backlash (j_θ)

The maximum angle that allows the gear to move when the other mating gear is held stationary.

(4) Radial backlash (j_r)

The radial Backlash is the shrinkage (displacement) in the stated center distance when it is set so the meshed tooth flanks of the paired gears get contact each other.

(5) Axial Backlash (j_x)

The axial backlash is the shrinkage (displacement) in the stated center distance when a pair of bevel gears is set so the meshed tooth flanks of the paired gears contact each other.

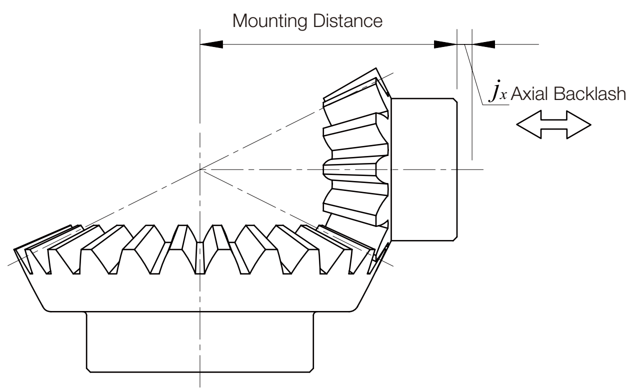


Fig. 6.2 Axial Backlash of a Bevel Gear

6.2 Backlash Relationships

Table 6.1 reveals relationships among backlashes and the fundamental equations. While bevel gears are of cone shaped gears, axial backlash is considered instead of radial backlash.

Table 6.1 Relationships among backlashes

Gear Mesh	Type of Gear Meshes	Circumferential Backlash j_t	Normal Backlash j_n	Angular Backlash j_θ	Radial Backlash j_r	Axial Backlash j_x
Parallel Axes Gears	Spur gear	$\dfrac{j_n}{\cos\alpha_n \cos\beta}$	$j_t \cos\alpha_n \cos\beta$	$\dfrac{360^\circ j_t}{\pi d}$	$\dfrac{j_n}{2\sin\alpha_n}$	
	Helical gear					
Intersecting Axes Gears	Straight bevel gear	$\dfrac{j_n}{\cos\alpha_n \cos\beta_m}$	$j_t \cos\alpha_n \cos\beta_m$			$\dfrac{j_n}{2\sin\alpha_n \sin\delta}$
	Spiral bevel gear					
Nonparallel & Nonintersecting Axis Gears	Screw Gear	$\dfrac{j_n}{\cos\alpha_n \cos\beta}$	$j_t \cos\alpha_n \cos\beta$		$\dfrac{j_n}{2\sin\alpha_n}$	
	Worm	$\dfrac{j_n}{\cos\alpha_n \sin\gamma}$	$j_t \cos\alpha_n \sin\gamma$			
	Wormwheel	$\dfrac{j_n}{\cos\alpha_n \cos\gamma}$	$j_t \cos\alpha_n \cos\gamma$			

(1) Backlash of Parallel Axes Gear Mesh

Table 6.2 shows calculation examples for backlashes and the center distance of spur and helical gear meshes. By adjusting the center distance (radial backlash), backlash can be controlled.

Table 6.2 Spur and Helical Gear Mesh

No.	Specifications	Symbol	Formula	Spur gear		Helical gear (Normal)	
1	Transverse module	m_t	Set value	2		2	
2	Normal pressure angle	α_n		20°		18°43'	
3	Transverse pressure angle	α_t		20°		20°	
4	No. of teeth	z		20	40	20	40
5	Spiral angle	β		0		21°30'	
7	Normalbacklash	j_n		0.150		0.150	
6	Reference diameter	d	zm_t	40	80	40	80
8	Circumferential backlash	j_t	$\dfrac{j_n}{\cos\alpha_n \cos\beta}$	0.160		0.170	
9	Angular backlash (°)	j_θ	$\dfrac{360° j_t}{\pi d}$	0.457°	0.229°	0.488°	0.244°
10	Radial backlash	j_r	$\dfrac{j_n}{2\sin\alpha_n}$	0.219		0.234	

(2) Backlash of Intersecting Axes Gear Mesh

Table 6.3 shows calculation examples for backlashes and the mounting distance of bevel gear meshes. The common way to control backlash of bevel gear meshes is to adjust the mounting distance (axial backlash) by adding shims. When adjusting the mounting distance, it is important to keep proper tooth contact in consideration of the gears and pinions in balance.

Table 6.3 Bevel Gear Mesh

No.	Specifications	Symbol	Formula	Straight bevel gear		Spiral bevel gear	
				Pinion	Gear	Pinion	Gear
1	Shaft angle	Σ	Set value	90°		90°	
2	Module	m		2		2	
3	Normal pressure angle	α_n		20°		20°	
4	No. of teeth	z		20	20	20	40
5	Mean spiral angle	β_m		0		35°	
6	Normal backlash	j_n		0.150		0.150	
7	Reference diameter	d	zm	40	40	40	80
8	Pitch angle	$\delta_1 \cdot \delta_2$	$\tan^{-1}\left(\dfrac{z_1}{z_2}\right)$ $\quad \Sigma - \delta_1$	45°	45°	26°34'	63°26'
9	Circumferential backlash	j_t	$\dfrac{j_n}{\cos\alpha_n \cos\beta_m}$	0.160		0.195	
10	Angular backlash (°)	j_θ	$\dfrac{360° j_t}{\pi d}$	0.457°	0.457°	0.558°	0.279°
11	Axial backlash	j_x	$\dfrac{j_n}{2\sin\alpha_n \sin\delta}$	0.310	0.310	0.490	0.245

(3) Backlash of Nonparallel and Nonintersecting Axes Mesh

Table 6.4 shows calculation examples for backlashes and the mounting distance of worm gear meshes.

A Worm gear pair has a different circumferential backlash for each drive and driven gear (worm and wheel) and it is a feature of a worm gear pair.

Table 6.4 Worm Gear Pair Meshes

No.	Specifications	Symbol	Formula	Worm gear pair	
				Worm	Wheel
1	Shaft angle	Σ	Set value	90°	
2	Axial / Transverse module	$m_x \cdot m_t$		2	
3	Normal pressure angle	α_n		20°	
4	No. of teeth	z		1	20
6	Reference diameter (Worm)	d_1		31	—
5	Normal backlash	j_n		0.150	
7	Reference diameter (Wheel)	d_2	$z_2 m_t$	—	40
8	Lead angle	γ	$\tan^{-1}\left(\dfrac{m_x z_1}{d_1}\right)$	3°41'	
9	Circumferential backlash	j_{t1}	$\dfrac{j_n}{\cos\alpha_n \sin\gamma}$	2.480	—
		j_{t2}	$\dfrac{j_n}{\cos\alpha_n \cos\gamma}$	—	0.160
10	Angular backlash (°)	j_θ	$\dfrac{360° j_t}{\pi d}$	9.165°	0.458°
11	Radial backlash	j_r	$\dfrac{j_n}{2\sin\alpha_n}$	0.219	

Table 6.5 Calculation examples for backlash screw gear meshes.

Table 6.5 Screw Gear Mesh

No.	Specifications	Symbol	Formula	Screw gear	
				Pinion	Gear
1	Shaft angle	Σ	Set value	90°	
2	Normal module	m_n		2	
3	Normal pressure angle	α_n		20°	
4	No. of teeth	z		10	20
5	Spiral angle	β		45°	45°
7	Normal backlash	j_n		0.150	
6	Reference diameter	d	$\dfrac{zm_n}{\cos\beta}$	28.284	56.569
8	Circumferential backlash	j_t	$\dfrac{j_n}{\cos\alpha_n \cos\beta}$	0.226	0.226
9	Angular backlash (°)	j_θ	$\dfrac{360° j_t}{\pi d}$	0.915°	0.457°
10	Radial backlash	j_r	$\dfrac{j_n}{2\sin\alpha_n}$	0.219	

6.3 Tooth Thickness and Backlash

There are two ways to produce backlash. One is to enlarge the center distance. The other is to reduce the tooth thickness. The latter is much more popular than the former. We are going to discuss more about the way of reducing the tooth thickness.

In SECTION 5, we have discussed the standard tooth thickness s_1 and s_2. In the meshing of a pair of gears, if the tooth thickness of pinion and gear were reduced by Δs_1 and Δs_2, they would produce a backlash of Δs_1 and Δs_2 in the direction of the pitch circle. Let the magnitude of Δs_1 and Δs_2 be 0.1., We know that α= 20°, then:

$$j_t = \Delta s_1 + \Delta s_2 = 0.1 + 0.1 = 0.2$$

We can convert it into the backlash on normal direction j_n:

$$j_n = j_t \cos\alpha = 0.2 \times \cos 20° = 0.1879$$

Let the backlash on the center distance direction be j_r, then:

$$j_r = \frac{j_t}{2\tan\alpha} = \frac{0.2}{2 \times \tan 20°} = 0.2747$$

These express the relationship among several kinds of backlashes. In application, one should consult the JIS standard. There are two JIS standards for backlash – one is JIS B 1703-76（Suspended standard) for spur gears and helical gears, and the other is JIS B 1705-73 for bevel gears. All these standards regulate the standard backlashes in the direction of the pitch circle j_t or j_{tt}. These standards can be applied directly, but the backlash beyond the standards may also be used for special purposes. When writing tooth thicknesses on a drawing, it is necessary to specify, in addition, the tolerances on the thicknesses as well as the backlash.

For example:

Tooth thickness	$3.141^{-0.050}_{-0.100}$
Backlash	0.100 ～ 0.200

Since the tooth thickness directly relates to backlash, the tolerances on the thickness will become a very important factor.

6.4 Gear Train and Backlash

The discussions so far involved a single pair of gears. Now, we are going to discuss two stage gear trains and their backlash. In a two stage gear train, as Figure 6.3 shows, j_{t1} and j_{t4} represent the backlashes of first stage gear train and second stage gear train respectively.

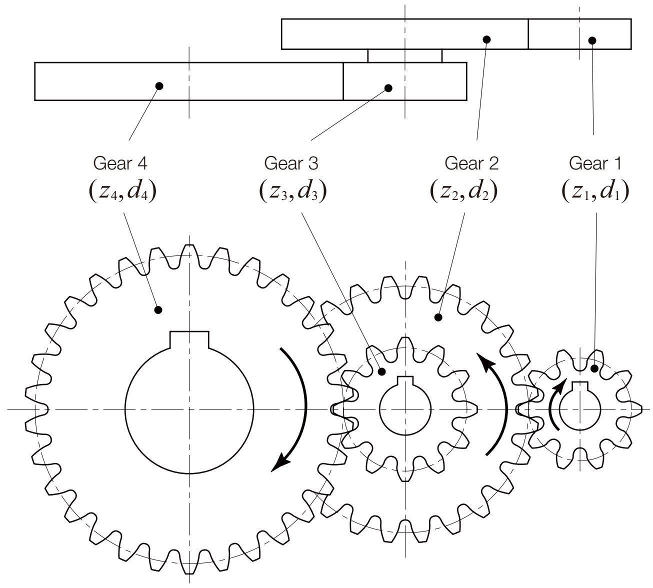


Fig.6.3 Overall accumulated backlash of two stage gear train

If number one gear were fixed, then the accumulated backlash on number four gear j_{tT4} would be as follows:

$$j_{tT4} = j_{t1}\frac{d_3}{d_2} + j_{t4} \qquad (6.1)$$

This accumulated backlash can be converted into rotation in degrees:

$$j_\theta = j_{tT4}\frac{360}{\pi d_4}\ \text{(degrees)} \qquad (6.2)$$

The reverse case is to fix number four gear and to examine the accumulated backlash on number one gear j_{tT1}.

$$j_{tT1} = j_{t4}\frac{d_2}{d_3} + j_{t1} \qquad (6.3)$$

This accumulated backlash can be converted into rotation in degrees:

$$j_\theta = j_{tT1}\frac{360}{\pi d_1}\ \text{(degrees)} \qquad (6.4)$$

6.5 Method of Reducing Backlash (Zero Backlash Gears)

Low backlash or zero-backlash is the performance required for high-precision gear applications. In order to meet special needs, precision gears are used more frequently than ever before. This section introduces methods of reducing or eliminating backlash.

(1) Use of Gears with less tooth thinning (Common Method)

By processing gears which have less amount of tooth thinning than common gears, and by using them with the center distance or mounting distance fixed at normal values, it enables to reduces backlash. This method cannot be used to make the backlash zero, but it is the most simple way and applicable to many types of gears. If you use the gear with low runout, you can reduce the backlash variation. Zero-backlash is concerned. It should be considered carefully that the gear may not rotated smoothly if the generated backlash value is zero.

(2) Use of gears adjustable for small backlash

A method to use gears to adjust for low backlash. Zero-backlash can not be generated with this method.

(a) Control backlash by adjustment of the center distance

This method can be applied to spur, helical, screw and worm gears. By shortening the center distance of the gear, this enables adjustment of the radial plays and reduce the backlash. The adjustment of the center distance is complicated.

(b) Control backlash by adjustment of mounting distance

For bevel gears, shortening of the mounting distance of the gear, enables to control axial plays and reduce the backlash. The adjustment of the center distance is rather complicated, if the mounting distance of only one of the paired bevel gears is adjusted, this creates bad tooth contact. The mounting distance of each meshed gear should be adjusted with equally., this method is generally made by adjusting shims.

(c) Control backlash by separating the gear into two parts

This method is applicable for most types of gears. By separating a gear in two parts, and by adjusting and fixing the phase relationships between the tooth position of each, generates low backlash. This is shown in Fig. 6.4.

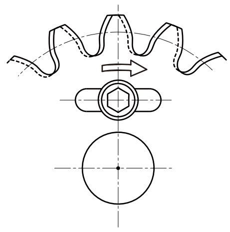


Fig. 6.4 Gear separated in two parts. (Fixed)

For helical gears or worm gears, there is a way to adjust the phase relationships between the tooth position of each meshed gear by moving one of the paired gears (1) in an axial direction. Fig. 6.5 shows the basis.

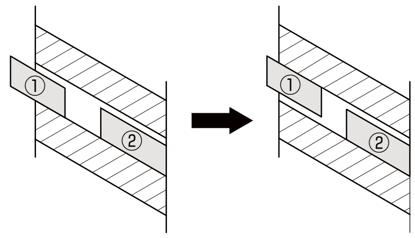


Fig. 6.5 Backlash Adjustment of Helical Gear

(d) Tapered gears (Spur gear and tapered racks)

Tapered gears are also called conical gears. Since tapered gears are a cone shaped gear having continuously-shifted teeth, the tooth profile/tooth thickness are continuously transformed. Fig. 6.6 shows the tooth profile of a tapered spur gear. Since the tooth thickness of the meshed tooth varies if the taper gear is moved in axial direction, this enables you to adjust backlash. The shim adjustment is a simple and easy way to move the tapered gear in axial direction.

Unlike bevel gears, moving the tapered gear in axial direction involves no change in tooth contact and this is an advantage of tapered gears.

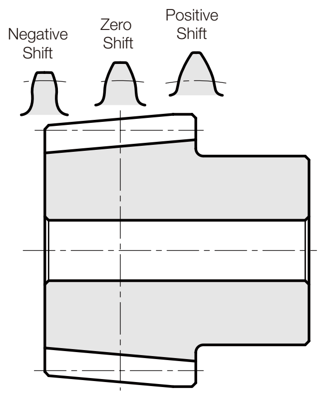


Fig. 6.6 Tooth Profile of Tapered Spur Gear

(e) Duplex Lead Worm Gear Pair

A Duplex lead worm gear differs in module between the right and left tooth surface. While the pitch of the right and left tooth surface also differs, the tooth thickness varies continuously. By shifting the worm axially, the tooth thickness at the working point varies, and can be used to adjust the backlash of the duplex lead worm gears. There are some methods to adjust the worm in the axial direction. The simple and secure way is shim adjustment, in the same way as any other type of gears. Zero-backlash is not favorable, as the worm gear mesh requires a certain amount of backlash to avoid the depletion of lubricant on the tooth surface.

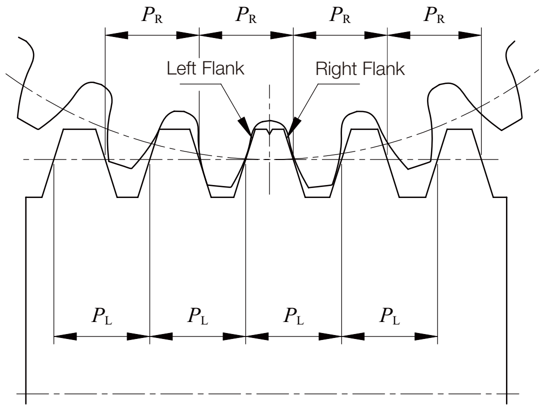


Fig. 6.7 Basic Concept of the Duplex Lead Worm Gear

Figure 6.7 presents the basic concept of a duplex lead worm gear pair. (For more detail, please see page 418)

(3) Gears which have Zero-Backlash

This type of gear has a structure that can forcibly remove backlash by external force. While this structure involves double flank meshing, it should be carefully maintained to avoid the depletion of lubricant. This structure is not suitable for gears, which have a large amount of slippage on the tooth surface when transmitting power, such as worm gear or screw gears.
If the depletion of the lubricant occurs on the tooth surface causing large slippage, there is danger of abrasion.

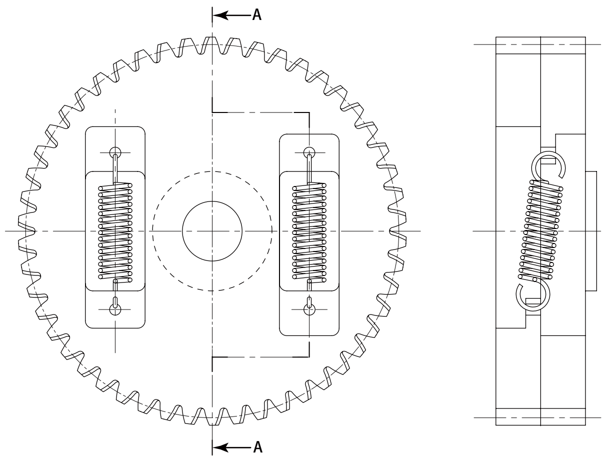


Fig. 6.8 Scissors Gear (with Coil Springs)

Scissors Gear with Zero Circumferential Backlash

By applying spring force to the tightly held teeth of the mating gear, with the gear separated in two parts, the backlash is removed. Figure 6.8 shows the structure.

7 Gear Accuracy

Gears are one of the basic elements used to transmit power and position. As designers, we desire them to meet various demands:

① Maximum power capability
② Minimum size.
③ Minimum noise (silent operation).
④ Accurate rotation/position

To meet various levels of these demands requires appropriate degrees of gear accuracy. This involves several gear features.

7.1 Accuracy of Spur and Helical Gears

Gear accuracy of spur and helical gears, is described in accordance with the following JIS standards.

JIS B 1702−1 : 1998 Cylindrical gears - ISO system of accuracy - Part 1:
Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth. (This specification describes 13 grades of gear accuracy grouped from 0 through 12, - 0, the highest grade and 12, the lowest grade).

JIS B 1702−2 : 1998 Cylindrical gears - ISO system of accuracy - Part 2: Definitions and allowable values of deviations relevant to radial composite deviations and runout information. (This specification consists of 9 grades of gear accuracy grouped from 4 through 12, - 4, the highest grade and 12, the lowest grade).

These new standards for gear accuracy differ from the former standards of JIS B 1702-1976 in various ways. For example, the gear accuracy used to be classified into nine grades (0 to 8) in the former standards. To distinguish new standards from old ones, each of the grades under the new standards has the prefix "N".

(1) Single Pitch Deviation (f_{pt})

The deviation between actual measured pitch value between any adjacent tooth surface and theoretical circular pitch.

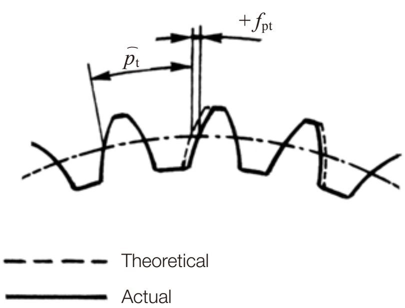


Fig.7.1 Single pitch deviation f_{pt}

(2) Total Cumulative Pitch Deviation (F_p)

Difference between theoretical summation over any number of teeth interval, and summation of actual pitch measurement over the same interval.

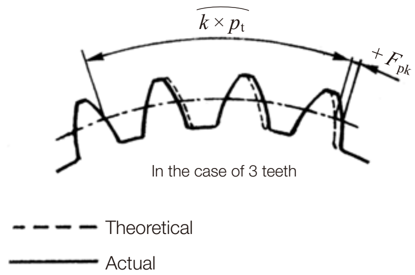


Fig.7.2 Total cumulative pitch deviation

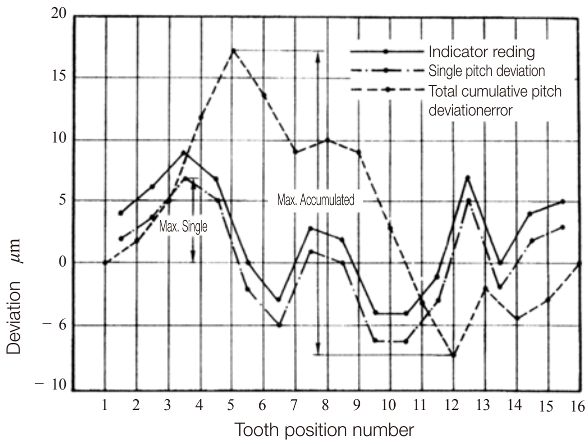


Fig.7.3 Examples of pitch deviation for a 15 tooth gear

(3) Total Profile Deviation (F_α)

Total profile deviation represents the distance (F_α) shown in Figure 7.4. Actual profile chart is lying in between upper design chart and lower design chart.

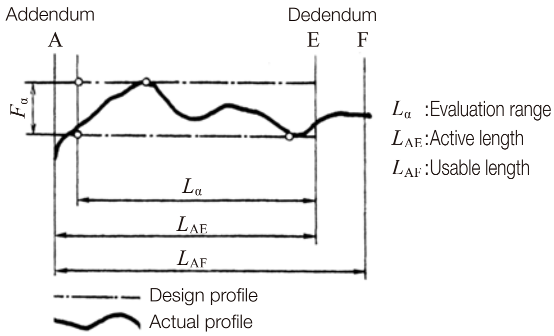


Fig.7.4 Total profile deviation F_α

(4) Total Helix Deviation (F_β)

Total helix deviation represents the distance (F_β) shown in Figure 7.5. The actual helix chart is lying in between upper helix chart and lower helix chart. Total helix deviation results in poor tooth contact, particularly concentrating contact to the tip area. Modifications, such as tooth crowning and end relief can alleviate this deviation to some degree.

Shown in Figure 7.6 is an example of a chart measuring total profile deviation and total helix deviation using a Zeiss UMC 550 tester.

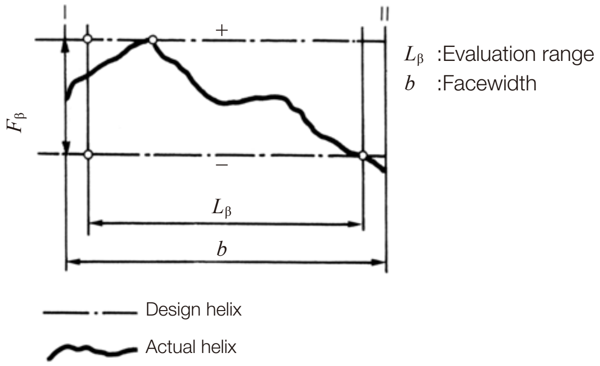


Fig.7.5 Total helix deviation F_β

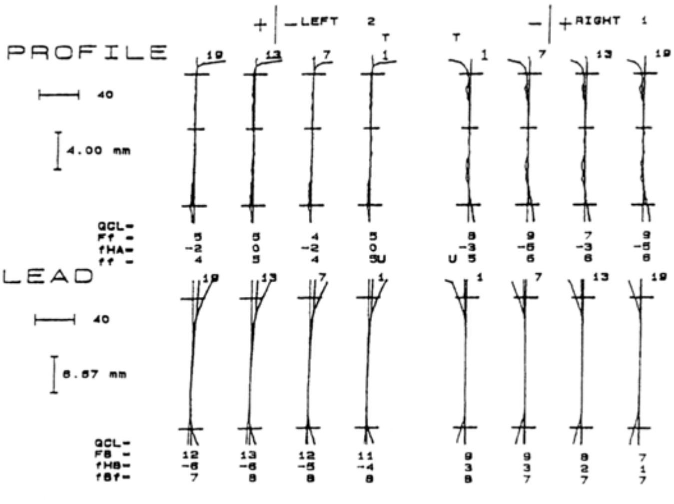


Fig.7.6 An example of a chart measuring total profile deviation and total helix deviation

(5) Total Radial Composite Deviation (F_i'')

Total radial composite deviation represents variation in center distance when product gear is rotated one revolution in tight mesh with a master gear.

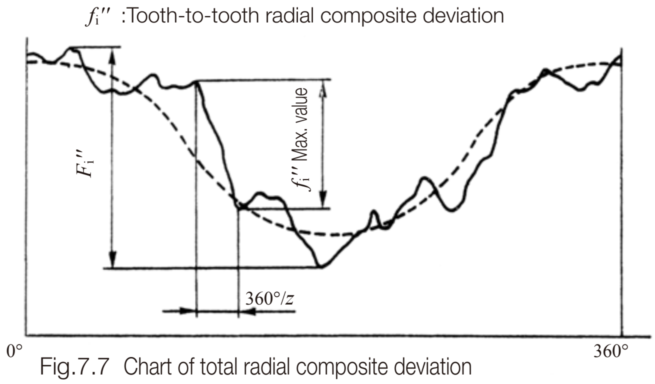


Fig.7.7 Chart of total radial composite deviation

(6) Runout Error of Gear Teeth (F_r)

Most often runout error is measured by indicating the position of a pin or ball inserted in each tooth space around the gear and taking the largest difference.

Runout causes a number of problems, one of which is noise. The source of this error is most often insufficient accuracy and ruggedness of the cutting arbor and tooling system. And, therefore, it is very important to pay attention to these cutting arbor and tooling system to reduce runout error. Shown in Fig. 7.8 is the chart of runout. The values of runout includes eccentricity.

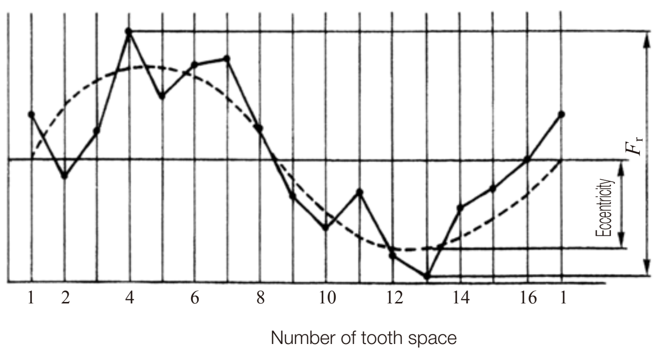


Fig. 7.8 Runout error of a 16-tooth gear

Please see page 620 to 625 in referring to the selected standard values for each allowable error.

7.2 Accuracy of Bevel Gears

JIS B 1704 : 1978 regulates the specification of a bevel gear's accuracy. It also groups bevel gears into 9 grades, from 0 to 8.
There are 4 types of allowable errors:

(1) Single pitch error.
(2) Pitch variation error
(3) Accumulative pitch error.
(4) Runout error of teeth (pitch circle).

These are similar to the spur gear errors.

① Single pitch error

The deviation between actual measured pitch value between any adjacent teeth and the theoretical circular pitch at the mean cone distance.

② Pitch variation error

Absolute pitch variation between any two adjacent teeth at the mean cone distance.

③ Accumulative pitch error

Difference between theoretical pitch sum of any teeth interval, and the summation of actual measured pitches for the same teeth interval at the mean cone distance.

④ Runout error of teeth

This is the maximum amount of tooth runout in the radial direction, measured by indicating a pin or ball placed between two teeth at the central cone distance.

Table 7.1 presents equations for allowable values of these various errors.

Table 7.1 Equations for allowable single pitch error, Accumulative pitch error and pitch cone runout error, (μm)

Grade	Single pitch error	Accumulative pitch error	Runout error of pitch cone
JIS 0	$0.4W + 2.65$	$1.6W + 10.6$	$2.36\sqrt{d}$
1	$0.63W + 5.0$	$2.5W + 20.0$	$3.6\sqrt{d}$
2	$1.0W + 9.5$	$4.0W + 38.0$	$5.3\sqrt{d}$
3	$1.6W + 18.0$	$6.4W + 72.0$	$8.0\sqrt{d}$
4	$2.5W + 33.5$	$10.0W + 134.0$	$12.0\sqrt{d}$
5	$4.0W + 63.0$	–	$18.0\sqrt{d}$
6	$6.3W + 118.0$	–	$27.0\sqrt{d}$
7	–	–	$60.0\sqrt{d}$
8	–	–	$130.0\sqrt{d}$

where W : Tolerance unit

$$W = \sqrt[3]{d} + 0.65m \quad (\mu m)$$

d : Reference Diameter (mm)

The allowable pitch variation error value is defined as;
Single pitch error tolerance x k-value
Table 7.2 shows the k-value. The k-value varies depending on the tolerance value of a single pitch error.

Table 7.2 k-values

Single pitch error (μm)	Pitch variation error k
Below 70	1.3
Over 70 Below 100	1.4
Over 100 Below 150	1.5
Over 150	1.6

Besides the above errors, there are seven specifications for bevel gear blank dimensions and angles, plus an eighth that concerns the cut gear set:

① The tolerance of the blank tip diameter and the crown to back surface distance.
② The tolerance of the outer cone angle of the gear blank.
③ The tolerance of the cone surface runout of the gear blank.
④ The tolerance of the side surface runout of the gear blank.
⑤ The feeler gauze size to check the flatness of blank back surface.
⑥ The tolerance of the shaft runout of the gear blank.
⑦ The tolerance of the shaft bore dimension deviation of the gear blank.
⑧ The tooth contact.

Item ⑧ relates to cutting of the two mating gears' teeth. The tooth contact must be full and even across the profiles. This is an important criterion that supersedes all other blank requirements.

Please see page 722 to 723 in referring to selected date of each allowable error.

8 Mounting Accuracy

Even if the gear has high accuracy, if the gear is not mounted properly it is not possible to avoid problems regarding bad tooth contact noise, wear, and breakage.

8.1 Accuracy of Center Distance

Error in the center distance influences the backlash of the gear mesh. If the center distance value increases, the backlash value is increased. As the result, gear teeth can not mesh deeply enough each other, and the contact ratio decreases. If the center distance value decreases, the backlash value also decreases. Gears may not rotate if the backlash decreases too much.

Table 8.1 shows the center distance tolerance of spur and helical gears. The tolerance values in this table are quoted from JGMA1101-01(2000), and are applicable for involute spur and helical gears, made of iron and steel.

Table 8.1 Center Distance Tolerance of Spur and Helical Gears $\pm f_a$

Unit: μm

Center Distance (mm)		Accuracy Grade of Gears			
More than	Less than	N3,N4	N5,N6	N7,N8	N9,N10
5	20	6	10	16	26
20	50	8	12	20	31
50	125	12	20	32	50
125	280	16	26	40	65
280	560	22	35	55	88

8.2 Axial Parallelism

The accuracy of two parallel axis is composed with parallelism error and shaft offset error. These errors influence the tooth contact in the tooth trace direction. It may result in bad tooth contact occurring at the tip of tooth width. Increase of the error involves decreasing the backlash or causing of noise by tooth breakage.
Table 8.2 and 8.3 shows Shaft parallelism error and offset error tolerance of spur and helical gears, where data was selected from JGMA1102-01(2000).

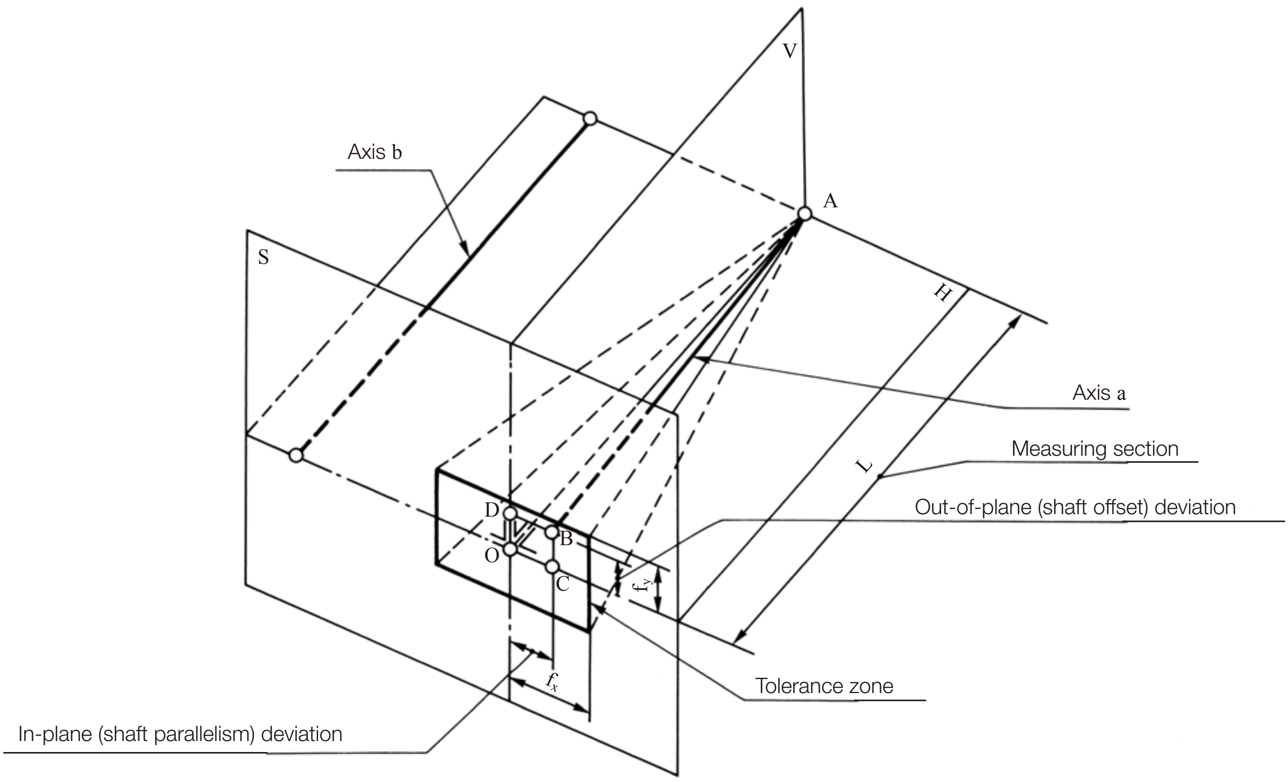


Fig. 8.1 Shaft Parallelism Error and Shaft Offset Error

Table 8.2　Allowable in-plane deviation with respect to parallelism of axes per facewidth　f_x

Unit: μm

Reference diameter d (mm)	Facewidth b (mm)	Accuracy grades					
		N5	N6	N7	N8	N9	N10
5 d 20	$4 \leqq b \leqq 10$	6.0	8.5	12	17	24	35
	$10 < b \leqq 20$	7.0	9.5	14	19	28	39
20 < d 50	$4 \leqq b \leqq 10$	6.5	9.0	13	18	25	36
	$10 < b \leqq 20$	7.0	10	14	20	29	40
	$20 < b \leqq 40$	8.0	11	16	23	32	46
	$40 < b \leqq 80$	9.5	13	19	27	38	54
50 < d 125	$4 \leqq b \leqq 10$	6.5	9.5	13	19	27	38
	$10 < b \leqq 20$	7.5	11	15	21	30	42
	$20 < b \leqq 40$	8.5	12	17	24	34	48
	$40 < b \leqq 80$	10	14	20	28	39	56
125 < d 280	$4 \leqq b \leqq 10$	7.0	10	14	20	29	40
	$10 < b \leqq 20$	8.0	11	16	22	32	45
	$20 < b \leqq 40$	9.0	13	18	25	36	50
	$40 < b \leqq 80$	10	15	21	29	41	58
	$80 < b \leqq 160$	12	17	25	35	49	69
280 < d 560	$10 < b \leqq 20$	8.5	12	17	24	34	48
	$20 < b \leqq 40$	9.5	13	19	27	38	54
	$40 < b \leqq 80$	11	15	22	31	44	62

Table 8.3　Allowable out-of-plane deviation with respect to parallelism of axes per facewidth　f_y

Unit: μm

Reference diameter d (mm)	Facewidth b (mm)	Accuracy grades					
		N5	N6	N7	N8	N9	N10
$5 \leqq d \leqq 20$	$4 \leqq b \leqq 10$	3.1	4.3	6.0	8.5	12	17
	$10 < b \leqq 20$	3.4	4.9	7.0	9.5	14	19
$20 < d \leqq 50$	$4 \leqq b \leqq 10$	3.2	4.5	6.5	9.0	13	18
	$10 < b \leqq 20$	3.6	5.0	7.0	10	14	20
	$20 < b \leqq 40$	4.1	5.5	8.0	11	16	23
	$40 < b \leqq 80$	4.8	6.5	9.5	13	19	27
$50 < d \leqq 125$	$4 \leqq b \leqq 10$	3.3	4.7	6.5	9.5	13	19
	$10 < b \leqq 20$	3.7	5.5	7.5	11	15	21
	$20 < b \leqq 40$	4.2	6.0	8.5	12	17	24
	$40 < b \leqq 80$	4.9	7.0	10	14	20	28
$125 < d \leqq 280$	$4 \leqq b \leqq 10$	3.5	5.0	7.0	10	14	20
	$10 < b \leqq 20$	4.0	5.5	8.0	11	16	22
	$20 < b \leqq 40$	4.5	6.5	9.0	13	18	25
	$40 < b \leqq 80$	5.0	7.5	10	15	21	29
	$80 < b \leqq 160$	6.0	8.5	12	17	25	35
$280 < d \leqq 560$	$10 < b \leqq 20$	4.3	6.0	8.5	12	17	24
	$20 < b \leqq 40$	4.8	6.5	9.5	13	19	27
	$40 < b \leqq 80$	5.5	7.5	11	15	22	31

8.3 Features of Tooth Contact

Tooth contact is critical to noise, vibration, efficiency, strength, wear and life. To obtain good contact, the designer must give proper consideration to the following features:

- Modifying the tooth shape
  Improve tooth contact by crowning or end relief.
- Using higher precision gear
  Specify higher accuracy by design. Also, specify that the manufacturing process is to include grinding or lapping.
- Controlling the accuracy of the gear assembly
  Specify adequate shaft parallelism and perpendicularity of the gear housing (box or structure).

The features above are all related to the production of gears/ gearboxes, or to the accuracy of modification. In spite of efforts of prevention, tooth contact problems still may occur at the final inspection before mounting, in some cases. If this happens, tooth contact of spur and helical gears can reasonably be controlled by shifting the gear in axial direction.
Proper tooth contact is one of the elements in providing gear accuracy and very important for bevel and worm gear pairs. Compared to spur or helical gears, it is more difficult to inspect gear accuracy of bevel gears and worm gear pairs. Consequently, final inspection of bevel and worm mesh tooth contact in assembly provides a quality criteria for control.
JGMA1002-01（2003）classifies tooth contact into three levels, A, B, C, as presented in Table 8.4.

Table 8.4 Levels of tooth contact

Level	Types of gear	Levels of tooth contact: Tooth width direction	Levels of tooth contact: Tooth height direction
A	Cylindrical gears	More than 70%	More than 40%
	Bevel gears	More than 50%	
	Worm wheels		
B	Cylindrical gears	More than 50%	More than 30%
	Bevel gears	More than 35%	
	Worm wheels		
C	Cylindrical gears	More than 35%	More than 20%
	Bevel gears	More than 25%	
	Worm wheels	More than 20%	

The percentage in Table 8.4 considers only the effective width and height of teeth.

8.3.1 Tooth Contact of a Bevel Gear

It is important to check the tooth contact of a bevel gear both during manufacturing and again in final assembly. The method is to apply a colored dye and observe the contact area after running. Usually some load is applied, either the actual or applied braking, to realize a realistic contact condition. Ideal contact favors the toe end under no or light load, as shown in Figure 8.2; and, as load is increased to full load, contact shifts to the central part of the tooth width.

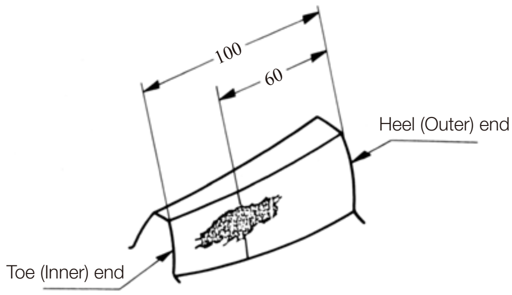


Fig. 8.2 Central toe contact

Even when a gear is ideally manufactured, it may reveal poor tooth contact due to lack of precision in housing or improper mounting position, or both. Usual major faults are:

① Shafts are not intersecting, but are skew (Offset error)
② Shaft angle error of gearbox.
③ Mounting distance error.

Errors ① and ② can be corrected only by reprocessing the housing/mounting. Error ③ can be corrected by adjusting the gears in an axial direction. All three errors may be the cause of improper backlash.

(1) The Offset Error of Shaft Alignment

If a gearbox has an offset error, then it will produce crossed contact, as shown in Figure 8.3. This error often appears as if error is in the gear tooth orientation.

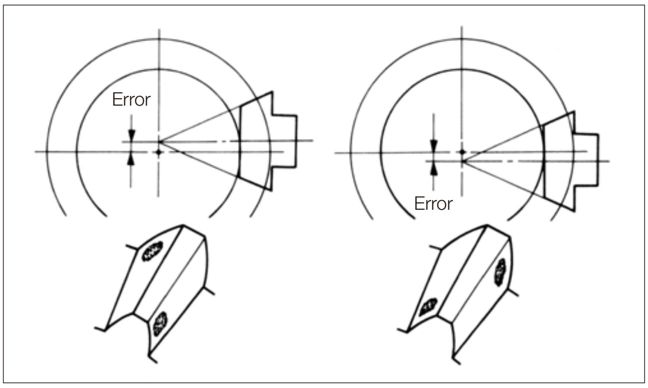


Fig. 8.3 Poor tooth contact due to offset error of shafts

(2) The Shaft Angle Error of Gear Box

As Figure 8.4 shows, the tooth contact will move toward the toe end if the shaft angle error is positive; the tooth contact will move toward the heel end if the shaft angle error is negative.

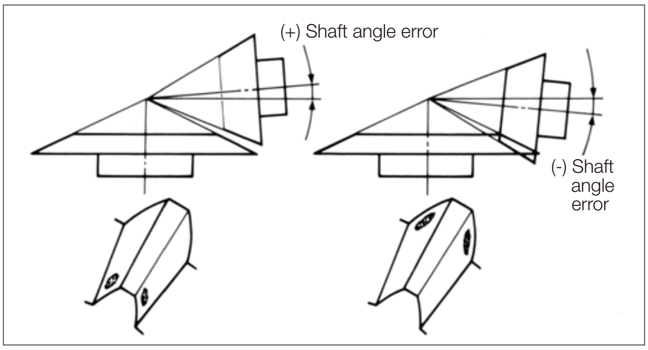


Fig. 8.4 Poor tooth contact due to shaft angle error

(3) Mounting Distance Error

When the mounting distance of the pinion is a positive error, the contact of the pinion will move towards the tooth root, while the contact of the mating gear will move toward the top of the tooth. This is the same situation as if the pressure angle of the pinion is smaller than that of the gear. On the other hand, if the mounting distance of the pinion has a negative error, the contact of the pinion will move toward the top and that of the gear will move toward the root. This is similar to the pressure angle of the pinion being larger than that of the gear. These errors may be diminished by axial adjustment with a backing shim.

The various contact patterns due to mounting distance errors are shown in Figure 8.5.

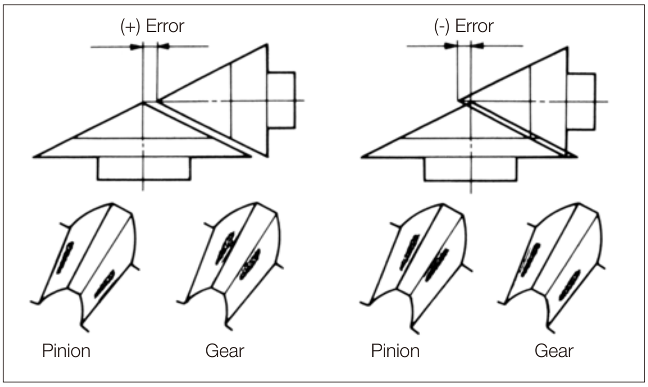


Fig .8.5 Poor tooth contact due to error in mounting distance

Mounting distance error will cause a change of backlash; positive error will increase backlash; and negative, decrease. Since the mounting distance error of the pinion affects the tooth contact greatly, it is customary to adjust the gear rather than the pinion in its axial direction.

8.3.2 Tooth Contact of a Worm Gear Pair

There is no specific Japanese standard concerning worm gearing, except for some specifications regarding tooth contact in JGMA1002-01 (2003).
Therefore, it is the general practice to test the tooth contact and backlash with a tester. Figure 8.6 shows the ideal contact for a worm mesh.

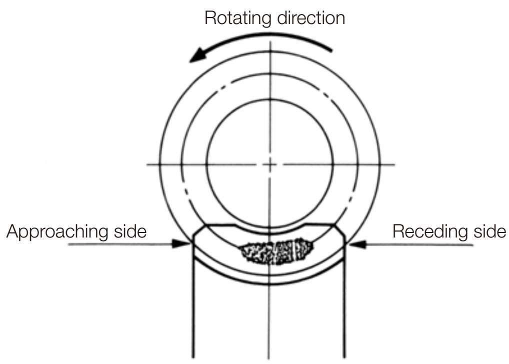


Fig. 8.6 Ideal tooth contact of worm gear pair

From Figure 8.6, we realize that the ideal portion of contact inclines to the receding side.

Because the clearance in the approaching side is larger than on the receding side, the oil film is established much easier in the approaching side. However, an excellent worm wheel in conjunction with a defective gearbox will decrease the level of tooth contact and the performance. There are three major factors, besides the gear itself, which may influence the tooth contact:

① Shaft Angle Error.
② Center Distance Error.
③ Locating Distance Error of Worm Wheel.

Errors ① and ② can only be corrected by remaking the housing. Error ③ may be decreased by adjusting the worm wheel along the axial direction. These three errors introduce varying degrees of backlash.

(1) Shaft Angle Error

If the gear box has a shaft angle error, then it will produce crossed contact as shown in Figure 8.7.
A helix angle error will also produce a similar crossed contact.

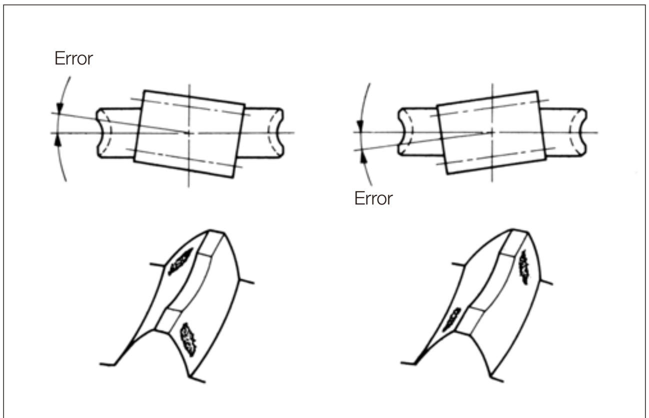


Fig. 8.7 Poor tooth contact due to shaft angle error

(2) Center Distance Error

Even when exaggerated center distance errors exist, as shown in Figure 8.8, the results are crossed contact. Such errors not only cause bad contact but also greatly influence backlash.
A positive center distance error causes increased backlash. A negative error will decrease backlash and may result in a tight mesh, or even make it impossible to assemble.

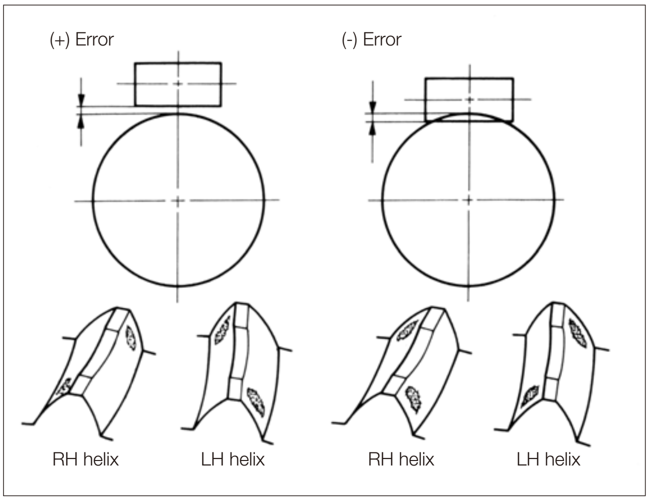


Fig. 8.8 Poor tooth contact due to center distance error

(3) Locating Distance Error

Figure 8.9 shows the resulting poor contact from locating distance error of the worm wheel. From the figure, we can see the contact shifts toward the worm wheel tooth's edge. The direction of shift in the contact area matches the direction of worm wheel locating error. This error affects backlash, which tends to decrease as the error increases. The error can be diminished by micro-adjustment of the worm wheel in the axial direction.

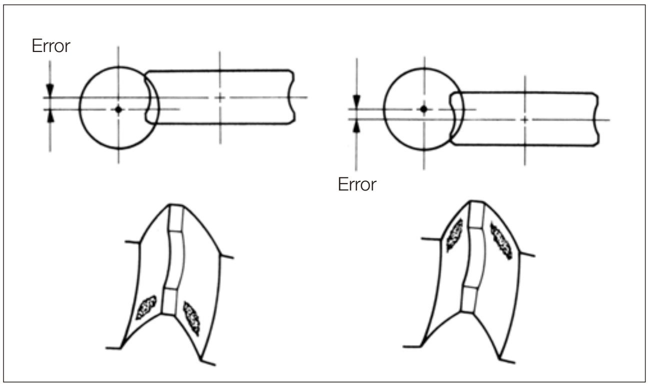


Fig.8.9 Poor tooth contact due to mounting distance error

9 Gear Materials

In accordance to their usage, gears are made of various types of materials, such as iron-based materials, nonferrous metals, or plastic materials. The strength of gears differs depending on the type of material, heat treatment or quenching applied.

9.1 Types of Gear Materials

Table 9.1 lists mechanical properties and characteristics of gear materials most commonly used.

Table 9.1 Types of Gear Materials

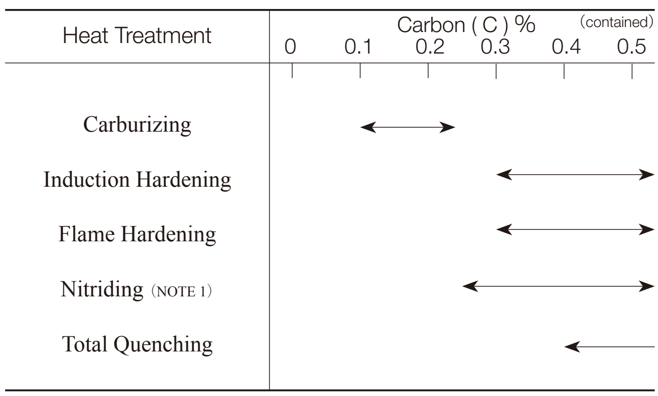
Material	JIS Material No.	Tensile Strength N/mm^2	Elongation (%) More than	Drawability (%) More than	Hardness HB	Characteristics, heat treatments applied
Carbon Steel for Structural Machine Usage	S15CK	More than 490	20	50	143 - 235	Low-carbon steel. High hardness obtained by Carburizing.
	S45C	More than 690	17	45	201 - 269	Most commonly used medium-carbon steel. Thermal refined / induction hardened
Alloy steel for Machine Structural Use	SCM435	More than 930	15	50	269 - 331	Medium-carbon alloy steel (C content: 0.3 – 0.7%). Thermal refined and induction hardened. High strength (High bending strength / High surface durability). Used in gear manufacturing, except for worm wheels.
	SCM440	More than 980	12	45	285 - 352	
	SNCM439	More than 980	16	45	293 - 352	
	SCr415	More than 780	15	40	217 - 302	Low-carbon Alloy Steel (C content below 0.3%). Surface-hardening treatment applied (Carburizing, Nitriding, Carbo-nitriding, etc.) High strength (Bending strength / Surface durability).
	SCM415	More than 830	16	40	235 - 321	
	SNC815	More than 980	12	45	285 - 388	
	SNCM220	More than 830	17	40	248 - 341	
	SNCM420	More than 980	15	40	293 - 375	
Rolled Steel for General Structures	SS400	More than 400	—	—	—	Low strength. Low cost.
Gray Cast Iron	FC200	More than 200	—	—	Less than 223	Lower strength than steel. Suitable for bulk production.
Nodular Graphite Cast Iron	FCD500-7	More than 500	7	—	150 ～ 230	Ductile Cast Iron with high strength. Used in the manufacturing of large casting gears.
Stainless Steel	SUS303	More than 520	40	50	Less than 187	Has more machinability than SUS304. Increases seizure resistant.
	SUS304	More than 520	40	60	Less than 187	Most commonly used stainless Steel. Used for food processing machines etc.
	SUS316	More than 520	40	60	Less than 187	Has corrosion resistance against salty seawater, better than SUS304.
	SUS420J2	More than 540	12	40	More than 217	Martensitic stainless steel, quenching can be applied.
	SUS440C	—	—	—	More than 58HRC	High hardness can be obtained by quenching. High surface durability.
Nonferrous Metals	C3604	335	—	—	More than 80HV	Free-Cutting Brass. Used in manufacturing of small gears.
	CAC502	295	10	—	More than 80	Phosphor bronze casting. Suitable for worm wheels.
	CAC702	540	15	—	120 More than	Aluminum-bronze casting. Used for worm wheels etc.
Engineering Plastics	MC901	96	—	—	120HRR	Used for machined gears. Lightweight. Anti-rust.
	MC602ST	96	—	—	120HRR	
	M90	62	—	—	80 HRR	Used for injection-molded gears. Suitable for bulk production at low cost. Applied for use with light load.

9.2 Heat Treatments

Heat treatment is a process that controls the heating and cooling of a material, performed to obtain required structural properties of metal materials. Heating methods include normalizing, annealing quenching, tempering, and surface hardening.

Heat treatment is performed to enhance the properties of the steel. as the hardness increases by applying successive heat treatments, the gear strength increases along with it; the tooth surface strength also increases drastically. As shown in Table 9.2, heat treatments differ depending on the quantity of carbon (C) contained in the steel.

Table 9.2 Heat Treatments



NOTE 1. For nitriding, it is necessary that the material contains one or more alloy elements, such as Al, Cr, Mo. or V.

(1) Normalizing

Normalizing is a heat treatment applied to the microstructure of the small crystals of steel to unify the overall structure. This treatment is performed to relieve internal stress or to resolve inconsistent fiber structure occurred by the forming processing such as rolling.

(2) Annealing

Annealing is a heat treatment applied to soften steel, to adjust crystalline structure, to relieve internal stress, and to modify for cold-working and cutting performance. There are several types of annealing in accordance with the application, such as Full Annealing, Softening, Stress Relieving, Straightening Annealing and Intermediate Annealing.

① Full Annealing
Annealing to relieve internal stress without changing the structure.

② Straightening Annealing
Annealing to fix deformation occurred in steel, or other materials. The treatment is performed by applying load.

③ Intermediate Annealing
Annealing applied in the process of cold-working, applied to soften the work-hardened material, so to make the next process easier.

(3) Quenching

Quenching is a treatment on steel, applying rapid cooling after heating at high temperature. There are several types of quenching in accordance with cooling conditions; water quenching, oil quenching, and vacuum quenching. It is essential to apply tempering after quenching.

(4) Tempering

Tempering is a heat treatment, applying cooling at a proper speed. After performing quench hardening, the material is heated again, then, tempering is applied. Tempering must be performed after quenching. Quenching is applied to adjust hardness, to add toughness, and to relieve internal stress. There are two types of tempering, one is high-temperature tempering, and the other is low-temperature tempering. Applying the tempering at higher temperature, the more toughness is obtained, although the hardness decreases.
For thermal refining, high-temperature tempering is performed.
For induction hardening or carburizing, the require tempering performed after surface-hardening treatment is, low-temperature tempering.

(5) Thermal Refining

Thermal Refining is a heat treatment applied to adjust hardness/strength/toughness of steel. This treatment involves quenching and high-temperature tempering, in combination. After thermal refining is performed, the hardness is adjusted by these treatments to increase the metals machineable properties.

The target hardness for thermal refining are:
S45C (Carbon Steel for Machine Structural Use) 200 - 270 HB
SCM440 (Alloy Steel for Machine Structural Use) 230 - 270 HB

(6) Carburizing

Carburizing is a heat treatment performed especially to harden the surface in which carbon is present and penetrates the surface. The surface of low-carbon steel is carburized (Carbon penetration) and in a state of high carbon, where quenching is required. Low-temperature tempering is applied after quenching to adjust the hardness.
Not only the surface, but the inner material structure is also somewhat hardened by some level of carburizing, however, it is not as hard as the surface.
If a masking agent is applied on a part of the surface, carbon penetration is prevented and the hardness is not changed.
The target hardness on the surface and the hardened depth are:

- Quench Hardness 55 - 63HRC (reference value)
- Effective Hardened Depth 0.3 - 1.2 mm (reference value)

Gears are deformed by carburizing, and the precision is decreased. To improve precision, gear grinding is necessary.

(7) Induction Hardening

Induction Hardening is a heat treatment performed to harden the surface by induction-heating of the steel, composed of 0.3% carbon. For gear products, induction hardening is effective for hardening tooth areas including tooth surface and the tip, however, the root may not be hardened in some cases.
Generally, the precision of gears declines from deformation caused by induction hardening.
For induction hardening of S45C products, please refer to the values below.

- Quench Hardness 45 - 55 HRC
- Effective Hardened Depth 1 - 2 mm

(8) Flame Hardening

Flame Hardening is a surface-hardening treatment performed by flame heating. This treatment is usually performed on the surface for partial hardening of iron and steel.

(9) Nitriding

Nitriding is a heat treatment performed to harden the surface by introducing nitrogen into the surface of steel. If the steel alloy includes aluminum, chrome, and molybdenum, it improves nitriding and the hardness can be obtained. A representative nitride steel is SACM645 (Aluminium chromium molybdenum steel).

(10) Total Quenching

A heat treatment by heating the entire steel material to the core, and then cooling rapidly afterwards, where not only the surface is hardened, the core part is also hardened.

10 Strength and Durability of Gears

The strength of gears are generally calculated by considering the bending strength and surface durability. In the case of using gears under severe conditions, the scoring resistance is also considered.
This section introduces the formulas for strength calculation, based on excerpts from JGMA (Japan Gear Manufacturers Association's Standards). For detailed information, please refer to the following JGMA Standards:

JGMA Standards

JGMA 401−01 : 1974 Bending Strength Formulas for Spur Gears and Helical Gears
JGMA 402−01 : 1975 Surface Durability Formulas for Spur Gears and Helical Gears
JGMA 403−01 : 1976 Bending Strength Formulas for Bevel Gears
JGMA 404−01 : 1977 Surface Durability Formulas for Bevel Gears
JGMA 405−01 : 1978 The Strength Formulas for Worm Gears

Japan Gear Manufacturers Association
Kikai Shinko Kaikan Room No. 208, 5-8, Shiba Koen
3-chome, Minato-ku, Tokyo Tel 03(3431)1871・1872

10.1 Bending Strength of Spur and Helical Gears

JGMA 401−01 : 1974

Generally, bending strength and durability specifications are applied to spur and helical gears (including double helical and internal gears) to beused in industrial machines in the following range:

Module	m	1.5 - 25 mm
Pitch diameter	d_0	25 - 3200 mm
Tangential speed	v	25 m/s or slower
Rotational speed	n	3600 rpm or slower

(1) Conversion Formulas

The equations that relate transmitted tangential force at the pitch circle, F_t (kgf), power P (kW), and torque, T (kgf・m) are basic to the calculations.
The relations are as follows:

$$F_t = \frac{102P}{v} = \frac{1.95 \times 10^6 P}{d_b n} = \frac{2000T}{d_b} \quad (10.1)$$

$$P = \frac{F_t v}{102} = \frac{10^{-6}}{1.95} F_t d_b n \quad (10.2)$$

$$T = \frac{F_t d_b}{2000} = \frac{974P}{n} \quad (10.3)$$

Where v : Tangential speed of working pitch circle (m/s)

$$v = \frac{d_b n}{19100}$$

d_b : Working pitch diameter (mm)
n : Rotational speed (rpm)

(2) Bending Strength Equations

In order to satisfy the bending strength, the transmitted tangential force at the working pitch circle, F_t, is not to exceed the allowable tangential force at the working pitch circle, F_{tlim}, that is calculated taking into account the allowable bending stress at the root.

$$F_t \leqq F_{tlim} \quad (10.4)$$

At the same time, the actual bending stress at the root, σ_F, that is calculated on the basis of the transmitted tangential force at the working pitch circle, F_t, must be less than the allowable bending stress at the root, σ_{Flim}.

$$\sigma_F \leqq \sigma_{Flim} \quad (10.5)$$

Equation (10.6) presents the calculation of F_{tlim} (kgf).

$$F_{tlim} = \sigma_{Flim} \frac{m_n b}{Y_F Y_\varepsilon Y_\beta} \left(\frac{K_L K_{FX}}{K_V K_O} \right) \frac{1}{S_F} \quad (10.6)$$

Equation (10.6) can be converted into stress by Equation (10.7) (kgf/mm²).

$$\sigma_F = F_t \frac{Y_F Y_\varepsilon Y_\beta}{m_n b} \left(\frac{K_V K_O}{K_L K_{FX}} \right) S_F \quad (10.7)$$

(3) Determination of Factors

(3)-1 Facewidth b (mm)

If the gears in a pair have different facewidth, let the wider one be b_w and the narrower one be b_s.
And if:

$b_w - b_s \leqq m_n$ b_w and b_s can be put directly into Equation (10.6).

$b_w - b_s > m_n$ the wider one would be changed to $b_s + m_n$ and the narrower one, b_s, would be unchanged.

NOTE: Regarding the facewidth of round racks, see 10.2 (3) - 1.

(3)-2 Tooth Profile Factor Y_F

The tooth profile factor Y_F is obtainable from Figure 10.1 based on the equivalent number of teeth, z_V, and profile shift coefficient, x, if the gear has a standard tooth profile with pressure angle $\alpha_n = 20°$, per JIS B 1701. Figure 10.1 also indicates (a) theoretical undercut limit, and (b) narrow tooth top limit. These will be helpful in determining gear specifications. For internal gears, obtain the factor by considering the equivalent racks.

(3)-3 Load Sharing Factor, Y_ε

Load sharing factor, Y_ε, is the reciprocal of transverse contact ratio, ε_α.

$$Y_\varepsilon = \frac{1}{\varepsilon_\alpha} \quad (10.8)$$

Fig.10.1 Chart showing tooth profile factor

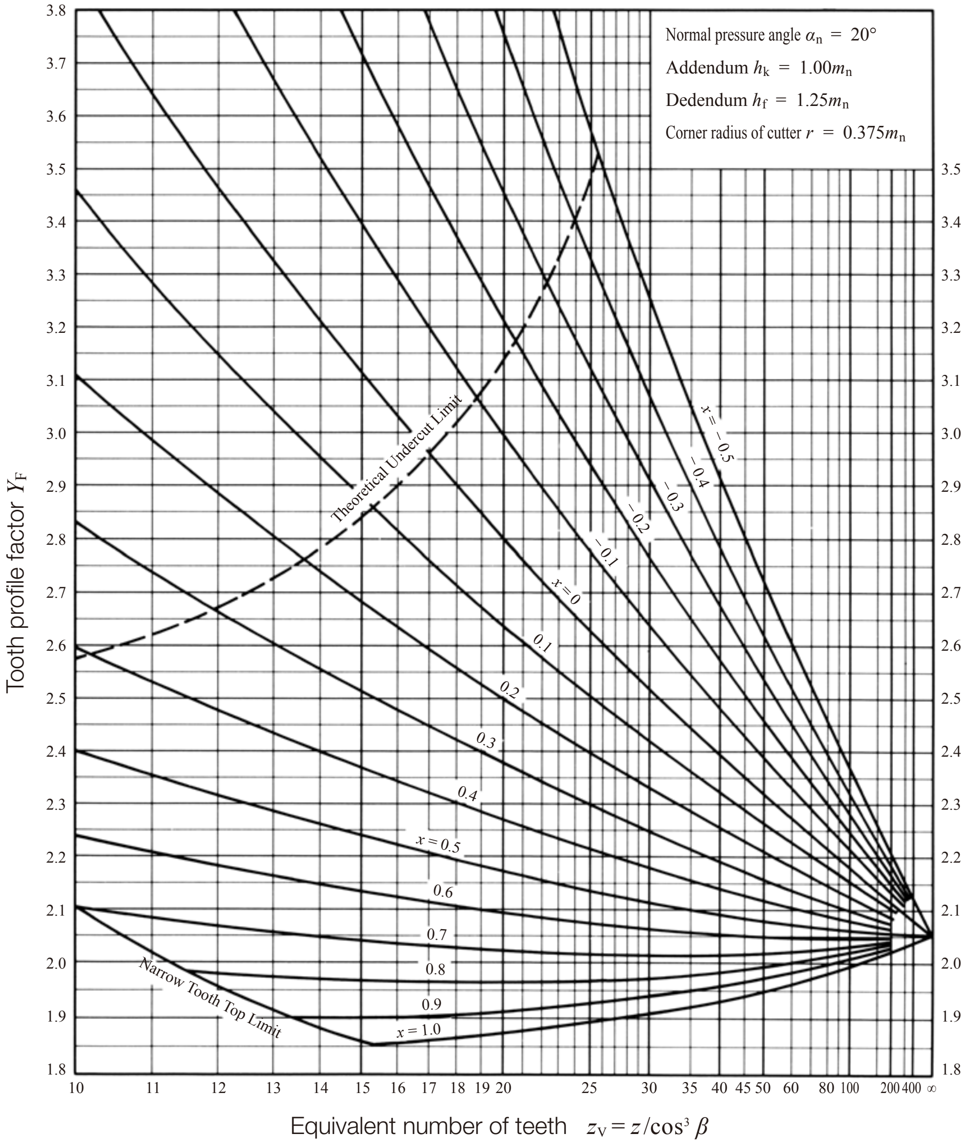


Table 10.1 Transverse contact ratio of standard spur gears, ε_α

($\alpha_0 = 20°$)

No. of teeth	17	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	110	120
17	1.514																			
20	1.535	1.557																		
25	1.563	1.584	1.612																	
30	1.584	1.605	1.633	1.654																
35	1.603	1.622	1.649	1.670	1.687															
40	1.614	1.635	1.663	1.684	1.700	1.714														
45	1.625	1.646	1.674	1.695	1.711	1.725	1.736													
50	1.634	1.656	1.683	1.704	1.721	1.734	1.745	1.755												
55	1.642	1.664	1.691	1.712	1.729	1.742	1.753	1.763	1.771											
60	1.649	1.671	1.698	1.719	1.736	1.749	1.760	1.770	1.778	1.785										
65	1.655	1.677	1.704	1.725	1.742	1.755	1.766	1.776	1.784	1.791	1.797									
70	1.661	1.682	1.710	1.731	1.747	1.761	1.772	1.781	1.789	1.796	1.802	1.808								
75	1.666	1.687	1.714	1.735	1.752	1.765	1.777	1.786	1.794	1.801	1.807	1.812	1.817							
80	1.670	1.691	1.719	1.740	1.756	1.770	1.781	1.790	1.798	1.805	1.811	1.817	1.821	1.826						
85	1.674	1.695	1.723	1.743	1.760	1.773	1.785	1.794	1.802	1.809	1.815	1.821	1.825	1.830	1.833					
90	1.677	1.699	1.726	1.747	1.764	1.777	1.788	1.798	1.806	1.813	1.819	1.824	1.829	1.833	1.837	1.840				
95	1.681	1.702	1.729	1.750	1.767	1.780	1.791	1.801	1.809	1.816	1.822	1.827	1.832	1.836	1.840	1.844	1.847			
100	1.683	1.705	1.732	1.753	1.770	1.783	1.794	1.804	1.812	1.819	1.825	1.830	1.835	1.839	1.843	1.846	1.850	1.853		
110	1.688	1.710	1.737	1.758	1.775	1.788	1.799	1.809	1.817	1.824	1.830	1.835	1.840	1.844	1.848	1.852	1.855	1.858	1.863	
120	1.693	1.714	1.742	1.762	1.779	1.792	1.804	1.813	1.821	1.828	1.834	1.840	1.844	1.849	1.852	1.856	1.859	1.862	1.867	1.871
RACK	1.748	1.769	1.797	1.817	1.834	1.847	1.859	1.868	1.876	1.883	1.889	1.894	1.899	1.903	1.907	1.911	1.914	1.917	1.922	1.926

$$\varepsilon_\alpha = \frac{\sqrt{r_{k1}^2 - r_{g1}^2} + \sqrt{r_{k2}^2 - r_{g2}^2} - a \sin\alpha_b}{\pi m \cos\alpha_o}$$

Transverse Contact Ratio is calculated as follows.

For Spur Gears：

$$\varepsilon_\alpha = \frac{\sqrt{r_{k1}^2 - r_{g1}^2} + \sqrt{r_{k2}^2 - r_{g2}^2} - a \sin \alpha_b}{\pi m \cos \alpha_0}$$

For Helical Gears：

$$\varepsilon_\alpha = \frac{\sqrt{r_{k1}^2 - r_{g1}^2} + \sqrt{r_{k2}^2 - r_{g2}^2} - a \sin \alpha_{bs}}{\pi m_s \cos \alpha_s} \quad (10.9)$$

Where：
r_k:Tip diameter (mm) α_b: Working pressure angle（degree）
r_g:Reference radius (mm) α_{bs}: Transverse working pressure angle（degree）
a: Center distance (mm) α_0: Reference pressure angle（degree）
α_s: Reference transverse pressure angl（degree）

Table 10.1 shows the transverse contact ratio ε_α of a standard spur gear ($\alpha_0 = 20°$)

(3)-4 Helix Angle Factor, Y_β

Helix angle factor, Y_β , can be obtained from Equation

$$\left.\begin{array}{lll} 0 \leqq \beta \leqq 30° \text{ then} & Y_\beta = 1 - \dfrac{\beta}{120} \\ \beta \geqq 30° \quad \text{then} & Y_\beta = 0.75 \end{array}\right\} \quad (10.10)$$

(3)-5 Life Factor, K_L

We can choose the proper life factor, K_L, from Table 10.2. The number of cyclic repetitions means the total loaded meshing during its lifetime.

Table 10.2 Life factor

No. of cyclic repetitions	Hardness (1) H_B120 ～ 220	Hardness (2) H_B221 or over	Gears w. carburizing/nitriding
10000 or fewer	1.4	1.5	1.5
Approx. 100000	1.2	1.4	1.5
Approx. 10^6	1.1	1.1	1.1
10^7 or greater	1.0	1.0	1.0

NOTES（1）Cast steel gears apply to this column.
（2）For induction hardened gears, use the core hardness.

(3)-6 Size Factor of Root Stress, K_{FX}

Generally, this factor, K_{FX}, is unity.

$$K_{FX} = 1.00 \quad (10.11)$$

(3)-7 Dynamic Load Factor, K_V

Dynamic load factor, K_V, can be obtained from Table 10.3 based on the precision of the gear and the tangential speed at working pitch circle.

Table 10.3 Dynamic load factor, K_V

Precision grade of gears from JIS B 1702		Tangential speed at working pitch circle（m/s）						
Tooth profile		1 or under	Over 1 to 3 incl.	Over 3 to 5 incl.	Over 5 to 8 incl.	Over 8 to 12 incl.	Over 12 to 18 incl.	Over 18 to 25 incl.
Unmodified	Modified							
	1	—	—	1.0	1.0	1.1	1.2	1.3
1	2	—	1.0	1.05	1.1	1.2	1.3	1.5
2	3	1.0	1.1	1.15	1.2	1.3	1.5	
3	4	1.0	1.2	1.3	1.4	1.5		
4	—	1.0	1.3	1.4	1.5			
5	—	1.1	1.4	1.5				
6	—	1.2	1.5					

(3)-8 Overload Factor K_O

Overload factor, K_O, can be obtained from Equation

$$K_O = \frac{\text{Actual tangential force}}{\text{Nominal tangential force, } F_t} \quad (10.12)$$

If tangential force is unknown, Table 10.4 provides guiding values. Load grades on affected machinery are introduced on page 572, as reference.

Table 10.4 Overload Factor, K_O

Impact from Prime Mover	Impact from Load Side of Machine		
	Uniform Load	Medium Impact Load	Heavy Impact Load
Uniform Load (Motor, Turbine, Hydraulic Motor)	1.0	1.25	1.75
Light Impact Load (Multicylinder Engine)	1.25	1.5	2.0
Medium Impact Load (Single Cylinder Engine)	1.5	1.75	2.25

(3)-9 Safety Factor for Bending Failure, S_F

Safety factor, S_F, is too complicated to be determined precisely. Usually, it is set to at least 1.2.

(3)-10 Allowable Bending Stress at Root, $\sigma_{F\lim}$

For a unidirectionally loaded gear, the allowable bending stresses at the root, $\sigma_{F\lim}$, are shown in Tables 10.5 to 10.9. In these tables, the value of $\sigma_{F\lim}$ is the quotient of the fatigue limit under pulsating tension divided by the stress concentration factor 1.4. If the load is bidirectional, and both sides of the tooth are equally loaded, the value of allowable bending stress, $\sigma_{F\lim}$, should be taken as 2/3 of the given value in the table. The core hardness means the hardness at the center region of the root.

Table 10.5 Gears without surface hardening

	Material (Arrows indicate the ranges)	Hardness H_B	Hardness H_V	Tensile strength lower limit kgf/mm^2 (Reference)	σ_{Flim} kgf/mm^2
Cast steel gear	SC37			37	10.4
	SC42			42	12
	SC46			46	13.2
	SC49			49	14.2
	SCC3			55	15.8
				60	17.2
Normalizing carbon steel gear		120	126	39	13.8
		130	136	42	14.8
		140	147	45	15.8
	S25C	150	157	48	16.8
		160	167	51	17.6
		170	178	55	18.4
	S35C	180	189	58	19
		190	200	61	19.5
	S43C	200	210	64	20
	S48C	210	221	68	20.5
	S53C	220	231	71	21
	S58C	230	242	74	21.5
		240	252	77	22
		250	263	81	22.5
Quenched and tempered carbon steel gear		160	167	51	18.2
		170	178	55	19.4
		180	189	58	20.2
		190	200	61	21
	S35C	200	210	64	22
		210	221	68	23
		220	231	71	23.5
		230	242	74	24
	S43C, S48C	240	252	77	24.5
		250	263	81	25
	S53C	260	273	84	25.5
	S58C	270	284	87	26
		280	295	90	26
		290	305	93	26.5
Quenched and tempered alloy steel gear		220	231	71	25
		230	242	74	26
		240	252	77	27.5
		250	263	81	28.5
	SMn443	260	273	84	29.5
		270	284	87	31
		280	295	90	32
	SNC836	290	305	93	33
	SCM435	300	316	97	34
	SCM440	310	327	100	35
	SNCM439	320	337	103	36.5
		330	347	106	37.5
		340	358	110	39
		350	369	113	40
		360	380	117	41

Table 10.6 Induction hardened gears

		Material (Arrows indicate the ranges)	Heat treatment before induction hardening	Core hardness H_B	Core hardness H_V	Surface Hardness (1) H_V	σ_{Flim} kgf/mm²
Hardened throughout	Structual carbon steel		Normalized	160	167	More than 550	21
		S43C		180	189	〃	21
		S48C		220	231	〃	21.5
				240	252	〃	22
			Quenched and tempered	200	210	More than 550	23
				210	221	〃	23.5
		S43C		220	231	〃	24
		S48C		230	242	〃	24.5
				240	252	〃	25
				250	263	〃	25
	Structural alloy steel		Quenched and tempered	230	242	More than 550	27
				240	252	〃	28
		SCM440		250	263	〃	29
				260	273	〃	30
		SMn443		270	284	〃	31
				280	295	〃	32
		SNCM439		290	305	〃	33
		SCM435		300	316	〃	34
		SNC836		310	327	〃	35
				320	337	〃	36.5
Hardened except root area							75% of the above

Remarks: If a gear is not quenched completely, or not evenly, or has quenching cracks, the σ_{Flim} will drop dramatically.

NOTE(1):If the hardness after quenching is relatively low, the value of σFlim should be that given in Table 10.5.

Table 10.7 Carburized and quenched gears

	Material (Arrows indicate the ranges)	Core hardness H_B	Core hardness H_V	σ_{Flim} kgf/mm²
Structural carbon steel		140	147	18.2
	S15C	150	157	19.6
	S15CK	160	167	21
		170	178	22
		180	189	23
		190	200	24
Structural alloy steel		220	231	34
		230	242	36
		240	252	38
		250	263	39
		260	273	41
		270	284	42.5
	SCM415, SNC415	280	295	44
		290	305	45
	SCM420	300	316	46
		310	327	47
		320	337	48
	SNCM420, SNC815	330	347	49
		340	358	50
		350	369	51
		360	380	51.5
		370	390	52

NOTE (2)
The table on the left only applies to those gears which have adequate carburized depth and surface hardness. If the carburized depth is relatively thin, the value of σ_{Flim} should be stated for quenched/tempered gears, having no surface hardened.

Table 10.8 Nitrided Gears Excerpted from JGMA403-01(1976)

Material	Surface Hardness (Reference)	Core Hardness H_B	Core Hardness H_V	σ_{Flim} kgf/mm^2
Structural alloy steel except nitriding steel	H_V 650 or more	220	231	30
		240	252	33
		260	273	36
		280	295	38
		300	316	40
		320	337	42
		340	358	44
		360	380	46
Nitriding steel SACM645	H_V 650 or more	220	231	32
		240	252	35
		260	273	38
		280	295	41
		300	316	44

NOTE (1)
The table on the left only applies to those gears which have adequate nitrided depth. If the nitrided depth is relatively thin, the value of σ_{Flim} should be stated for gears which have no surface hardened.

Table 10. 9 Stainless Steel and Free-Cutting Brass Gears Excerpted from JGMA6101-02 (2007)

Material	Hardness	Yield Point MPa	Tensile Strength MPa	σ_{Flim} MPa
Stainless Steel SUS304	Less than 187HB	More than 206 (Durability)	More than 520	103
Free-Cutting Brass C3604	More than 80HV	—	More than 333	39.3

【Reference】Load Grades on Affected Machinery Quoted from JGMA402-01 (1975)

Affected Machinery	Load Grade	Affected Machinery	Load Grade
Stirring machine	M	Food machinery	M
Blower	U	Hammermill	H
Brewing and distilling machine	U	Hoist	M
Automotive machinery	M	Machine tool	H
Clarifier	U	Metal working machinery	H
Sorting machine	M	Tumbling mill	M
Porcelain machine (Medium load)	M	Tumbler	H
Porcelain machine (Heavy load)	H	Blender	M
Compression Machine	M	Petroleum Refinery	M
Conveyer (Uniform load)	U	Papermaking machine	M
Conveyer (Non-uniform / heavy load)	M	Peeling machine	H
Crane	U	Pump	M
Crushing machine	H	Rubber machinery (Medium load)	M
Dredging boat (Medium load)	M	Rubber machinery (Heavy load)	H
Dredging boat (Heavy load)	H	Water treatment machine (Light load)	U
Elevator	U	Water treatment machine (Medium load)	M
Extruding machine	U	Screen (Sifter)	U
Fan (Household use)	U	Screen (Sand strainer)	M
Fan (Industrial use)	M	Sugar refinery machinery	M
Supplying machine	M	Textile machinery	M
Supplying machine (reciprocated)	H		

NOTE1. This sheet was created in reference to AGMA 151.02
2. In this sheet, symbols are used to classify what load grades are:
U: Uniform load, M: Medium load and
H: Heavy load
3. This sheet indicates general tendency of load grades. For use in heavy load, one-higher-grade should be adopted. For details, please refer to the AGMA standard mentioned in NOTE 1.

(4) Example of Calculation

Spur gear design details

No.	Item	Symbol	Unit	Pinion	Gear
1	Normal module	m_n	mm	2	
2	Normal pressure angle	α_n	Degree	20°	
3	Reference cylinder helix angle	β		0°	
4	Number of teeth	z		20	40
5	Center distance	a	mm	60	
6	Profile shift coefficient	x		+ 0.15	− 0.15
7	Pitch diameter	d_0	mm	40.000	80.000
8	Working pitch diameter	d_b		40.000	80.000
9	Facewidth	b		20	20
10	Precision grade			JIS 5 (Without tooth modification)	JIS 5 (Without tooth modification)
11	Manufacturing method			Hobbing	
12	Surface roughness			12.5S	
13	Rotational speed	n	rpm	1500	750
14	Tangential speed	v	m/s	3.142	
15	Direction of load			Unidirectional	
16	Duty cycle		Cycles	10^7 cycles or over	
17	Material			SCM415	
18	Heat treatment			Carburizing and quenching	
19	Surface hardness			H_V 600 − 640	
20	Core hardness			H_B 260 − 280	
21	Effective case depth		mm	0.3 − 0.5	

Bending Strength Factors of Spur Gear

No.	Item	Symbol	Unit	Pinion	Gear
1	Allowable bending stress at root	σ_{Flim}	kgf/mm²	42.5	
2	Normal module	m_n	mm	2	
3	Facewidth	b		20	
4	Tooth profile factor	Y_F		2.568	2.535
5	Load sharing factor	Y_ε		0.619	
6	Helix angle factor	Y_β		1.0	
7	Life factor	K_L		1.0	
8	Size factor of root stress	K_{FX}		1.0	
9	Dynamic load factor	K_V		1.5	
10	Overload factor	K_O		1.0	
11	Safety factor	S_F		1.2	
12	Allowable tangential force on working pitch circle	F_{tlim}	kgf	594.1	601.9

10.2 Surface Durability of Spur and Helical Gears

JGMA 402-01 : 1975

The following equations can be applied to both spur and helical gears, including double helical and internal gears, used in power transmission. The general range of application is:

Module	m	1.5 - 25 mm
Pitch diameter	d_0	25 -3200mm
Tangential speed	v	25 m/s or less
Rotational speed	n	3600 rpm or less

(1) Conversion Formulas

The equations that relate tangential force at the pitch circle, F_t (kgf), power, P (kW), and torque, T (kgf・m) are basic to the calculations. The relations are:

$$F_t = \frac{102P}{v_0} = \frac{1.95\times10^6 P}{d_0 n} = \frac{2000T}{d_0} \quad (10.12)$$

$$P = \frac{F_t v_0}{102} = \frac{10^{-6}}{1.95} F_t d_0 n \quad (10.13)$$

$$T = \frac{F_t d_0}{2000} = \frac{974P}{n} \quad (10.14)$$

Where

v_0 : Tangential speed of working pitch circle (m/s) $= \dfrac{d_0 n}{19100}$

d_0 : Working pitch diameter (mm)

n : Rotational speed (rpm)

(2) Surface Durability Equations

In order to satisfy the surface durability, the transmitted tangential force at the reference pitch circle, F_t, is not to exceed the allowable tangential force at the reference pitch circle, $F_{t\lim}$, that is calculated taking into account the allowable Hertz stress.

$$F_t \leqq F_{t\lim} \quad (10.15)$$

At the same time, the actual Hertz stress, σ_H, that is calculated on the basis of the tangential force at the reference pitch circle, F_t, should not exceed the allowable Hertz stress, $\sigma_{H\lim}$.

$$\sigma_H \leqq \sigma_{H\lim} \quad (10.16)$$

The allowable tangential force, $F_{t\lim}$ (kgf), at the reference pitch circle, can be calculated from Equation (10.17)

$$F_{t\lim} = \sigma_{H\lim}{}^2 d_{01} b_H \frac{i}{i\pm1} \left(\frac{K_{HL} Z_L Z_R Z_V Z_W K_{HX}}{Z_H Z_M Z_\varepsilon Z_\beta}\right)^2 \frac{1}{K_{H\beta} K_V K_O} \frac{1}{S_H{}^2} \quad (10.17)$$

The Hertz stress σ_H (kgf/mm²) is calculated from Equation (10.18)

$$\sigma_H = \sqrt{\frac{F_t}{d_{01} b_H} \frac{i\pm1}{i}} \; \frac{Z_H Z_M Z_\varepsilon Z_\beta}{K_{HL} Z_L Z_R Z_V Z_W K_{HX}} \sqrt{K_{H\beta} K_V K_O} \; S_H \quad (10.18)$$

The "+" symbol in Equations (10.17) and (10.18) applies to two external gears in mesh, whereas the "-" symbol is used for an internal gear and an external gear mesh. For the case of a rack and a gear, the quantity $\dfrac{i}{i\pm1}$ becomes 1.

(3) Determination of Factors

(3)-1 Effective Facewidth in Calculating Surface Strength b_H (mm)

When gears with wider facewidth mate with gears with thinner facewidth, take thinner the facewidth for the calculation of surface strength b_H.

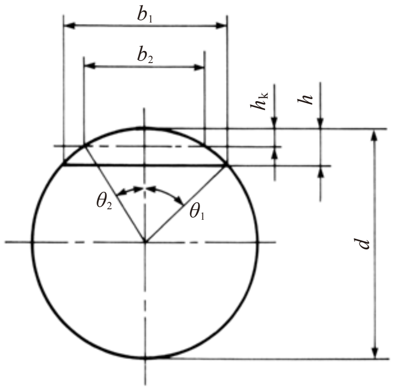
When gears are end relieved, the effective facewidth should not include the relieved portions.

Supplement Facewidth of round racks

In order to obtain the values of the allowable forces shown in the dimensional table, the calculations were made based on condition that the facewidth was:

b_1 - in the case of bending strength

b_2 - in the case of surface durability:



$$b_1 = d \times \sin\theta_1$$

$$\theta_1 = \cos^{-1}\left(1 - \frac{2h}{d}\right)$$

$$b_2 = d \times \sin\theta_2$$

$$\theta_2 = \cos^{-1}\left(1 - \frac{2h_k}{d}\right)$$

Where

h_k = addendum

h = tooth depth

d = outside diameter

(3)-2 Zone Factor, Z_H

The zone factor, Z_H, is defined as:

$$Z_H = \sqrt{\frac{2\cos\beta_g \cos\alpha_{bs}}{\cos^2\alpha_s \sin\alpha_{bs}}} = \frac{1}{\cos\alpha_s}\sqrt{\frac{2\cos\beta_g}{\tan\alpha_{bs}}} \qquad (10.19)$$

Where $\beta_g = \tan^{-1}(\tan\beta\cos\alpha_s)$

β_g : Base helix angle (degrees)

α_{bs}: Working transverse pressure angle (degrees)

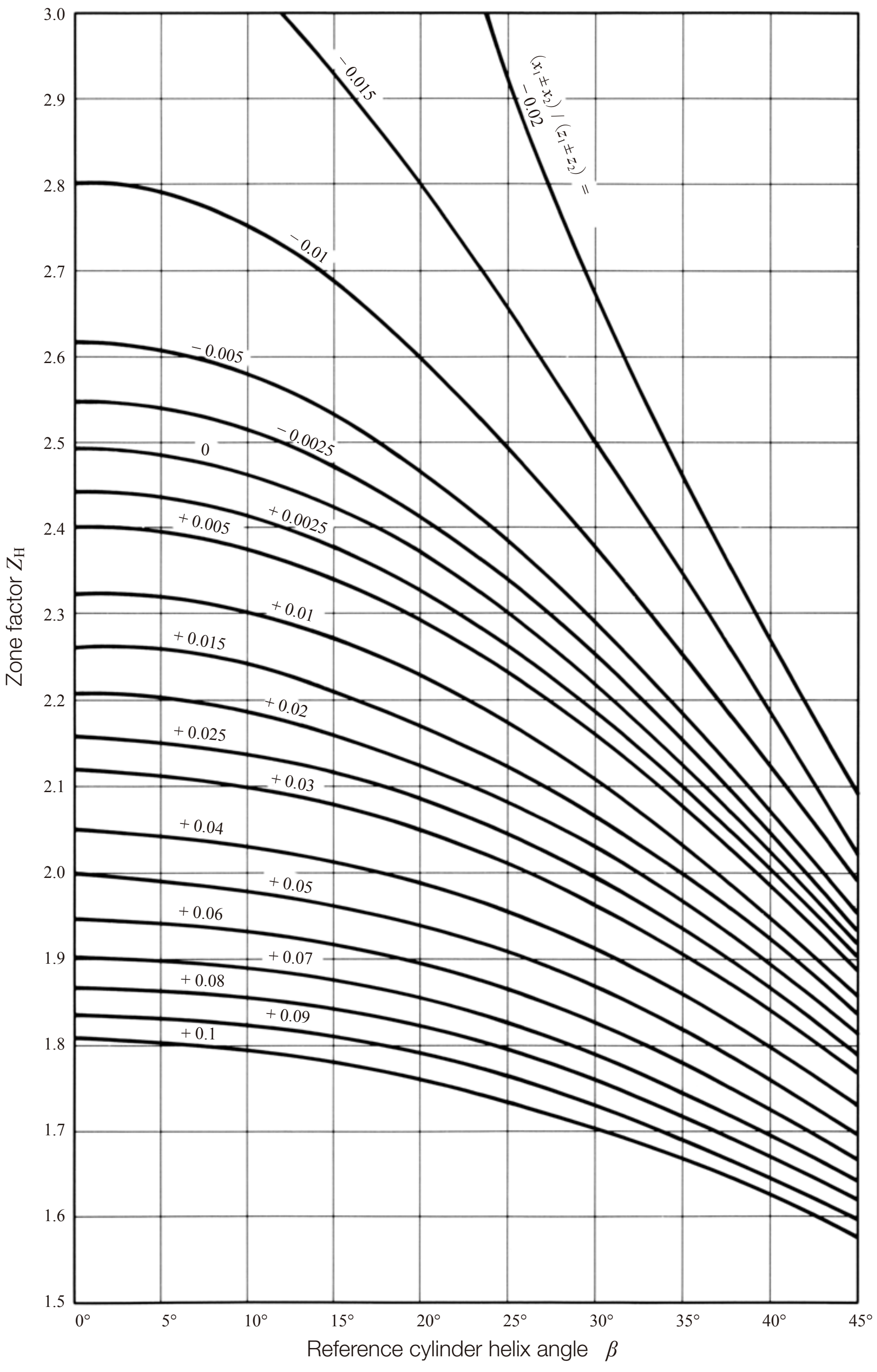
α_s : Transverse pressure angle (degrees)

The zone factors are presented in Figure 10.2 for tooth profiles per JIS B 1701, pressure angle $\alpha_n = 20°$, profile shift coefficient x_1 and x_2, numbers of teeth z_1 and z_2, and helix angle β_0.

Re: "±" symbol in Figure 10.2

The "+" symbol applies to external gear meshes, whereas the "-" is used for internal gear and external gear meshes.

Fig.10.2 Zone factor, Z_H



(3)-3 Material Factor, Z_M

The material factor, Z_M is determined from:

$$Z_M = \sqrt{\frac{1}{\pi\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)}} \qquad (10.20)$$

Where ν : Poisson's ratio

E : Young's modulus (kgf/mm²)

Table 10.9 contains several combinations of material and their material factor, Z_M.

Table 10.9 Material factor, Z_M

Gear				Meshing gear				Material factor Z_M (kgf/mm²)$^{0.5}$
Material	Symbol	Young's modulus E kgf/mm²	Poisson's ratio ν	Material	Symbol	Young's modulus E kgf/mm²	Poisson's ratio ν	
Structural steel	※(1)	21000	0.3	Structural steel	※(1)	21000	0.3	60.6
				Cast steel	SC	20500		60.2
				Ductile cast iron	FCD	17600		57.9
				Gray cast iron	FC	12000		51.7
Cast steel	SC	20500		Cast iron	SC	20500		59.9
				Ductile cast iron	FCD	17600		57.6
				Gray cast iron	FC	12000		51.5
Ductile cast iron	FCD	17600		Ductile cast iron	FCD	17600		55.5
				Gray cast iron	FC	12000		50.0
Gray cast iron	F C	12000		Gray cast	FC	12000		45.8

NOTE (1)※ Structural steels are S ~ C、SNC、SNCM、SCr、SCM etc.

(3)-4 Contact Ratio Factor, Z_ε

Contact ratio factor can be determined from:

Spur gear : $Z_\varepsilon = 1.0$

Helical gear : $\varepsilon_\beta \leqq 1$

$$Z_\varepsilon = \sqrt{1-\varepsilon_\beta+\frac{\varepsilon_\beta}{\varepsilon_\alpha}}$$

When $\varepsilon_\beta > 1$

$$Z_\varepsilon = \sqrt{\frac{1}{\varepsilon_\alpha}} \qquad (10.21)$$

Where ε_α : Transverse contact ratio

ε_β : Overlap ratio

$$\varepsilon_\beta = \frac{b_H \sin\beta}{\pi m_n} \qquad (10.21a)$$

(3)-5 Helix Angle Factor, Z_β

This is a difficult parameter to evaluate. Therefore, it is assumed to be 1.0 unless better information is available.

$$Z_\beta = 1.0 \qquad (10.22)$$

(3)-6 Life Factor, K_{HL}

Table 10.10 indicates the life factor, K_{HL}

Table10.10 Life factor, K_{HL}

Duty cycles	Life factor
10,000 or fewer	1.5
Approx. 100,000	1.3
Approx. 10^6	1.15
10^7or greater	1.0

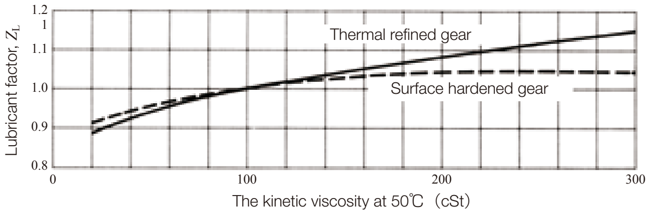
NOTE 1. The duty cycle is the number meshing cycles during a lifetime.
2. Although an idler has two meshing points in one cycle, it is still regarded as one repetition.
3. For bidirectional gear drives, the larger loaded direction is taken as the number of cyclic loads.

When the number of cycles is unknown, K_{HL} is assumed to be 1.0.

(3)-7 Lubricant Factor, Z_L

The lubricant factor, Z_L is based upon the lubricant's kinematic viscosity at 50℃ , cSt . See Figure 10.3.

Fig. 10.3 Lubricant factor, Z_L



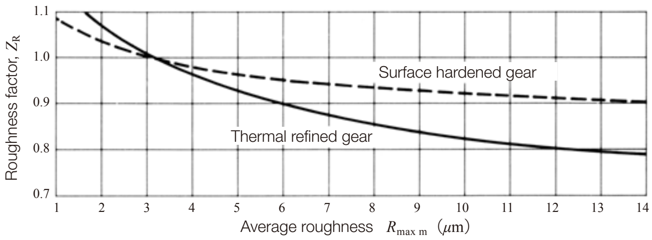
NOTE: Thermal refined gears include quenched and tempered gears and normalized gears.

(3)-8 Surface Roughness Factor, Z_R

The surface roughness factor, Z_R is obtained from Figure 10.4 on the basis of the average roughness R_{maxm} (μm). The average roughness, R_{maxm} is calculated by Equation (10.23) using the surface roughness values of the pinion and gear, R_{max1} and R_{max2}, and the center distance, a, in mm.

$$R_{maxm} = \frac{R_{max1} + R_{max2}}{2} \sqrt[3]{\frac{100}{a}} \quad (\mu\text{m}) \qquad (10.23)$$

Fig.10.4 Surface roughness factor, Z_R

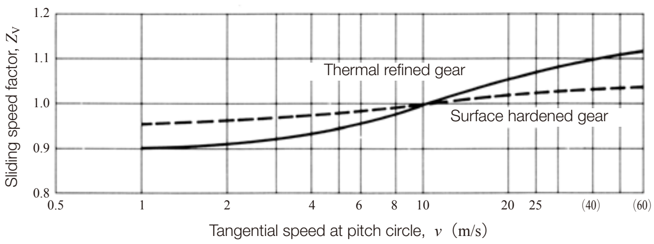


NOTE: Thermal refined gears include quenched and tempered gears and normalized gears.

(3)-9 Lubrication speed factor, Z_V

The lubrication speed factor, Z_V, relates to the tangential speed of the pitch circle, v (m/s). See Figure 10.5.

Fig.10.5 Lubrication speed factor, Z_V



NOTE: Thermal refined gears include quenched and tempered gears and normalized gears.

(3)-10 Hardness Ratio Factor, Z_W

The hardness ratio factor, Z_W, applies only to the gear that is in mesh with a pinion which is quenched and ground. The hardness ratio factor, Z_W, is calculated by Equation (10.24).

$$Z_W = 1.2 - \frac{H_{B2} - 130}{1700} \qquad (10.24)$$

Where H_{B2} : Brinell hardness of gear range:

$$130 \leqq H_{B2} \leqq 470$$

If a gear is out of this range, the Z_W is assumed to be 1.0.

(3)-11 Size Factor, K_{HX}

Because the conditions affecting this parameter are often unknown, the factor is usually set at 1.0.

$$K_{HX} = 1.0 \qquad (10.25)$$

(3)-12 Longitudinal Load Distribution Factor, $K_{H\beta}$

The longitudinal load distribution factor, $K_{H\beta}$, is obtainable from:

① When tooth contact under load is not predictable:

This case relates to the method of gear shaft support, and to the ratio, b/d_{01}, of the gear facewidth b, to the pitch diameter, d_{01}. See Table 10.11.

Table10.11 Longitudinal load distribution facor, $K_{H\beta}$

$\frac{b}{d_{01}}$	Method of gear shaft support			
	Bearings on both ends			Bearing on one end
	Gear equidistant from bearings	Gear close to one end (Rugged shaft)	Gear close to one end (Weak shaft)	
0.2	1.0	1.0	1.1	1.2
0.4	1.0	1.1	1.3	1.45
0.6	1.05	1.2	1.5	1.65
0.8	1.1	1.3	1.7	1.85
1.0	1.2	1.45	1.85	2.0
1.2	1.3	1.6	2.0	2.15
1.4	1.4	1.8	2.1	—
1.6	1.5	2.05	2.2	—
1.8	1.8	—	—	—
2.0	2.1	—	—	—

NOTE:
1. The b means effective facewidth of spur and helical gears. For double helical gears, b is facewidth including central groove.
2. Tooth contact must be good under no load.
3. The values in this table are not applicable to gears with two or more mesh points, such as an idler.

② When tooth contact under load is good.

When tooth contact under load is good, and in addition, when a proper running-in is conducted, the factor is in a narrower range, as specified below:

$$K_{H\beta} = 1.0 \sim 1.2 \qquad (10.26)$$

(3)-13 Dynamic Load Factor, K_V

Dynamic load factor, K_V, is obtainable from Table 10.3 according to the gear's precision grade and pitch circle tangential speed, v_0.

(3)-14 Overload Factor, K_O

The overload factor, K_O, is obtained from either Equation (10.12) or Table 10.4.

(3)-15 Safety Factor for Pitting, S_H

The causes of pitting involves many environmental factors and usually is difficult to precisely define. Therefore, it is advised that a factor of at least 1.15 be used.

Table 10.12 Gears without surface hardening - allowable Hertz stress

	Material (Arrows indicate the ranges)	Surface hardness H_B	Surface hardness H_V	Lower limit of tensile strength kgf/mm2 (Reference)	σ_{Hlim} kgf/mm²
Cast steel	SC37			37	34
	SC42			42	35
	SC46			46	36
	SC49			49	37
	SCC3			55	39
				60	40
Normalizing structural steel		120	126	39	41.5
		130	136	42	42.5
		140	147	45	44
	S25C	150	157	48	45
		160	167	51	46.5
		170	178	55	47.5
	S35C	180	189	58	49
	S43C	190	200	61	50
		200	210	64	51.5
	S48C	210	221	68	52.5
	S53C	220	231	71	54
	S58C	230	242	74	55
		240	253	77	56.5
		250	263	81	57.5
Quenched and tempered structural steel		160	167	51	51
		170	178	55	52.5
		180	189	58	54
		190	200	61	55.5
		200	210	64	57
	S35C	210	221	68	58.5
		220	231	71	60
		230	242	74	61
		240	252	77	62.5
	S43C	250	263	81	64
	S48C	260	273	84	65.5
	S53C	270	284	87	67
	S58C	280	295	90	68.5
		290	305	93	70
		300	316	97	71
		310	327	100	72.5
		320	337	103	74
		330	347	106	75.5
		340	358	110	77
		350	369	113	78.5

(3)-16 Allowable Hertz Stress, σ_{Hlim}

The values of allowable Hertz stress, σ_{Hlim}, for various gear materials are listed in Tables 10.12 through 10.16. Values for hardness not listed can be estimated by interpolation. Surface hardness is defined as the hardness in the pitch circle region.

Table 10.12 Gears without surface hardening - allowable Hertz stress (Continued from page 674)

Material (Arrows indicate the ranges)		Surface hardness H_B	Surface hardness H_V	Lower limit of tensile strength kgf/mm2 (Reference)	σ_{Hlim} kgf/mm²
Quenched and tempered structural alloy steel		220	231	71	70
		230	242	74	71.5
		240	252	77	73
		250	263	81	74.5
		260	273	84	76
		270	284	87	77.5
	SMn443	280	295	90	79
		290	305	93	81
		300	316	97	82.5
	SNC836	310	327	100	84
	SCM435	320	337	103	85.5
	SCM440	330	347	106	87
		340	358	110	88.5
	SNCM439	350	369	113	90
		360	380	117	92
		370	391	121	93.5
		380	402	126	95
		390	413	130	96.5
		400	424	135	98

Table 10.13 Gears with induction hardening - allowable Hertz stress

Material		Heat treatment before induction hardening	Surface hardness H_V (Quenched)	σ_{Hlim} kgf/mm²
Structural carbon steel	S43C S48C	Normalized	420	77
			440	80
			460	82
			480	85
			500	87
			520	90
			540	92
			560	93.5
			580	95
			600 and above	96
		Quenched and tempered	500	96
			520	99
			540	101
			560	103
			580	105
			600	106.5
			620	107.5
			640	108.5
			660	109
			680 and above	109.5
Structural alloy steel	SMn443 SCM435 SCM440 SNC836 SNCM439	Quenched and tempered	500	109
			520	112
			540	115
			560	117
			580	119
			600	121
			620	123
			640	124
			660	125
			680 and above	126

Table 10.14 Carburized and quenched gears - allowable Hertz stress

	Material	Effective case depth (1)	Surface hardness H_V	σ_{Hlim} kgf/mm²
Structural carbon steel	S15C S15CK	Relatively shallow See NOTE (1)A	580	115
			600	117
			620	118
			640	119
			660	120
			680	120
			700	120
			720	119
			740	118
			760	117
			780	115
			800	113
Structural alloy steel	SCM415 SCM420 SNC420 SNC815 SNCM420	Relatively shallow See NOTE (1)A	580	131
			600	134
			620	137
			640	138
			660	138
			680	138
			700	138
			720	137
			740	136
			760	134
			780	132
			800	130
		Relatively thick See NOTE (1)B	580	156
			600	160
			620	164
			640	166
			660	166
			680	166
			700	164
			720	161
			740	158
			760	154
			780	150
			800	146

NOTE (1) Gears with thin effective case depth have "A" row values in the following Table. For thicker depths, use "B" values. The effective case depth is defined as the depth which has the hardness greater than H_V513 (HRC50). The effective case depth of ground gears is defined as the residual layer depth after grinding to final dimensions.

Module		1.5	2	3	4	5	6	8	10	15	20	25
Depth (mm)	A	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.9	1.2	1.5	1.8
	B	0.3	0.3	0.5	0.7	0.8	0.9	1.1	1.4	2.0	2.5	3.4

REMARKS: For two gears with large numbers of teeth in mesh, the maximum shear stress point occurs in the inner part of the tooth beyond the carburized depth. In such a case, a larger safety factor, S_H, should be used.

Table 10.15 Gears with nitriding - allowable Hertz stress ([1])

Material		Surface hardness (Reference)	σ_{Hlim} kgf/mm²	
Nitriding steel	SACM645 etc.	H_V650 or over	Standard processing time	120
			Extra long processing time	130 ～ 140

NOTE: (1) In order to ensure the proper strength, this table applies only to those gears which have adequate depth of nitriding. Gears with insufficient nitriding or where the maximum shear stress point occurs much deeper than the nitriding depth should have a larger safety factor, S_H.

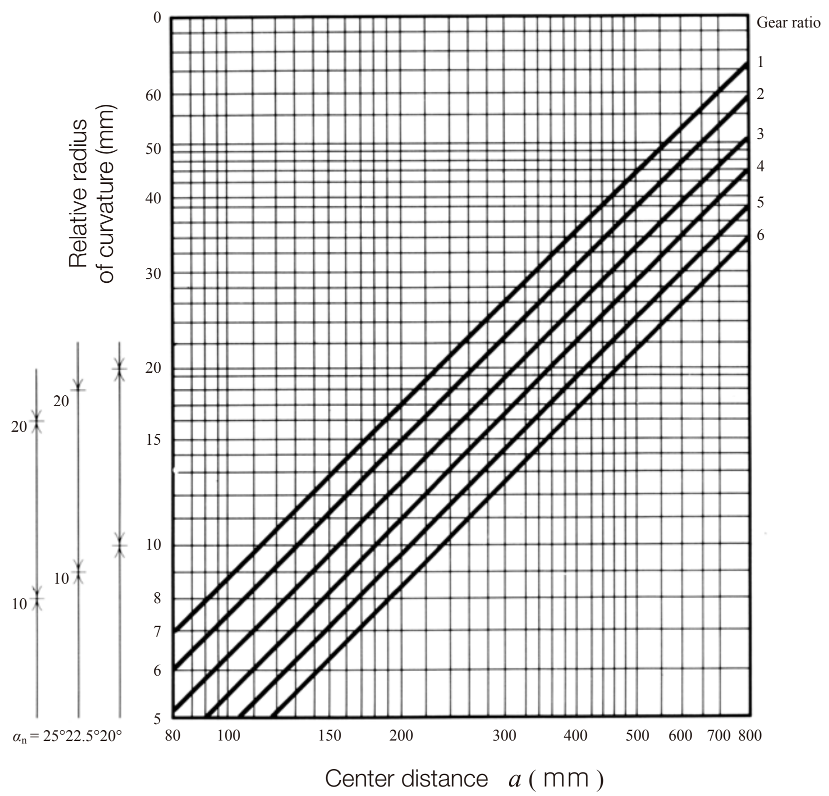
Table 10.16 Gears with soft nitriding ([1])

Material	Nitriding time (h)	σ_{Hlim} kgf/mm²		
		Relative radius of curvature (mm) ([2])		
		10 or less	10 ～ 20	20 or more
Structural steel or alloy steel	2	100	90	80
	4	110	100	90
	6	120	110	100

NOTE([1]) Applicable to salt bath soft nitriding and gas soft nitriding gears. ([2]) Relative radius of curvature is obtained from Figure 1.6.

REMARKS：The center area is assumed to be properly thermal refined.

Fig. 10.6 Relative radius of curvature



(4) Example of Calculation

Spur gear design details

No.	Item	Symbol	Unit	Pinion	Gear
1	Normal module	m_n	mm	2	
2	Normal pressure angle	α_n	Degree	20°	
3	Reference cylinder helix angle	β		0°	
4	Number of teeth	z		20	40
5	Center distance	a	mm	60	
6	Profile shift coefficient	x		+ 0.15	− 0.15
7	Pitch diameter	d_0	mm	40.000	80.000
8	Working pitch diameter	d_b		40.000	80.000
9	Facewidth	b		20	20
10	Precision grade			JIS (Without tooth modification)	JIS (Without tooth modification)
11	Manufacturing method			Hobbing	
12	Surface roughness			12.5S	
13	Rotational speed	n	rpm	1500	750
14	Tangential speed	v	m/s	3.142	
15	Kinematic viscosity of lubricant		cSt	100	
16	Direction of load			Gears are supported on one end (Tooth contact is unpredictable)	
17	Duty cycle		Cycle	10^7 cycles or over	
17	Material			SCM415	
18	Heat treatment			Carburizing	
19	Surface hardness			H_V 600 − 640	
20	Core hardness			H_B 260 − 280	
21	Effective case depth		mm	0.3 − 0.5	

Surface durability factors calculation of spur gear

No.	Item	Symbol	Unit	Pinion	Gear
1	Allowable Hertz stress	σ_{Hlim}	kgf/mm²	164	
2	Pitch diameter of pinion	d_{01}	mm	40	
3	Effective facewidth	b_H		20	
4	Gear ratio(z_2/z_1)	i		2	
5	Zone factor	Z_H		2.495	
6	Material factor	Z_M	$(kgf/mm^2)^{0.5}$	60.6	
7	Contact ratio factor	Z_ε		1.0	
8	Helix angle factor	Z_β		1.0	
9	Life factor	K_{HL}		1.0	
10	Lubricant factor	Z_L		1.0	
11	Surface roughness factor	Z_R		0.90	
12	Lubrication speed factor	Z_V		0.97	
13	Hardness ratio factor	Z_W		1.0	
14	Size factor	K_{HX}		1.0	
15	Load distribution factor	$K_{H\beta}$		1.025	
16	Dynamic load factor	K_V		1.5	
17	Overload factor	K_O		1.0	
18	Safety factor for pitting	S_H		1.15	
19	Allowable tangential force on reference pitch circle	F_{tlim}	kgf	233.8	233.8

10.3 Bending Strength of Bevel Gears JGMA 403-01 : 1976

This information is valid for bevel gears which are used for power transmission in general industrial machines. The applicable ranges are:

Transverse module	m	1.5 ～ 25 mm
Pitch diameter	d_0	1600 mm or less (for straight bevel gears) 1000 mm or less (for spiral bevel gears)
Tangential speed:	v	25 m/s or less
Rotational speed:	n	3600 rpm or less

(1) Conversion Formulas

In calculating strength, transmitted tangential force at the pitch circle, F_{tm} (kgf) , power, P (kW) , and torque, T (kgf・m) , are the design criteria. Their basic relationships are expressed in the following Equations.

$$F_{tm} = \frac{102P}{v_m} = \frac{1.95\times10^6 P}{d_m n} = \frac{2000T}{d_m} \qquad (10.27)$$

$$P = \frac{F_{tm} v_m}{102} = 5.13\times10^{-7} F_{tm} d_m n \qquad (10.28)$$

$$T = \frac{F_{tm} d_m}{2000} = \frac{974P}{n} \qquad (10.29)$$

Where v_m : Tangential speed at the central pitch circle (m/s)

$$= \frac{d_m n}{19100}$$

d_m : Central pitch diameter (mm)

$$= d_0 - b\sin\delta_0$$

(2) Bending Strength Equations

The tangential force, F_{tm}, acting at the central pitch circle should be less than the allowable tangential force, F_{tmlim}, which is based upon the allowable bending stress at the root σ_{Flim}.

That is:

$$F_{tm} \leqq F_{tmlim} \qquad (10.30)$$

The bending stress at the root, σ_F, which is derived from F_{tm} should not exceed the allowable bending stress σ_{Flim}.

$$\sigma_F \leqq \sigma_{Flim} \qquad (10.31)$$

The tangential force at the central pitch circle, F_{tmlim}(kgf) is obtained from Equation (10.32).

$$\left.\begin{aligned} F_{tmlim} = 0.85\cos\beta_m \sigma_{Flim} mb \frac{R_a - 0.5b}{R_a} \\ \frac{1}{Y_F Y_\varepsilon Y_\beta Y_C}\left(\frac{K_L K_{FX}}{K_M K_V K_O}\right)\frac{1}{K_R} \end{aligned}\right\} \qquad (10.32)$$

Where β_m : Mean spiral angle (degrees)

m : Transverse module (mm)

R_a : Cone distance (mm)

The bending strength at the root, σ_F (kgf/mm²) , is calculated from Equation (10.33).

$$\left.\begin{aligned} \sigma_F = F_{tm}\frac{Y_F Y_\varepsilon Y_\beta Y_C}{0.85\cos\beta_m mb}\frac{R_a}{R_a - 0.5b} \\ \left(\frac{K_M K_V K_O}{K_L K_{FX}}\right)K_R \end{aligned}\right\} \qquad (10.33)$$

(3) Determination of Various Coefficients

(3) -1 Facewidth b

The term b is defined as the facewidth on the pitch cone. For the meshed pair, the narrower one is used for strength calculations.

(3) -2 Tooth Profile Factor, Y_F

The tooth profile factor, Y_F, can be obtained in the following manner: Using Figures 10.8 and 10.9, determine the value of the radial tooth profile factor, Y_{F0}. And then, from Figure 10.7 obtain the correction factor, C, for axial shift. Finally, calculate Y_F by Equation 10.34.

$$Y_F = CY_{F0} \qquad (10.34)$$

Should the bevel gear pair not have any axial shift, the tooth profile factor, Y_F, is simply Y_{F0}.

The equivalent number of teeth, z_v, and the profile shift coefficient, x, when using Figures 10.8 and 10.9 is obtainable from Equation (10.35).

$$\left.\begin{aligned} z_v &= \frac{z}{\cos\delta_0\cos^3\beta_m} \\ x &= \frac{h_k - h_{k0}}{m} \end{aligned}\right\} \qquad (10.35)$$

Where

h_k : Addendum at outer end(mm)

h_{k0} : Addendum of standard form(mm)

m : Transverse module(mm)

s : Outer transverse circular tooth thickness(mm)

The axial shift factor, K, is computed from the formula:

$$K = \frac{1}{m}\left\{s - 0.5\pi m - \frac{2(h_k - h_{k0})\tan\alpha_n}{\cos\beta_m}\right\} \qquad (10.36)$$

Fig.10.7 Correction factor for axial shift, C

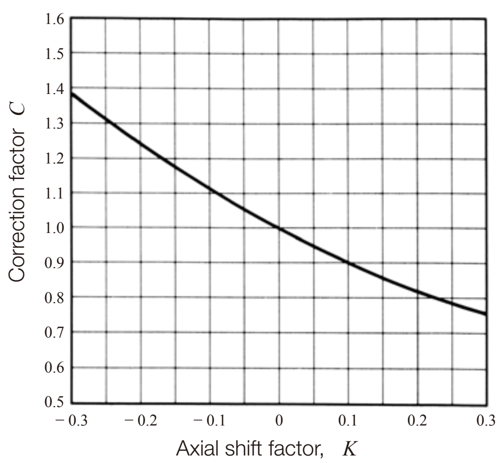


Fig.10.8 Tooth profile factor, Y_{F0} (Straight bevel gear)

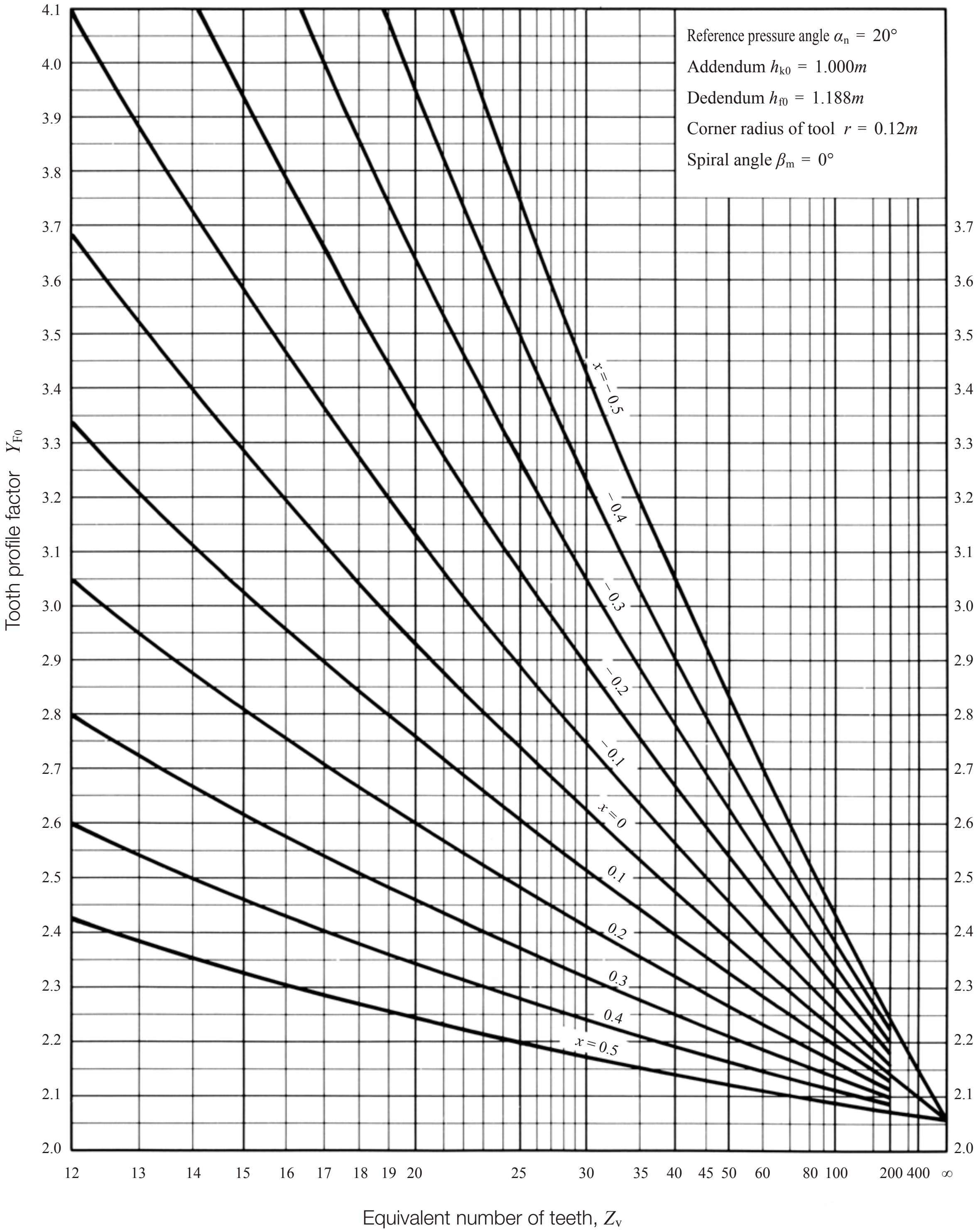
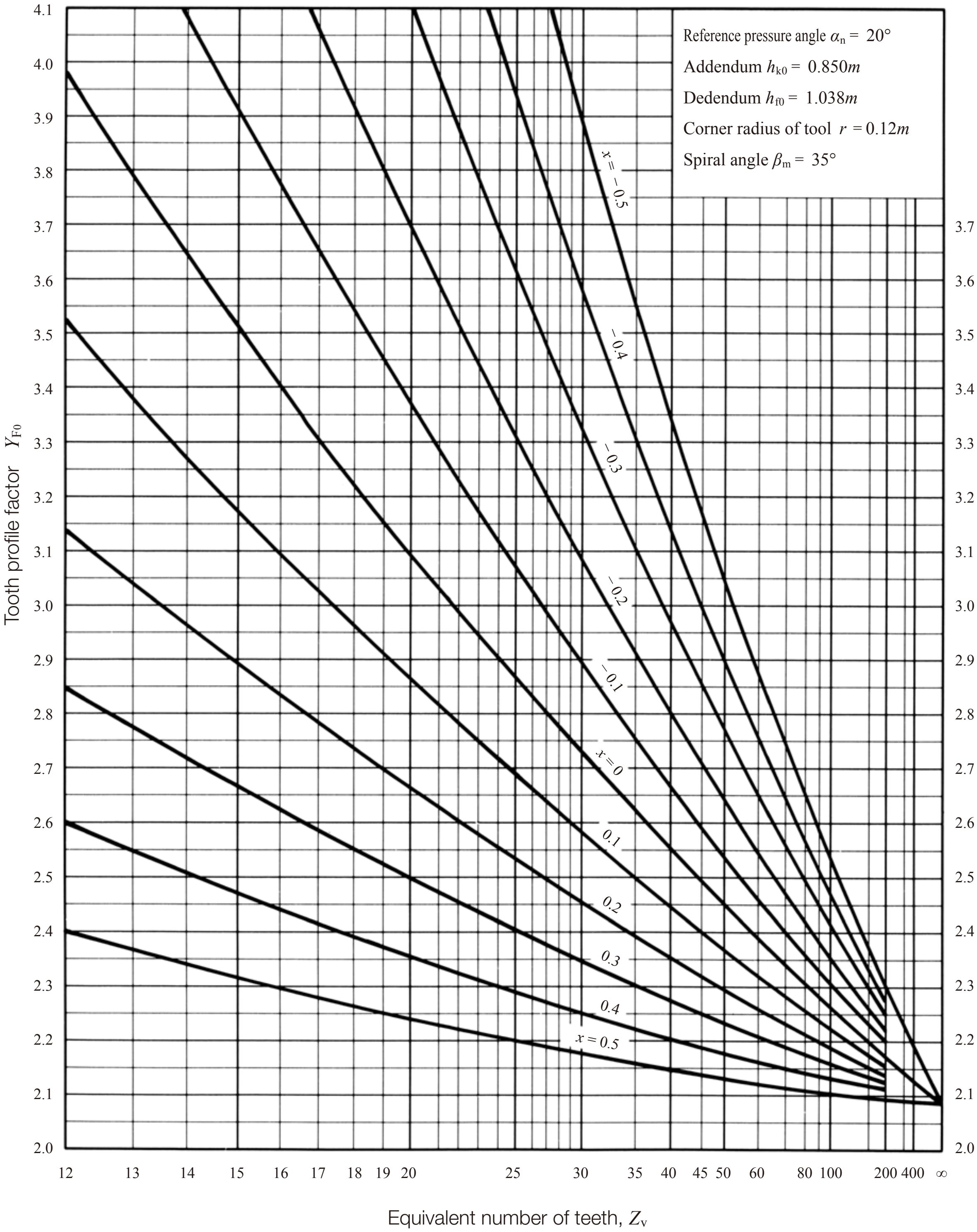


Fig.10.9 Tooth profile factor, Y_{F0} (Spiral bevel gear)



(3) -3 Load Sharing Factor, Y_ε

Load sharing factor, Y_ε, is the reciprocal of transverse contact ratio, ε_α.

$$Y_\varepsilon = \frac{1}{\varepsilon_\alpha} \qquad (10.37)$$

The transverse contact ratio, ε_α, for a straight bevel gear mesh is:

$$\varepsilon_\alpha = \frac{\sqrt{R_{vk1}^2 - R_{vg1}^2} + \sqrt{R_{vk2}^2 - R_{vg2}^2} - (R_{v1} + R_{v2}) \sin \alpha_0}{\pi m \cos \alpha_0}$$

And the transverse contact ratio for spiral bevel gear is:

$$\varepsilon_\alpha = \frac{\sqrt{R_{vk1}^2 - R_{vg1}^2} + \sqrt{R_{vk2}^2 - R_{vg2}^2} - (R_{v1} + R_{v2}) \sin \alpha_s}{\pi m \cos \alpha_s} \qquad (10.38)$$

See Tables 10.17 - 10.19 for some calculating examples of transverse contact ratio for various bevel gear pairs.

Where:

R_{vk} :Tip diameter on back cone for equivalent spur gear (mm)

$$R_{vk} = R_v + h_k = r_0 \sec\delta_0 + h_k$$

R_{vg} :Reference radius on back cone for equivalent spur gear(mm)

Helical gears $= R_v \cos\alpha_0 = r_0 \sec\delta_0 \cos\alpha_0$

Spiral bevel gears $= R_v \cos\alpha_s = r_0 \sec\delta_0 \cos\alpha_s$

R_v :Back cone distance(mm) $= r_0 \sec\delta_0$

r_0 :Pitch radius(mm) $= 0.5 z m$

h_k :Addendum at outer end(mm)

α_0 :Reference pressure angle (degree)

α_s :Mean transverse pressure angle (degree) $= \tan^{-1}(\tan\alpha_n / \cos\beta_m)$

α_n :Reference normal pressure angle (degree)

Table 10.17 The transverse contact ratio for Gleason's straight bevel gear, ε_α ($\Sigma = 90°$、$\alpha_0 = 20°$)

z_2 \ z_1	12	15	16	18	20	25	30	36	40	45	60
12	1.514										
15	1.529	1.572									
16	1.529	1.578	1.588								
18	1.528	1.584	1.597	1.616							
20	1.525	1.584	1.599	1.624	1.640						
25	1.518	1.577	1.595	1.625	1.650	1.689					
30	1.512	1.570	1.587	1.618	1.645	1.697	1.725				
36	1.508	1.563	1.579	1.609	1.637	1.692	1.732	1.758			
40	1.506	1.559	1.575	1.605	1.632	1.688	1.730	1.763	1.775		
45	1.503	1.556	1.571	1.600	1.626	1.681	1.725	1.763	1.781	1.794	
60	1.500	1.549	1.564	1.591	1.615	1.668	1.710	1.751	1.773	1.796	1.833

Table 10.18 The transverse contact ratio for standard straight bevel gear, ε_α ($\Sigma = 90°$、$\alpha_0 = 20°$)

z_2 \ z_1	12	15	16	18	20	25	30	36	40	45	60
12	1.514										
15	1.545	1.572									
16	1.554	1.580	1.588								
18	1.571	1.595	1.602	1.616							
20	1.585	1.608	1.615	1.628	1.640						
25	1.614	1.636	1.643	1.655	1.666	1.689					
30	1.634	1.656	1.663	1.675	1.685	1.707	1.725				
36	1.651	1.674	1.681	1.692	1.703	1.725	1.742	1.758			
40	1.659	1.683	1.689	1.702	1.712	1.734	1.751	1.767	1.775		
45	1.666	1.691	1.698	1.711	1.721	1.743	1.760	1.776	1.785	1.794	
60	1.680	1.707	1.714	1.728	1.739	1.762	1.780	1.796	1.804	1.813	1.833

Table 10.19 The transverse contact ratio for Gleason's spiral bevel gear, ε_α ($\Sigma = 90°$、$\alpha_0 = 20°$、$\beta_m = 35°$)

z_2 \ z_1	12	15	16	18	20	25	30	36	40	45	60
12	1.221										
15	1.228	1.254									
16	1.227	1.258	1.264								
18	1.225	1.260	1.269	1.280							
20	1.221	1.259	1.269	1.284	1.293						
25	1.214	1.253	1.263	1.282	1.297	1.319					
30	1.209	1.246	1.257	1.276	1.293	1.323	1.338				
36	1.204	1.240	1.251	1.270	1.286	1.319	1.341	1.355			
40	1.202	1.238	1.248	1.266	1.283	1.316	1.340	1.358	1.364		
45	1.201	1.235	1.245	1.263	1.279	1.312	1.336	1.357	1.366	1.373	
60	1.197	1.230	1.239	1.256	1.271	1.303	1.327	1.349	1.361	1.373	1.392

(3) -4 Spiral Angle Factor, Y_β

The spiral angle factor, Y_β, is obtainable from Equation (10.39).

$$\left.\begin{array}{lll} \text{when} & 0 \leqq \beta_m \leqq 30 & Y_\beta = 1 - \dfrac{\beta_m}{120} \\ \text{when} & \beta_m \geqq 30^\circ & Y_\beta = 0.75 \end{array}\right\} \quad (10.39)$$

(3) -5 Cutter Diameter Effect Factor, Y_C

The cutter diameter effect factor, Y_C, can be obtained from Table 10.20 by the value of tooth flank length, $b/\cos\beta_m$ (mm), over cutter diameter.

If cutter diameter is not known, assume $Y_C = 1.0$.

Table 10.20 Cutter diameter effect factor, Y_C

Types of bevel gears	Relative size of cutter diameter			
	∞	6 times facewidth	5 times facewidth	4 times facewidth
Straight bevel gears	1.15	—	—	—
Spiral and zerol bevel gears	—	1.00	0.95	0.90

(3) -6 Life Factor, K_L

The life factor, K_L, is obtainable from Table 10.2.

(3) -7 Size Factor of Bending Stress at Root, K_{FX}

The size factor of bending stress at root, K_{FX}, can be obtained from Table 10.21 based on the transverse module, m.

Table 10.21 Size factor for bending strength, K_{FX}

Transverse module at outside diameter, m	Gears without hardened surface	Gears with hardened surface
From 1.5 to 5 incl.	1.0	1.0
Over 5 to 7 incl.	0.99	0.98
Over 7 to 9 incl.	0.98	0.96
Over 9 to 11 incl.	0.97	0.94
Over 10 to 13 incl.	0.96	0.92
Over 13 to 15 incl.	0.94	0.90
Over 15 to 17 incl.	0.93	0.88
Over 17 to 19 incl.	0.92	0.86
Over 19 to 22 incl.	0.90	0.83
Over 22 to 25 incl.	0.88	0.80

(3) -8 Longitudinal Load Distribution Factor, K_M

The longitudinal load distribution factor, K_M, is obtained from Table 10.22 or Table 10.23.

Table 10.22 Longitudinal load distribution factor, K_M for spiral bevel gears, zerol bevel gears and straight bevel gears with crowning

		Both gears supported on two sides	One gear supported on one end	Both gears supported on one end
Stiffness of shaft, gearbox etc.	Very stiff	1.2	1.35	1.5
	Average	1.4	1.6	1.8
	Somewhat weak	1.55	1.75	2.0

Table 10.23 Tooth Flank Load Distribution Factor K_M for Straight Bevel Gears without Crowing

		Both gears supported on two sides	One gear supported on one end	Both gears supported on one end
Stiffness of shaft, gearbox etc.	Very stiff	1.05	1.15	1.35
	Average	1.6	1.8	2.1
	Somewhat weak	2.2	2.5	2.8

(3) -9 Dynamic Load Factor, K_V

Dynamic load factor, K_V, is a function of the precision grade of the gear and the tangential speed at the outer pitch circle, as shown in Table 10.24.

Table 10.24 Dynamic load factor, K_V

Precision grade of gears from JIS B 1704	Tangential speed (m/s)						
	1 or less	Over 1 to 3 incl.	Over 3 to 5 incl.	Over 5 to 8 incl.	Over 8 to 12 incl.	Over 12 to 18 incl.	Over 18 to 25 incl.
1	1.0	1.1	1.15	1.2	1.3	1.5	1.7
2	1.0	1.2	1.3	1.4	1.5	1.7	
3	1.0	1.3	1.4	1.5	1.7		
4	1.1	1.4	1.5	1.7			
5	1.2	1.5	1.7				
6	1.4	1.7					

(3) -10 Overload Factor, K_O

The overload factor, K_O, can be computed from Equation (10.12) or obtained from Table 10.4, identical to the case of spur and helical gears.

(3) -11 Reliability Factor, K_R

The reliability factor, K_R, should be assumed to be as follows:

① General case $K_R = 1.2$

② When all other factors can be determined accurately: $K_R = 1.0$

③ When all or some of the factors cannot be known with certainty: $K_R = 1.4$

(3) -12 Allowable Bending Stress at Root, σ_{Flim}

The allowable bending stress at the root is obtained by a bending strength calculation for spur and helical gears as shown at < (3) -10 >.

(4) Example of Calculation

Gleason straight bevel gear design details

No.	Item	Symbol	Unit	Pinion	Gear
1	Shaft angle	Σ	Degree	90°	
2	Module	m	mm	2	
3	Pressure angle	α_0	Degree	20°	
4	Mean spiral angle	β_m		0°	
5	Number of teeth	z		20	40
6	Pitch diameter	d_0	mm	40.000	80.000
7	Pitch angle	δ_0	Degree	26.56505°	63.43495°
8	Cone distance	R_a	mm	44.721	
9	Facewidth	b		15	
10	Center reference diameter	d_m		33.292	66.584
11	Precision grade			JIS 3	JIS 3
12	Manufacturing method			Gleason No.104	
13	Surface roughness			12.5S	12.5S
14	Rotational speed	n	rpm	1500	750
15	Tangential speed	v	m/s	3.142	
16	Direction of load			Unidirectional	
17	Duty cycle		Cycle	More than 10^7 circles	
18	Gear support			Gears are supported on one end	
19	Stiffness of shaft and gear box			Normal stiffness	
20	Material			SCM415	
21	Heat treatment			Carburized	
22	Surface hardness			H_V 600 – 640	
23	Core hardness			H_B 260 – 280	
24	Effective case depth		mm	0.3 – 0.5	

Bending strength factors for Gleason straight bevel gear

No.	Item	Symbol	Unit	Pinion	Gear
1	Mean spiral angle	β_m	Degree	0°	
2	Allowable bending stress at root	σ_{Flim}	kgf/mm²	42.5	42.5
3	Module	m	mm	2	
4	Facewidth	b		15	
5	Cone distance	R_a		44.721	
6	Tooth profile factor	Y_F		2.369	2.387
7	Load sharing factor	Y_ε		0.613	
8	Spiral angle factor	Y_β		1.0	
9	Cutter diameter effect factor	Y_C		1.15	
10	Life factor	K_L		1.0	
11	Size factor	K_{FX}		1.0	
12	Longitudinal load distribution factor	K_M		1.8	1.8
13	Dynamic load factor	K_V		1.4	
14	Overload factor	K_O		1.0	
15	Reliability factor	K_R		1.2	
16	Allowable tangential force at central pitch circle	F_{tlim}	kgf	178.6	177.3

10.4 Surface Durability of Bevel Gear JGMA 404-01 : 1977

This information is valid for bevel gears which are used for power transmission in general industrial machines. The applicable ranges are:

Transverse module	m	1.5 - 25mm
Pitch diameter	d_0	1600mm or less (Straight bevel gear) 1000mm or less (Spiral bevel gear)
Tangential speed	v	25m/s or less
Rotational speed	n	3600rpm or less

(1) Basic Conversion Formulas

Equations (10.27), (10.28) and (10.29) in 1.3 "Bending strength of bevel gears" shall apply.

(2) Surface Durability Equations

In order to obtain a proper surface durability, the transmitted tangential force at the central pitch circle, F_{tm}, should not exceed the allowable tangential force at the central pitch circle, F_{tmlim}, based on the allowable Hertz stress σ_{Hlim}.

$$F_{tm} \leqq F_{tmlim} \tag{10.40}$$

Alternately, the Hertz stress, σ_H, which is derived from the transmitted tangential force at the central pitch circle should not exceed the allowable Hertz stress, σ_{Hlim}.

$$\sigma_H \leqq \sigma_{Hlim} \tag{10.41}$$

The allowable tangential force at the central pitch circle, F_{tmlim} (kgf), can be calculated from Equation (10.42).

$$F_{tmlim} = \left(\frac{\sigma_{Hlim}}{Z_M}\right)^2 \frac{d_{01}}{\cos\delta_{01}} \frac{R_a - 0.5b}{R_a} b \frac{i^2}{i^2+1} \left(\frac{K_{HL}Z_LZ_RZ_VZ_WK_{HX}}{Z_HZ_\varepsilon Z_\beta}\right)^2 \frac{1}{K_{H\beta}K_VK_O} \frac{1}{C_R^2} \tag{10.42}$$

The Hertz stress, σ_H (kgf/mm^2), is calculated from Equation (10.43).

$$\sigma_H = \sqrt{\frac{\cos\delta_{01}F_{tm}}{d_{01}b} \frac{i^2+1}{i^2} \frac{R_a}{R_a-0.5b}} \frac{Z_HZ_MZ_\varepsilon Z_\beta}{K_{HL}Z_LZ_RZ_VZ_WK_{HX}} \sqrt{K_{H\beta}K_VK_O}\, C_R \tag{10.43}$$

(3) Determinaion of Factors

(3) -1 Facewidth, b

This term is defined as the facewidth on the pitch cone.For a meshed pair, the narrower gear's b is to be used.

(3) -2 Zone Factor, Z_H

The zone factor, Z_H, is defined as:

$$Z_H = \sqrt{\frac{2\cos\beta_g}{\sin\alpha_s\cos\alpha_s}} \tag{10.44}$$

where β_m : Mean spiral angle

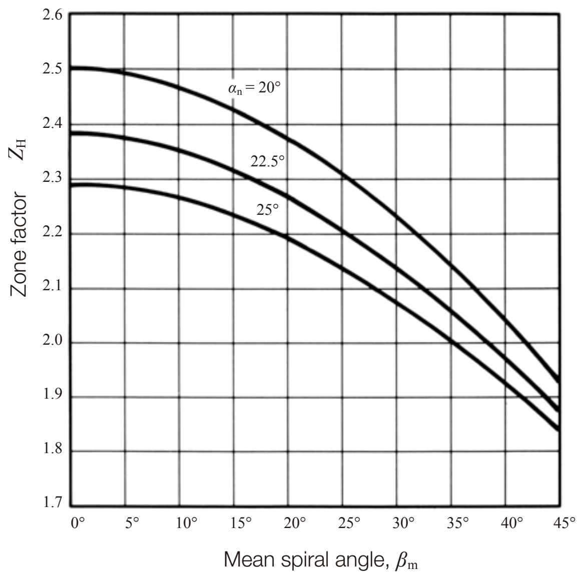
α_n : Normal reference pressure angle

α_s : Central transverse pressure angle $= \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta_m}\right)$

$\beta_g = \tan^{-1}(\tan\beta_m\cos\alpha_s)$

If the normal reference pressure angle, α_n , is 20°, 22.5° or 25° , the zone factor, Z_H, can be obtained from Figure 10.10.

Fig. 10.10 Zone factor, Z_H



(3) -3 Material Factor, Z_M

The material factor, Z_M, can be obtainable from Table 10.9 in 1.2 "Surface durability of spur and helical gear".

(3) -4 Contact Ratio Factor, Z_ε

The contact ratio factor, Z_ε, is calculated from the equations below.

$$\left.\begin{array}{ll} \text{Straight bevel gear : } Z_\varepsilon = 1.0 & \\ \text{Spiral bevel gear :} & \\ \quad \text{When } \varepsilon_\beta \leqq 1 & Z_\varepsilon = \sqrt{1 - \varepsilon_\beta + \dfrac{\varepsilon_\beta}{\varepsilon_\alpha}} \\ \quad \text{When } \varepsilon_\beta > 1 & Z_\varepsilon = \sqrt{\dfrac{1}{\varepsilon_\alpha}} \end{array}\right\} \tag{10.45}$$

Where ε_α : Transverse contact ratio

ε_β : Overlap ratio

$$\varepsilon_\beta = \frac{R_a}{R_a - 0.5b} \frac{b\tan\beta_m}{\pi m} \tag{10.45a}$$

(3) -5 Spiral Angle Factor, Z_β

Since it is difficult to prescribe the spiral angle factor, Z_β, because little is known about this factor, 1.0 is usually used.

$$Z_\beta = 1.0 \tag{10.46}$$

(3) -6 Life Factor, K_{HL}

The life factor for surface durability, K_{HL}, is obtainable from Table 10.10 in 1.2 "Surface durability of spur and helical gear".

(3) -7 Lubricant Factor, Z_L

The lubricant factor, Z_L, is found in Figure 10.3. See page 673.

(3) -8 Surface Roughness Factor, Z_R

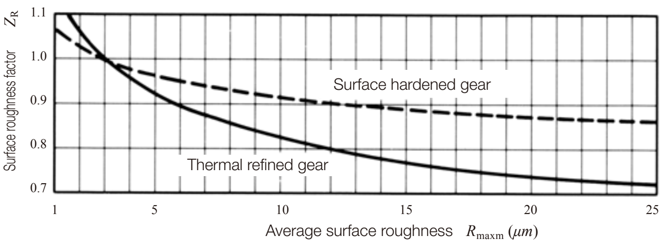
The surface roughness factor, Z_R, is obtainable from Figure 10.11 on the basis of average roughness, R_{maxm} (μm). The average surface roughness, R_{maxm}, is calculated by Equation (10.47) from surface roughness of the pinion and gear (R_{max1} and R_{max2}), and a (mm) .

$$R_{maxm} = \frac{R_{max1} + R_{max2}}{2} \sqrt[3]{\frac{100}{a}} \ (\mu m) \tag{10.47}$$

where $a = R_m(\sin \delta_{01} + \cos \delta_{01})$

$$R_m = R_a - b/2$$

Fig. 10.11 Surface roughness factor, Z_R



(3) -9 Lubrication speed factor, Z_V

The lubrication speed factor, Z_V, is obtained from Figure 10.5 . See page 673.

(3) -10 Hardness ratio factor, Z_W

The hardness ratio factor, Z_W, applies only to the gear that is in mesh with a pinion which is quenched and ground, and can be obtained from Equation (10.48) .

$$Z_W = 1.2 - \frac{H_{B2} - 130}{1700} \tag{10.48}$$

Where H_{B2} : Brinell hardness of the tooth flank of the gear

should be $130 \leqq H_{B2} \leqq 470$

If the gear's hardness is outside of this range, Z_W is assumed to be unity.

$$Z_W = 1.0 \tag{10.49}$$

(3) -11 Size Factor, K_{HX}

The size factor, K_{HX}, is assumed to be unity because, often, little is known about this factor.

$$K_{HX} = 1.0 \tag{10.50}$$

(3) -12 Longitudinal Load Distribution Factor, $K_{H\beta}$

The longitudinal load distribution factors are listed in Tables 10.25 and 10.26. If the gear and pinion are unhardened, the factors are to be reduced to 90% of the values in the table.

Table 10.25 Longitudinal load distribution factor for spiral bevel gears (zerol bevel gears included), and straight bevel gears with crowning , $K_{H\beta}$

Stiffness of shaft, gearbox etc.	Gear shaft support		
	Both gears supported on two sides	One gear supported on one end	Both gears supported on one end
Very stiff	1.3	1.5	1.7
Average	1.6	1.85	2.1
Somewhat weak	1.75	2.1	2.5

Table 10.26 Longitudinal load distribution factor for straight bevel gear without crowning , $K_{H\beta}$

Stiffness of shaft, gearbox etc.	Gear shaft support		
	Both gears supported on two sides	One gear supported on one end	Both gears supported on one end
Very stiff	1.3	1.5	1.7
Average	1.85	2.1	2.6
Somewhat weak	2.8	3.3	3.8

(3) -13 Dynamic Load Factor, K_V

The dynamic load factor, K_V, can be obtained from Table 10.24. See page 683.

(3) -14 Overload Factor, K_O

The overload factor, K_O, can be computed by Equation (10.12) or found in Table 10.4.

(3) -15 Reliability Factor, C_R

The general practice is to assume C_R to be at least 1.15.

(3) -16 Allowable Hertz Stress, σ_{Hlim}

The values of allowable Hertz stress, σ_{Hlim}, are given in Tables 10.12 through 10.16.

(4) Example of Calculation

Gleason straight bevel gear design details

No.	Item	Symbol	Unit	Pinion	Gear
1	Shaft angle	Σ	Degree	90°	
2	Module	m	mm	2	
3	Pressure angle	α_0	Degree	20°	
4	Mean spiral angle	β_m		0°	
5	Number of teeth	z		20	40
6	Pitch diameter	d_0	mm	40.000	80.000
7	Pitch angle	δ_0	Degree	26.56505°	63.43495°
8	Cone distance	R_a	mm	44.721	
9	Facewidth	b		15	
10	Center reference diameter	d_m		33.292	66.584
11	Precision grade			JIS 3	JIS 3
12	Manufacturing method			Gleason No.104	
13	Surface roughness			12.5S	12.5S
14	Rotational speed	n	rpm	1500	750
15	Tangential speed	v	m/s	3.142	
16	Kinematic viscosity of lubricant		cSt	100	
17	Gear support			Gears are supported on one end	
18	Stiffness of shaft and gear box			Normal stiffness	
19	Duty cycle		Cycle	10^7 cycles or over	
20	Material			SCM415	
21	Heat treatment			Carburized	
22	Surface hardness			H_V 600 – 640	
23	Core hardness			H_B 260 – 280	
24	Effective case depth		mm	0.3 – 0.5	

Surface durability factors of Gleason straight bevel gear

No.	Item	Symbol	Unit	Pinion	Gear
1	Allowable Hertz stress	σ_{Hlim}	kgf/mm²	164	
2	Pinion's pitch diameter	d_{01}	mm	40.000	
3	Pinion's pitch angle	δ_{01}	Degree	26.56505°	
4	Cone distance	R_a	mm	44.721	
5	Facewidth	b		15	
6	Gear ratio (z_2 / z_1)	i		2	
7	Zone factor	Z_H		2.495	
8	Material factor	Z_M	$(kgf/mm^2)^{0.5}$	60.6	
9	Contact ratio factor	Z_ε		1.0	
10	Spiral angle factor	Z_β		1.0	
11	Life factor	K_{HL}		1.0	
12	Lubrication factor	Z_L		1.0	
13	Surface roughness factor	Z_R		0.89	
14	Lubrication speed factor	Z_V		0.97	
15	Hardness ratio factor	Z_W		1.0	
16	Size factor	K_{HX}		1.0	
17	Longitudinal load ditribution factor	$K_{H\beta}$		2.1	
18	Dynamic load factor	K_V		1.4	
19	Overload factor	K_O		1.0	
20	Reliability factor	C_R		1.15	
21	Allowable tangential force on central pitch circle	F_{tlim}	kgf	101.3	101.3

10.5 Surface Durability of Cylindrical Worm Gearing

JGMA 405-01 : 1978

This information is applicable for worm gear pair drives that are used to transmit power in general industrial machines with the following parameters:

Axial module	m_a	1 ～ 25 mm
Pitch diameter of worm wheel	d_{02}	900mm or less
Sliding speed	v_s	30m/s or less
Rotational speed of worm wheel	n_2	600rpm or less

(1) Basic Formulas

(1) -1 Sliding speed (m/s)

$$v_s = \frac{d_{01}\, n_1}{19100 \cos \gamma_0} \tag{10.51}$$

(1) -2 Torque, tangential force and efficiency

① Worm as driver (Speed reducing)

$$\left.\begin{aligned} T_2 &= \frac{F_t\, d_{02}}{2000} \\ T_1 &= \frac{T_2}{i\eta_R} = \frac{F_t\, d_{02}}{2000 i\eta_R} \\ \eta_R &= \frac{\tan\gamma_0\left(1 - \tan\gamma_0 \dfrac{\mu}{\cos\alpha_n}\right)}{\tan\gamma_0 + \dfrac{\mu}{\cos\alpha_n}} \end{aligned}\right\} \tag{10.52}$$

Where

T_2 : Nominal torque of worm wheel (kgf·m)

T_1 : Nominal torque of worm (kgf·m)

F_t : Nominal tangential force on worm wheel's pitch circle (kgf)

d_{02} : Pitch diameter of worm wheel (mm)

i : Gear ratio $= z_2 / z_w$

η_R : Transmission efficiency, worm driving (not including bearing loss, lubricant agitation loss, etc.)

μ : Coefficient of friction

② Worm wheel as driver (Speed increasing)

$$\left.\begin{aligned} T_2 &= \frac{F_t\, d_{02}}{2000} \\ T_1 &= \frac{T_2\, \eta_I}{i} = \frac{F_t\, d_{02}\, \eta_I}{2000 i} \\ \eta_I &= \frac{\tan\gamma_0 - \dfrac{\mu}{\cos\alpha_n}}{\tan\gamma_0\left(1 + \tan\gamma_0 \dfrac{\mu}{\cos\alpha_n}\right)} \end{aligned}\right\} \tag{10.53}$$

Where η_I : Transmission efficiency, worm wheel driving (not including bearing loss, lubricant agitation loss, etc.)

Fig. 10.12 Coefficient of Friction

For lack of data, coefficient of friction, μ, of materials is very difficult to obtain. However, Table 10.27 indicates H. E. Merritt's offer as a reference.

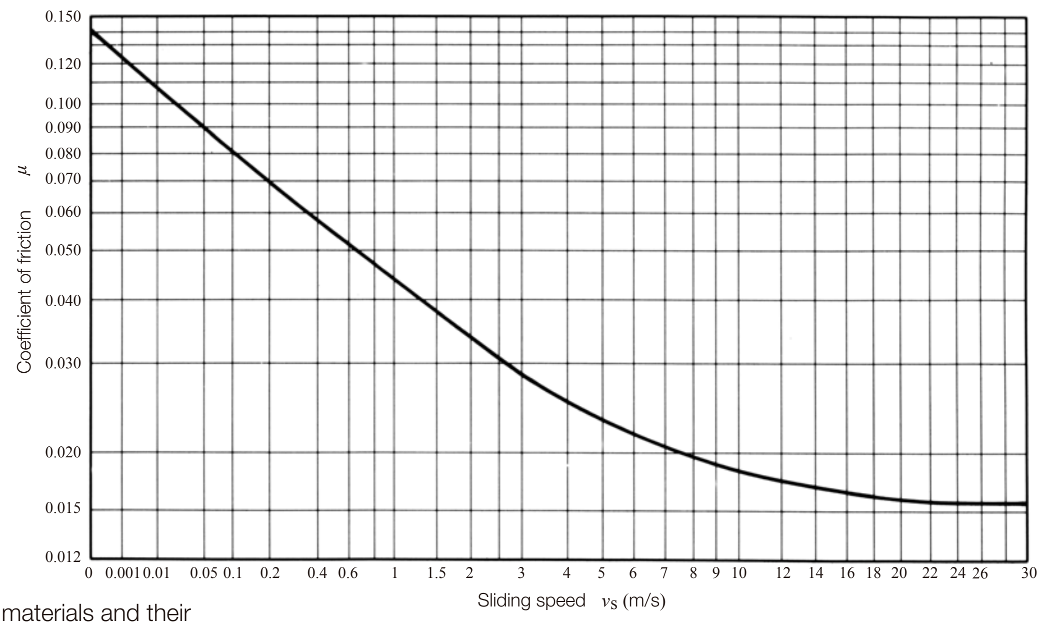


Table 10.27 Combination of materials and their coefficients of friction, μ

Combination of materials	Values of μ
Cast iron and phosphor bronze	μ in Figure 10.12 times 1.15
Cast iron and cast iron	μ in Figure 10.12 times 1.33
Quenched steel and aluminum alloy	μ in Figure 10.12 times 1.33
Steel and steel	μ in Figure 10.12 times 2.00

③ Coefficient of friction, μ

The coefficient of friction, μ, varies as sliding speed, v_S, changes. The combination of materials is important. For the case of a worm that is carburized and ground, and mated with a phosphorous bronze worm wheel, see Figure 10.12.

(2) Calculation of Allowable Load to Surface Durability

(2) -1 Calculation of Basic Load

Provided dimensions and materials of the worm gear pair are known, the allowable load is as follows:

Allowable tangential force F_{tlim} (kgf)

$$F_{\text{tlim}} = 3.82 K_v K_n S_{\text{clim}} Z\, d_{02}^{0.8} m_a \frac{Z_L Z_M Z_R}{K_C} \qquad (10.54)$$

Allowable worm wheel torque, T_{2lim} (kgf･m)

$$T_{\text{2lim}} = 0.00191 K_v K_n S_{\text{clim}} Z\, d_{02}^{1.8} m_a \frac{Z_L Z_M Z_R}{K_C} \qquad (10.55)$$

(2) -2 Calculation of Equivalent Load

Please note that in such cases, where the starting torque is not more than 200% of the rated torque (NOTE 1) and the frequency of starting is less than twice per hour, are regarded as 'no impact'. In all other cases, the equivalent load is to be calculated and compared to the basic load.

Equivalent load is then converted to an equivalent tangential force, F_{te} (kgf)

$$F_{\text{te}} = F_t K_h K_s \qquad (10.56)$$

and equivalent worm wheel torque, T_{2e} (kgf･m)

$$T_{2e} = T_2 K_h K_s \qquad (10.57)$$

[NOTE 1]

Rated torque denotes the torque on the worm wheel when the motor (or loader) is operating at rated load.

(2) -3 Determination of Load

① Under no impact condition, to have life expectancy of 26,000 hours, the following relationships must be satisfied:

$$F_t \leqq F_{\text{tlim}} \text{ or } T_2 \leqq T_{\text{2lim}} \qquad (10.58)$$

② For all other conditions:

$$F_{\text{te}} \leqq F_{\text{tlim}} \text{ or } T_{2e} \leqq T_{\text{2lim}} \qquad (10.59)$$

NOTE : If load is variable, the comprehensive load, T_{2C}, should be used as the criterion.

(3) Determination of Factors

(3) -1 Facewidth of Worm Wheel, b_2 (mm)

The facewidth of worm wheel, b_2, is defined as in Figure 10.13.

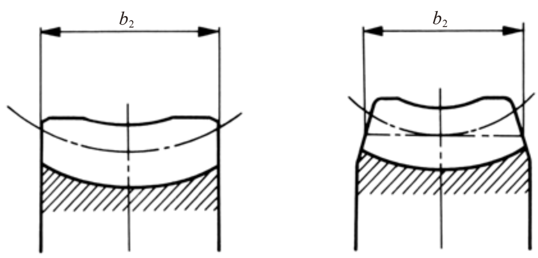


Fig. 0.13 Facewidth of worm wheel, b_2

(3) -2 Zone factor, Z

① If $b_2 < 2.3 m_a \sqrt{Q+1}$, then

$$Z = (\text{Basic zone factor}) \times \frac{b_2}{2 m_a \sqrt{Q+1}}$$

② If $b_2 \geqq 2.3 m_a \sqrt{Q+1}$, then

$$Z = (\text{Basic zone factor}) \times 1.15$$

(10.60)

Table 10.28 Basic zone factors

z_w \ Q	7	7.5	8	8.5	9	9.5	10	11	12	13	14	17	20
1	1.052	1.065	1.084	1.107	1.128	1.137	1.143	1.160	1.202	1.260	1.318	1.402	1.508
2	1.055	1.099	1.144	1.183	1.214	1.223	1.231	1.250	1.280	1.320	1.360	1.447	1.575
3	0.989	1.109	1.209	1.260	1.305	1.333	1.350	1.365	1.393	1.422	1.442	1.532	1.674
4	0.981	1.098	1.204	1.301	1.380	1.428	1.460	1.490	1.515	1.545	1.570	1.666	1.798

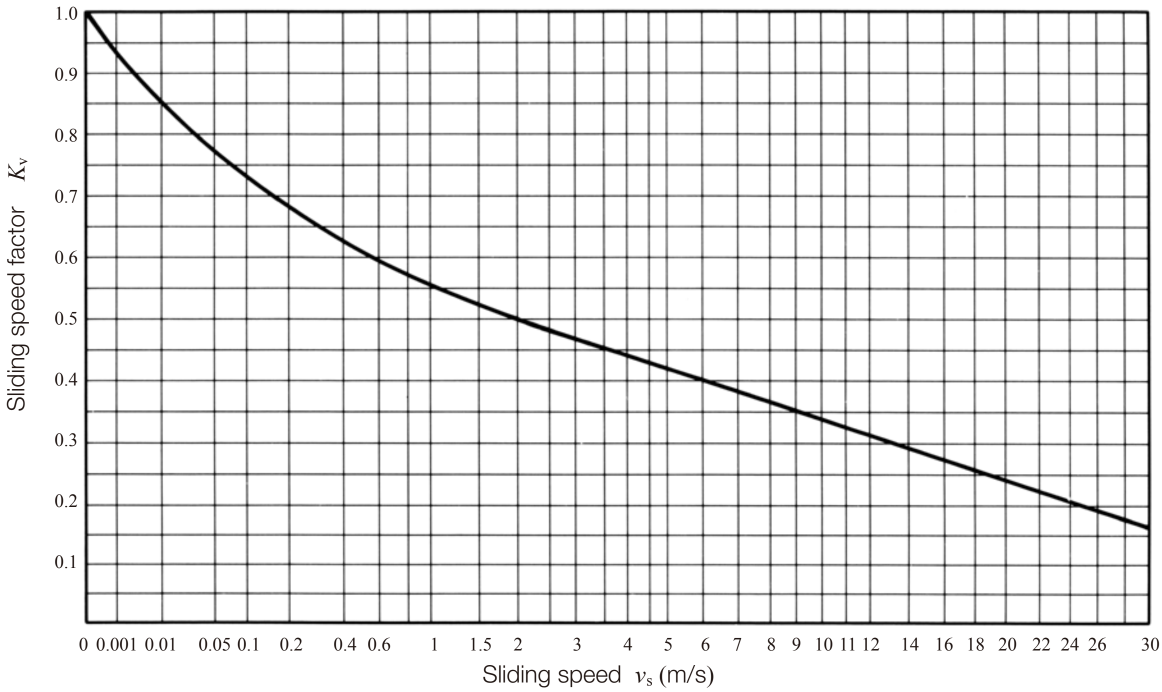
Where Q : Diameter factor $= \dfrac{d_{01}}{m_a}$

z_w : Number of worm threads

(3) -3 Sliding Speed Factor, K_v

Sliding speed factor, K_v, is obtainable from Figure 10.14, where the abscissa is the sliding speed, v_s.

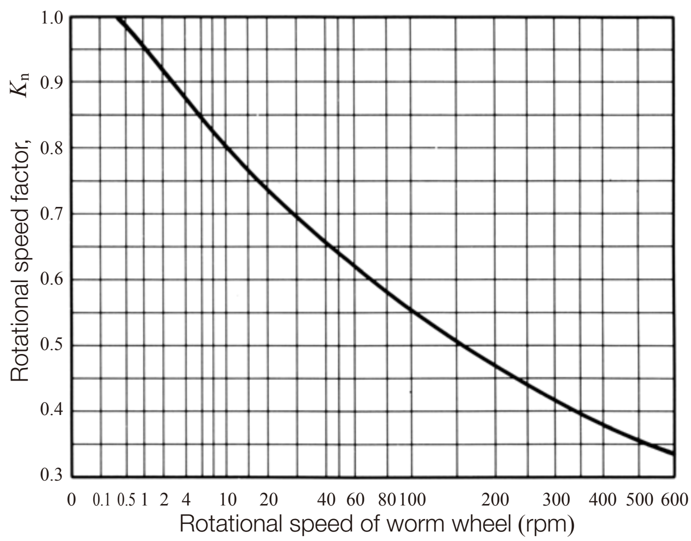
Fig. 10.14 Sliding speed factor, K_v



(3) -4 Rotational Speed Factor, K_n

The rotational speed factor, K_n, is presented in Figure 10.15 as a function of the worm wheel's rotational speed, n_2 (rpm).

Fig. 10.15 Rotational speed factor, K_n



(3) -5 Lubricant Factor, Z_L

Let Z_L = 1.0 if the lubricant is of proper viscosity and has extreme-pressure additives.

Some bearings in worm gearboxes may need a low viscosity lubricant. Then Z_L is to be less than 1.0. The recommended kinetic viscosity of lubrication is given in Table 10.29.

Table 10.29 Recommended kinetic viscosity of lubricant

Unit：cSt/37.8℃

Operating lubricant temperature		Sliding speed m/s		
Highest operating temperature	Lubricant temperature at start of operation	Less than 2.5	2.5 to 5	5 or more
0℃ to less than 10℃	−10℃ to less 0℃	110 ～ 130	110 ～ 130	110 ～ 130
	More than 0℃	110 ～ 150	110 ～ 150	110 ～ 150
10℃ to less than 30℃	More than 0℃	200 ～ 245	150 ～ 200	150 ～ 200
30℃ to less than 55℃	More than 0℃	350 ～ 510	245 ～ 350	200 ～ 245
55℃ to less than 80℃	More than 0℃	510 ～ 780	350 ～ 510	245 ～ 350
80℃ to less than 100℃	More than 0℃	900 ～ 1100	510 ～ 780	350 ～ 510

(3) -6 Lubrication Factor, Z_M

The lubrication factor, Z_M, is obtained from Table 10.30.

Table 10.30 Lubrication factor, Z_M

Sliding speed m/s	Less than10	10 to 14	14 or more
Oil bath lubrication	1.0	0.85	—
Forced circulation lubrication	1.0	1.0	1.0

(3) -7 Surface Roughness Factor, Z_R

The surface roughness factor, Z_R, is concerned with resistance to pitting of the working surfaces of the teeth. Since there is insufficient knowledge about this phenomenon, the factor is assumed to be 1.0.

$$Z_R = 1.0 \tag{10.61}$$

It should be noted that for Equation (10.61) to be applicable, surface roughness of the worm and worm wheel must be less than 3S and 12S respectively. If either is rougher, the factor is to be adjusted to a smaller value.

(3) -8 Tooth Contact Factor, K_C

Quality of tooth contact will affect load capacity dramatically. Since it is difficult to prescribe the tooth contact factor, it is usually regarded that K_C for Class A of JIS B 1741 is 1.0.

$$K_C = 1.0 \tag{10.62}$$

For Class B and C, K_C should be more than 1.0.

Table 1.31 gives the general values of K_C depending on the JIS tooth contact class.

Table 10.31 Classes of tooth contact and general values of tooth contact factor, K_C

Class	Proportion of tooth contact: Axial direction	Proportion of tooth contact: Radial direction	K_C
A	More than 50% of effective length of flank line	More than 40% of working tooth depth	1.0
B	More than 35% of effective length of flank line	More than 30% of working tooth depth	1.3 ～ 1.4
C	More than 20% of effective length of flank line	More than 20% of working tooth depth	1.5 ～ 1.7

(3) -9 Starting Factor, K_S

When starting torque is less than 200% of rated torque, K_S factor is per Table 10.32.

Table 10.32 Starting factor, K_S

Starting frequency per hour	Less than 2	2 ～ 5	5 ～ 10	more than 10
K_S	1.0	1.07	1.13	1.18

(3) -10 Time / Duty Factor, K_h

The time duty factor, K_h, is a function of the desired life and the impact environment. See Table 10.33. The expected lives in between the numbers shown in Table 10.33 can be interpolated.

Table 10.33 Time/duty factor, K_h

Impact from prime mover	Expected life	K_h Impact from load: Uniform load	Medium impact	Strong impact
Uniform load (Motor, turbine, hydraulic motor)	1500 hours	0.80	0.90	1.0
	5000 〃	0.90	1.0	1.25
	26000 〃 (1)	1.0	1.25	1.50
	60000 〃	1.25	1.50	1.75
Light impact (Multicylinder engine)	1500 hours	0.90	1.0	1.25
	5000 〃	1.0	1.25	1.50
	26000 〃 (1)	1.25	1.50	1.75
	60000 〃	1.50	1.75	2.0
Medium impact (Single cylinder engine)	1500 hours	1.0	1.25	1.50
	5000 〃	1.25	1.50	1.75
	26000 〃 (1)	1.50	1.70	2.0
	60000 〃	1.75	2.0	2.25

NOTE(1) For a machine that operates 10 hours a day, 260 days a year, this number corresponds to ten years of operating life.

(3) -11 Allowable Stress Factor, S_{clim}

Table 10.34 presents the allowable stress factor, S_{clim}, for various material combinations. Note that the table also specifies governing limits of sliding speed, which must be adhered to if scoring is to be avoided.

Table 10.34 Allowable stress factor for surface durability, S_{clim}

Material of worm wheel	Material of worm	S_{clim}	Sliding speed limit before scoring (1) m/s
Phosphor bronze centrifugal casting	Alloy steel carburized & quenched	1.55	30
	Alloy steel H_B400	1.34	20
	Alloy steel H_B250	1.12	10
Phosphor bronze chilled casting	Alloy steel carburized & quenched	1.27	30
	Alloy steel H_B400	1.05	20
	Alloy steel H_B250	0.88	10
Phosphor bronze sand molding or forging	Alloy steel carburized & quenched	1.05	30
	Alloy steel H_B400	0.84	20
	Alloy steel H_B250	0.70	10
Aluminum bronze	Alloy steel carburized & quenched	0.84	20
	Alloy steel H_B400	0.67	15
	Alloy steel H_B250	0.56	10
Brass	Alloy steel H_B400	0.49	8
	Alloy steel H_B250	0.42	5
Flake graphite ductile cast iron	Flake graphite ductile cast iron but with a higher hardness than the worm wheel	0.70	5
Cast iron(Perlitic)	Phosphor bronze casting and forging	0.63	2.5
	Cast iron but with a higher hardness than the worm wheel	0.42	2.5

NOTE(1) The value indicates the maximum sliding speed within the limit of the allowable stress factor, S_{clim}. Even when the allowable load is below the allowable stress level, if the sliding speed exceeds the indicated limit, there is danger of scoring gear surfaces.

(4) Examples of Calculation

Worm gear pair design details

No.	Item	Symbol	Unit	Worm	Worm wheel
1	Axial module	m_a	mm	2	
2	Normal pressure angle	α_n	Degree	20°	
3	No. of threads・No. of teeth	z_w・z_2		1	40
4	Pitch diameter	d_0	mm	28	80
5	Reference cylinder lead angle	γ_0	Degree	4.08562°	
6	Diameter factor	Q		14	—
7	Facewidth	b	mm	( )	20
8	Manufacturing method			Grinding	Hobbing
9	Surface roughness			3.2S	12.5S
10	Rotational speed	n	rpm	1500	37.5
11	Sliding speed	v_s	m/s	2.205	
12	Material			S45C	AℓBC2
13	Heat treatment			Induction hardening	—
14	Surface hardness			H_S 63 − 68	—

Surface durability factors and allowable force

No.	Item	Symbol	Unit	Worm wheel
1	Axial module	m_a	mm	2
2	Worm wheel pitch diameter	d_{02}		80
3	Zone factor	Z		1.5157
4	Sliding speed factor	K_v		0.49
5	Rotational speed factor	K_n		0.66
6	Lubricant factor	Z_L		1.0
7	Lubrication factor	Z_M		1.0
8	Surface roughness factor	Z_R		1.0
9	Tooth contact factor	K_C		1.0
10	Allowable stress factor	S_{clim}		0.67
11	Allowable tangential force	F_{tlim}	kgf	83.5

11 Design of Plastic Gears

11.1 The Properties of MC Nylon and Duracon

MC is the abbreviation of 'MONO CAST'. MC is essentially Polyamide 6 (Nylon).

Duracon is a crystalline thermoplastic otherwise called an acetal copolymer. It is one of the most popular engineering plastics.

The characteristics of these plastics are:

- ○ Ability to operate with minimum or no lubrication, due to their inherent lubricity.
- ○ Quietness of operation.
- ○ Lightweight. Excellent resistance to organic chemicals and alkalies.

On the other hand, these plastics are subject to greater dimensional instabilities, due to their larger coefficient of thermal expansion and moisture absorption. These should be taken into consideration when plastic parts are designed. Therefore the design engineer should be familiar with the limitations of plastic gears. It is usual that plastic gears are brought into use after a practical test.

(1) Mechanical Properties

Indicated in Table 11.1 are the mechanical properties under standardized test conditions. In regards to these mechanical properties, the strength tends to become less if the temperature rises.

Table11.1 Mechanical Properties of MC Nylon and Acetal Copolymer

Properties	Testing method ASTM	Unit	MC Nylon		Acetal Copolymer
			MC901	MC602ST	
Specific gravity	D −792	—	1.16	1.23	1.41
Tensile strength	D −638	MPa	96	96	61
Elongation	D −638	%	30	15	40
Tensile elastic modulus	D −638	MPa	3432	—	2824
Compression (Yield point)	D −695	MPa	103	—	—
Compressive strength (5% deformation point)	D −695	MPa	95	115	103 ※
Compressive elastic modulus	D −695	MPa	3530	4640	2700
Bending strength	D −790	MPa	110	140	89
Bending elastic modulus	D −790	MPa	3530	4640	2589
Poisson's ratio	—	—	0.4	—	0.35
Rockwell hardness	D −785	*R* scale	120	120	119
Shearing strength	D −732	MPa	70.9	—	54.9

NOTE 1 The data shown in the table are MC Nylon reference values measured at absolute dry.
NOTE 2 For Acetal Copolymer, compressive strength is “(10% deformation point)”.

(2) Thermal Properties

Compared to steel, plastic materials have larger dimensional changes from temperature change. Thermal properties of MC Nylon and Acetal Copolymer are indicated in Table 11.2.

Table11.2 Thermal Properties of MC Nylon and Acetal Copolymer

Properties	Testing method ASTM	Unit	MC Nylon		Acetal Copolymer
			MC901	M602ST	
Thermal conductivity	*C*−177	W/(m・k)	0.23	0.44	0.23
Coeff. of liner thermal expansion	*D*−696	10^{-5} / ℃	9.0	6.5	9.0
Specific heat	−	kJ/(kg・K)	1.67	—	1.46
Thermal deformation teperature 1.820MPa	*D*−648	℃	200	200	110
Thermal deformation temperature 0.445MPa	*D*−648	℃	215	215	158
Continuous working temperature	−	℃	120	150	95
Melting point	−	℃	222	222	165

NOTE 1 The data shown in the table are MC Nylon reference values measured at absolute dry.
NOTE 2 For use in low temperatures, consider the brittle temperature (-30C to –50C degrees) and determine in accordance with your experiences or tests performed.

◆ **Calculation Example for the Dimensional Change in a MC Nylon (MC901) Rack**

Assumed Product Model No. ：PR2-1000（Total Length: 1010 mm）
Assumed Condition Before Use:
• Atmospheric Temperature: 20℃ = Product Temperature: 20℃
• Total Length 1010 mm
Assumed Increase of Temperature
• 20℃ →40℃ Rise by 20℃
Coefficient of Linear Thermal Expansion:
• 9×10^{-5}/℃
Calculation Example:
Dimensional Change = Coefficient of linear thermal expansion × Length × Temperature difference
= 910^{-5}/℃ × 1010 mm × 20℃ (Temperature difference)
= 1.818 mm
This calculation indicates that a MC Nylon-made PR2-1000 Rack (Total length: 1010 mm) lengthens by 1.8 mm after a 20℃ temperature rise.

(3) Water Absorption Property

Mechanical properties and thermal properties of plastics deteriorate when plastics absorb moisture.

Table 11.3 indicates the water and moisture absorption properties of MC Nylon and Duracon.

Table11.3 Water and moisture absorption properties of MC Nylon and Duracon

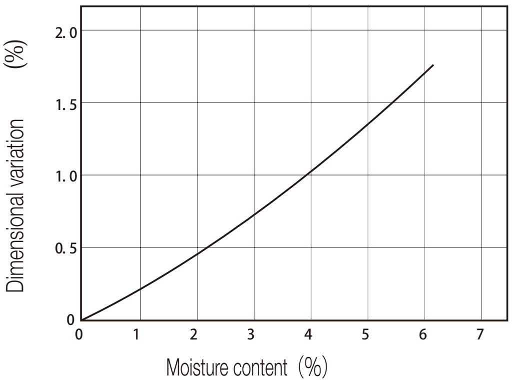
Conditions	Testing method ASTM	Unit	Nylon MC901	Durcon M90
Rate of water absorption (at room temp. in water, 24 hrs.)	$D-570$	%	0.8	0.22
Saturation absorption value (in water)			6.0	0.80
Saturation absorption value (in air, room temp.)			2.5 − 3.5	0.16

NOTE 1 As for 1.MC602ST, the rate of water absorption is approx. 90% of MC901.

Compared with MC Nylon, Duracon has less water absorbing property.

Dimensions of MC nylon gears change with moisture content. This may cause the sizes to vary from the time of purchase to the time of usage. The following figure and the chart show the moisture content and its effect on the dimensions of MC901 Nylon.

Fig.11.1 Moisture Content vs. Dimensional Variation of MC901



◆ Calculation Example for Dimensional Expansion in MC Nylon (MC901) Rack

Assumed Product Model No. : PR2-1000 (Total Length: 1010 mm)

Assumed Conditions Before Use
- With Water Absorption Rate at 1%
- Total Length: 1010 mm

Assumed Conditions After Swelling

Assumed when the water absorption rate increases to 3% at normal temperature.

Calculation Example

① From the data in Figure 11.1;
- It is determined that the dimensional expansion is 0.2%, as the water absorption rate is 1% before use.
- It is determined that the dimensional expansion is 0.75%, as the water absorption rate is 3% after swelling.

② The increment is calculated as follows:

0.75% − 0.2% = 0.55%

③ As total length is 1010 mm, and the dimensional expansion is determined as below;

1010 mm×0.55% = 5.555 mm

(4) Antichemical corrosion property

MC Nylon

Nylon MC901 has almost the same level of antichemical corrosion property as nylon resins. In general, it has a better antiorganic solvent property, but has a weaker antiacid property. The properties are as follows:

- For many nonorganic acids, even at low concentration at normal temperature, it should not be used without further tests.
- For nonorganic alkali at room temperature, it can be used to a certain level of concentration.
- For the solutions of nonorganic salts, it will be all right to apply them to a fairly high level of temperature and concentration.
- It has better antiacid ability and stability in organic acids than in nonorganic acids, except for formic acid.
- It is stable at room temperature in organic compounds of ester series and ketone series.
- It is stable at room temperature in aromatics.
- It is also stable in mineral oil, vegetable oil and animal oil, at room temperature.

Table 11.4 lists antichemical corrosion properties of Nylon resin. Please note that the data mentioned might differ depending on usage conditions, so pre-testing should be performed.

Table11.4 Chemical Resistance Properties of MC Nylon

(○ Hardly affected △ Possible to use under certain conditions ×Not suitable for use)

Diluted hydrochloric acid	△	Methyl acetate	○	Nitrobenzene	○
Concentrated hydrochloric acid	×	Ethyl acetate	○	Salicylic acid	○
Diluted sulfuric acid	△	Sodium acetate	○	Diduthylphthalate	○
Concentrated sulfuric acid	×	Aceton	○	Synchrohexane	○
Diluted nitric acid	△	Methyl acetate	○	Synchrohexanol	○
Concentrated nitric acid	×	Formaldehyde	○	Tetrahydrofuran	○
Diluted phosphoric acid	△	Acetaldehyde	○	(Epsilon)-caprolactam	○
Sodium hydroxide(50%)	○	Ether family	○	Petroleum ether	○
Ammonia water(10%)	○	Acetamide	○	Gasoline	○
Ammonia gas	○	Ethylenediamine	○	Diesel oil	○
Saline solution(10%)	○	Acrylnitrile	○	Lubricant oil	○
Potassium chloride	○	Carbon tetrachloride	○	Mineral oil	○
Calcium chloride	○	Ethylene chloride	○	Castor oil	○
Ammonium chloride	○	Ethylene chlorohydrin	○	Linseed oil	○
Sodium hypochlorite	×	Trichlorethylene(Tri-clene)	○	Silicon oil	○
Sodium sulfate	○	Benzene	○	Edible fat	○
Sodium thiosulfate	○	Toluene	○	Tallow	○
Sodium bisulfate	○	Phenol	△	Butter	○
Cupric sulfate	○	Aniline	△	Milk	○
Potassium dichromate (5%)	○	Benzaldehyde	△	Grape wine	○
Potassium permanganate	△	Benzonic acid	△	Fruit juice	○
Sodium carbonate	○	Chlorobenzene	○	Carbonate drink	○

Duracon

One of the outstanding features of Duracon is excellent resistance to organic chemicals and alkalies. However, it has the disadvantage of having a limited number of suitable adhesives. Its main properties are:

- Excellent resistance against inorganic chemicals. However, it will be corroded by strong acids such as nitric acid, hydrochloric acid and sulfuric acid.
- Household chemicals, such as synthetic detergents, have almost no effect on Duracon.
- Does not deteriorate even under long term operation in high temperature lubricating oil, except for some additives in high grade lubricants.
- With grease, it behaves the same as with oil lubricants.

In order to acquire knowledge about the resistance against other chemicals, plastic manufacturers' technical information manuals should be consulted.

11.2 Strength of Plastic Gears

(1) Bending strength of spur gears

MC Nylon

The allowable tangential force F (kgf) at the pitch circle of a MC Nylon spur gear can be obtained from the Lewis formula.

$$F = myb\sigma_b f \text{ (kgf)} \tag{11.1}$$

Where
m : Module (mm)
y : Tooth profile factor at pitch point (See Table 11.5)
b : Facewidth (mm)
σ_b : Allowable bending stress (kgf/mm²) (See Figure 11.2)
f : Speed factor (See Table 11.6)

Fig.11.2 Allowable bending stress σ_b

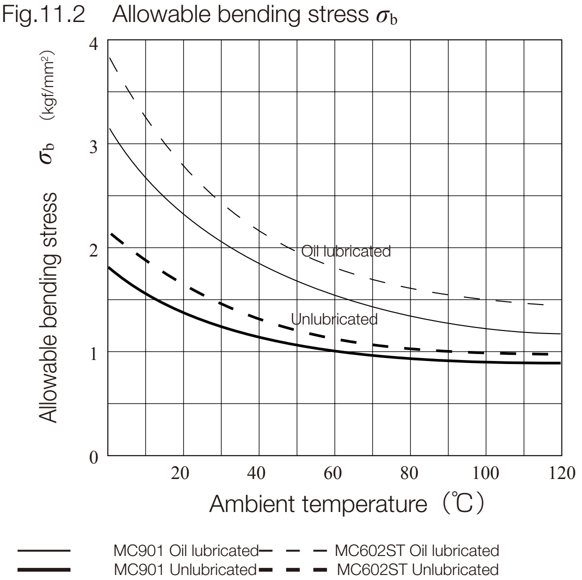


Table 11.5 Tooth profile factor y

No. of teeth	Tooth profile factor		
	14.5°	20°Full depth tooth	20°Stub tooth
12	0.355	0.415	0.496
14	0.399	0.468	0.540
16	0.430	0.503	0.578
18	0.458	0.522	0.603
20	0.480	0.544	0.628
22	0.496	0.559	0.648
24	0.509	0.572	0.664
26	0.522	0.588	0.678
28	0.535	0.597	0.688
30	0.540	0.606	0.698
34	0.553	0.628	0.714
38	0.556	0.651	0.729
40	0.569	0.657	0.733
50	0.588	0.694	0.757
60	0.604	0.722	0.774
75	0.613	0.735	0.792
100	0.622	0.757	0.808
150	0.635	0.779	0.830
300	0.650	0.801	0.855
Rack	0.660	0.823	0.881

Table11.6 Speed factor, f

Lubrication	Tangential speed m/s	Factor
Oil lubricated	Below 12	1.0
	More than 12	0.85
Unlubricated	Below 5	1.0
	More than 5	0.7

Duracon

The allowable tangential force F (kgf) at the pitch circle of a Duracon 90 spur gear can be obtained from the Lewis formula.

$$F = myb\sigma_b \tag{11.2}$$

Where
m : Module (mm)
y : tooth profile factor at pitch point (See Table 11.5)
b : Facewidth (mm)
σ_b : Allowable bending stress (kgf/mm²)

The allowable bending stress σ_b can be obtained from:

$$\sigma_b = \sigma_b \frac{K_V K_T K_L K_M}{C_S} \tag{11.3}$$

Where σ_b : Maximum allowable bending stress under standard condition (kgf/mm2) See Figure 11.3
C_S : Working factor (See Table 11.7)
K_V : Speed factor (See Figure 11.4)
K_T : Temperature factor (See Figure 11.5)
K_L : Lubrication factor (See Table 11.8)
K_M : Material factor (See Table 11.9)

Fig.11.3 Maximum allowable bending stress under standard conditions, σ_b

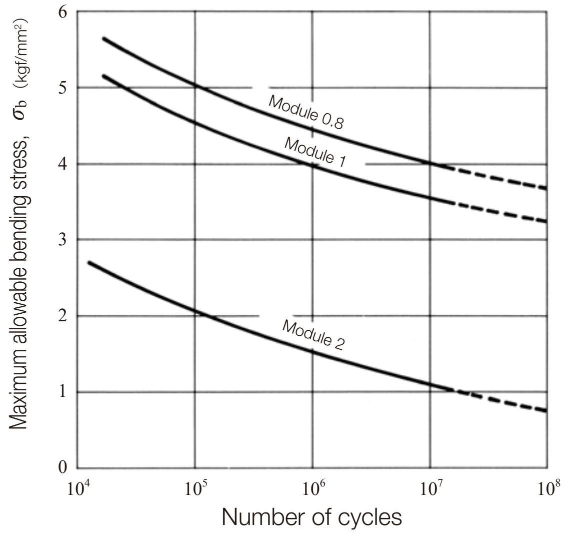


Fig.11.4 Speed factor, K_V

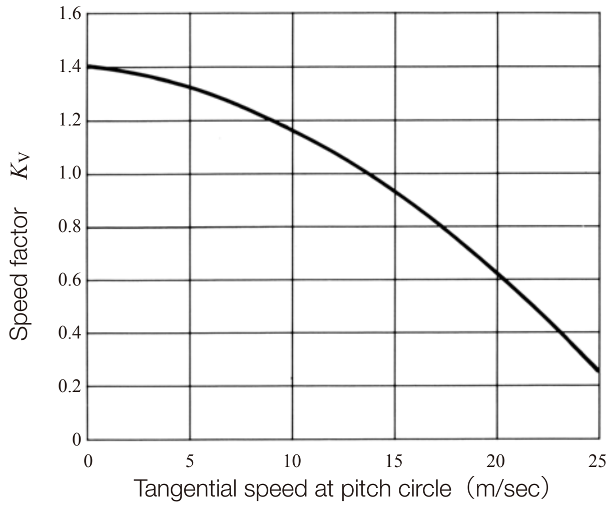


Fig. 11.5 Temperature factor, K_T

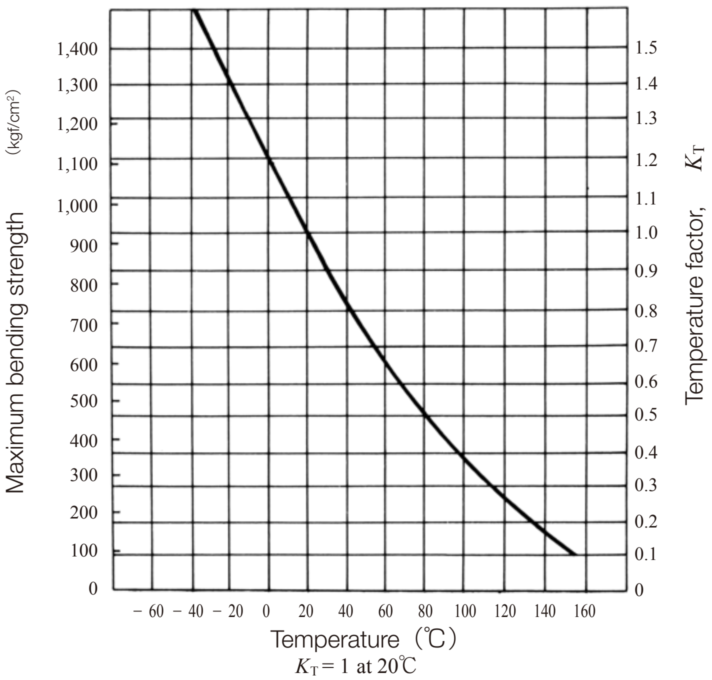


Table 11.7 Working factor, C_S

Types of load	Daily operating hours			
	24 hrs./day	8~10hrs./day	3 hrs./day	0.5 hr./day
Uniform load	1.25	1.00	0.80	0.50
Light impact	1.50	1.25	1.00	0.80
Medium impact	1.75	1.50	1.25	1.00
Heavy impact	2.00	1.75	1.50	1.25

Table 11.8 Lubrication factor, K_L

Lubrication	K_L
Initial grease lubrication	1
Continuous oil lubrication	1.5 − 3.0

Table 11.9 Material factor, K_M

Material combination	K_M
Duracon with metal	1
Duracon with duracon	0.75

Application Notes

In designing plastic gears, the effects of heat and moisture must be given careful consideration. The related problems are:

① Backlash

Plastic gears have larger coefficients of thermal expansion. Also, they have an affinity to absorb moisture and swell. Good design requires allowance for a greater amount of backlash than for metal gears.

② Lubrication

Most plastic gears do not require lubrication. However, temperature rise due to meshing may be controlled by the cooling effect of a lubricant as well as to reduce the of friction. Often, in the case of high-speed rotational speeds, lubrication is critical.

③ Plastic gear with a metal mate

If one of the gears of a mated pair is metal, there will be a heat sink that combats a high temperature rise. The effectiveness depends upon the particular metal, amount of metal mass, and rotational speed.

(2) Surface Durability of Spur Gears

Duracon

Duracon gears have less friction and wear in an oil lubrication condition. However, the calculation of durability must take into consideration a no-lubrication condition. The surface durability using Hertz stress, S_C , (kgf/mm2) is calculated by Equation (11.4).

$$S_C = \sqrt{\frac{F}{bd_{01}} \frac{i+1}{i}} \sqrt{\frac{1.4}{(1/E_1 + 1/E_2)\sin 2\alpha}} \qquad (11.4)$$

Where
F : Tangential force on tooth (kgf)
b : Facewidth (mm)
d_{01} : Pitch diameter of pinion (mm)
i : Gear ratio = z_2/z_1
E : Modulus of elasticity of material (kgf/mm²) (See Figure 11.6)
α : Pressure angle (degree)

Fig.11.6 Modulus of elasticity in bending of duracon

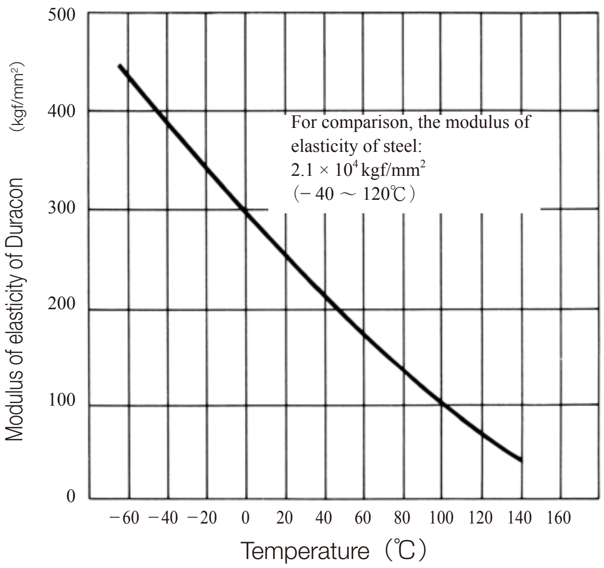
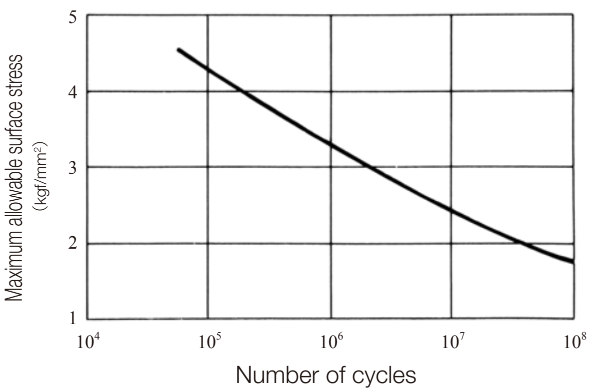


Fig.11.7 Maximum allowable surface stress - spur gears



If the value of Hertz contact stress, S_C, is calculated by Equation (11.4) and the value falls below the curve of Figure 11.7, then it is directly applicable as a safe design. If the calculated value falls above the curve, the Duracon gear is unsafe.
Figure 11.7 is based upon data for a pair of Duracon gears: m = 2, v = 12 m/s, and operating at room temperature. For working conditions that are similar or better, the values in Figure 11.7 can be used.

(3) Bending Strength of Plastic Bevel Gears

MC Nylon

The allowable tangential force, F (kgf) , at the pitch circle is calculated by Equation (11.5).

$$F = m \frac{R_a - b}{R_a} yb\sigma_b f \qquad (11.5)$$

where y : Tooth profile factor at pitch point, which is obtained from Table 11.5 by first computing the number of teeth of equivalent spur gear, z_v, using Equation (11.6).

$$z_v = \frac{z}{\cos\delta_0} \qquad (11.6)$$

R_a : Outer cone distance (mm)
δ_0 : Pitch cone angle (degree)

Other variables may be calculated the same way as for spur gears.

Duracon

The allowable tangential force, F (kgf) , on pitch circle of Duracon bevel gears can be obtained from Equation (11.7).

$$F = m \frac{R_a - b}{R_a} yb\sigma_b \qquad (11.7)$$

Where $\sigma_b = \sigma_b \dfrac{K_V K_T K_L K_M}{C_S}$

y : Tooth profile factor at pitch point, which is obtained from Table 11.5 by computing the equivalent number of teeth via Equation (11.6).

Other variables are obtained by using the equations for Duracon spur gears.

(4) Bending Strength of Worm Wheel

MC Nylon

Generally, the worm is much stronger than the worm wheel. Therefore, it is necessary to calculate the strength of only the worm wheel.

The allowable tangential force F (kgf) at the pitch circle of the worm wheel is obtained from Equation (11.8).

$$F = m_n y b \sigma_b f \text{ (kgf)} \qquad (11.8)$$

Where m_n : Normal module (mm)

y : Tooth profile factor at pitch point, which is obtained from Table 11.5 by first computing the equivalent number of teeth, z_v, using Equation (11.9).

$$z_v = \frac{z}{\cos^3 \gamma_0} \qquad (11.9)$$

Worm meshes have relatively high sliding velocities, which induces a high temperature rise. This causes a sharp decrease in strength and abnormal friction wear. Therefore, sliding speeds must be contained within recommendations of Table 11.10.

Table 11.10 Material combinations and limits of sliding speed

Material of worm	Material of worm wheel	Lubrication condition	Sliding speed
"MC"	"MC"	No lubrication	0.125m/s or less
Steel	"MC"	No lubrication	1 m/s or less
Steel	"MC"	Initial lubrication	1.5 m/s or less
Steel	"MC"	Continuous lubrication	2.5 m/s or less

Sliding speed, v_s, can be obtained from:

$$v_s = \frac{\pi d_1 n_1}{60000 \cos \gamma_0} \text{ (m/s)} \qquad (11.10)$$

Lubrication is vital in the case of plastic worm gear pair, particularly under high load and continuous operation.

(5) Strength of Plastic Keyway

Fastening of a plastic gear to the shaft is often done by means of a key and keyway.

Then, the critical thing is the stress σ (kgf/cm^2) imposed upon the keyway sides. This is calculated by Equation (11.11).

$$\sigma = \frac{2T}{dlh} \text{ (kgf/cm}^2\text{)} \qquad (11.11)$$

T : Transmitted torque (kgf・cm)
d : Diameter of shaft (cm)
l : Effective length of keyway (cm)
h : Depth of keyway (cm)

The maximum allowable surface pressure of MC901 is 200kgf/cm^2, and this must not be exceeded. Also, the keyway's corner must have a suitable radius to avoid stress concentration. The distance from the root of the gear to the bottom of the keyway should be at least twice the tooth depth.

Keyways are not to be used when the following conditions exist:

- Excessive keyway stress
- High ambient temperature
- Large outside diameter gears
- High impact

When above conditions prevail, it is expedient to use a metallic hub in the gear. Then, a keyway may be cut in the metal hub. A metallic hub can be fixed in the plastic gear by several methods:

- Press the metallic hub into the plastic gear, ensuring fastening with a knurl or screw.
- Screw fastened metal discs on each side of the plastic gear.
- Thermofuse the metal hub to the gear.

12 Gear Forces

When the gear mesh transmits power, forces act on the gear teeth. As shown in Figure 12.1, if the Z-axis of the orthogonal 3-axes denotes the gear shaft, forces are defined as follows:
The force that acts in the X-axis direction is defined as the tangential force F_t (N)
The force that acts in the Y-axis direction is defined as the radial force. F_r (N)
The force that acts in the Z-axis direction is defined as the axial force F_x (N) or thrust.
Analyzing these forces is very important when designing gears. In designing a gear, it is important to analyze these forces acting upon the gear teeth, shafts, bearings, etc.

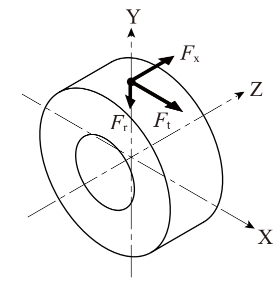


Fig. 12.1 Direction of Forces Acting on a Gear

Table 12.1 presents the equations for forces acting on gears. The unit of torque T and T_1 is N・m.

Table 12.1 Forces Acting upon a Gear

Types of gears		F_t : Tangential force	F_x : Axial force	F_r : Radial force
Spur gear		$F_t = \dfrac{2000T}{d}$	――――	$F_t \tan\alpha$
Helical gear			$F_t \tan\beta$	$F_t \dfrac{\tan\alpha_n}{\cos\beta}$
Straight bevel gear		$F_t = \dfrac{2000T}{d_m}$	$F_t \tan\alpha \sin\delta$	$F_t \tan\alpha \cos\delta$
Spiral bevel gear		d_m is the central reference diameter $d_m = d - b\sin\delta$	When convex surface is working :	
			$\dfrac{F_t}{\cos\beta_m}(\tan\alpha_n \sin\delta - \sin\beta_m \cos\delta)$	$\dfrac{F_t}{\cos\beta_m}(\tan\alpha_n \cos\delta + \sin\beta_m \sin\delta)$
			When concave surface is working :	
			$\dfrac{F_t}{\cos\beta_m}(\tan\alpha_n \sin\delta + \sin\beta_m \cos\delta)$	$\dfrac{F_t}{\cos\beta_m}(\tan\alpha_n \cos\delta - \sin\beta_m \sin\delta)$
Worm gear pair	Worm (Driver)	$F_{t1} = \dfrac{2000T_1}{d_1}$	$F_{t1} \dfrac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$	$F_{t1} \dfrac{\sin\alpha_n}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$
	Worm Wheel (Driven)	$F_{t2} = F_{t1} \dfrac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$	F_{t1}	
Screw gear $\left(\begin{array}{l}\Sigma = 90^\circ \\ \beta = 45^\circ\end{array}\right)$	Driver gear	$F_{t1} = \dfrac{2000T_1}{d_1}$	$F_{t1} \dfrac{\cos\alpha_n \sin\beta - \mu \cos\beta}{\cos\alpha_n \cos\beta + \mu \sin\beta}$	$F_{t1} \dfrac{\sin\alpha_n}{\cos\alpha_n \cos\beta + \mu \sin\beta}$
	Driven gear	$F_{t2} = F_{t1} \dfrac{\cos\alpha_n \sin\beta - \mu \cos\beta}{\cos\alpha_n \cos\beta + \mu \sin\beta}$	F_{t1}	

12.1 Forces in a Parallel Axis Gear Mesh

Figure 12.2 shows forces that act on the teeth of a helical gear. Larger helix angle of the teeth, has larger thrust (axial force). In case of spur gears, no axial force acts on teeth.

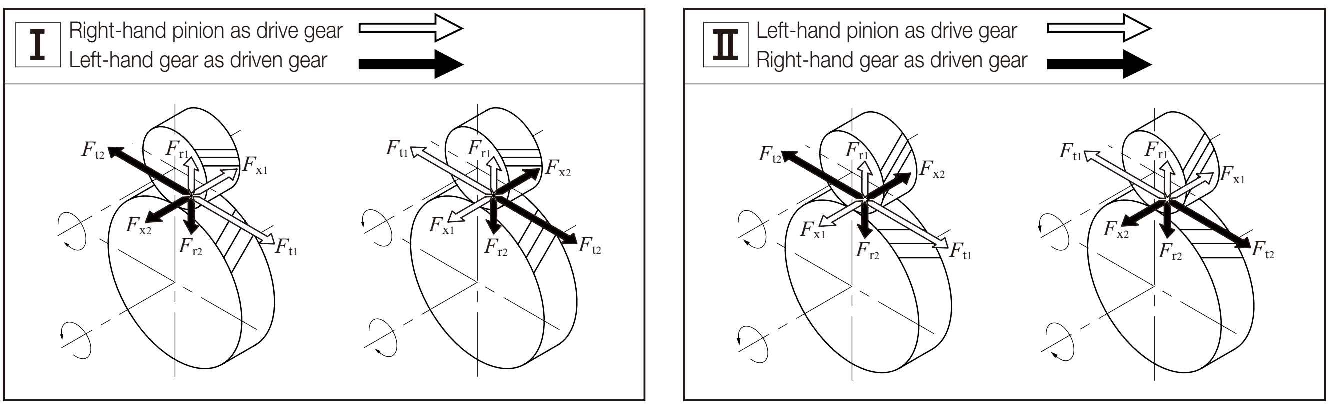


Fig.12.2 Direction of Forces acting on a Helical Gear Mesh

Table 12.2 Calculation Examples (Spur Gear)

No.	Specifications	Symbol	Unit	Formula	Spur Gear Pinion	Spur Gear Gear
1	Module	m	mm	Set Value	2	
2	Normal pressure angle	α_n	Degree		20°	
3	No, of teeth	z	—		20	40
4	Spiral angle	β	Degree		0	
5	Input torque	T_1	N·m		2	—
6	Reference diameter	d	mm	zm	40	80
7	Tangential force	F_t	N	$\frac{2000T}{d}$	100.0	
8	Axial force	F_x		—	0	
9	Radial force	F_r		$F_t \tan\alpha$	36.4	
10	Output torque	T_2	N·m	$\frac{F_t d_2}{2000}$	—	4

Table 12.3 Calculation Examples (Spiral Gear)

No.	Specifications	Symbol	Unit	Formula	Spur Gear Pinion	Spur Gear Gear
1	Transverse module	m_t	mm	Set Value	2	
2	Transverse pressure angle	α_t	Degree		20°	
3	No. of teeth	z	—		20	40
4	Spiral angle	β	Degree		21.5°	
5	Input torque	T_1	N·m		2	—
6	Normal pressure angle	α_n	Degree	$\tan^{-1}(\tan\alpha_t \cos\beta)$	18.70838°	
7	Reference diameter	d	mm	zm_t	40	80
8	Tangential force	F_t	N	$\frac{2000T}{d}$	100.0	
9	Axial force	F_x		$F_t \tan\beta$	39.4	
10	Radial force	F_r		$F_t \frac{\tan\alpha_n}{\cos\beta}$	36.4	
11	Output torque	T_2	N·m	$\frac{F_t d_2}{2000}$	—	4

12.2 Forces in an Intersecting Axis Gear Mesh

In the meshing of a pair of bevel gears with shaft angle $\Sigma = 90^\circ$, the axial force acting on drive gear equals the radial force acting on driven gear. Similarly, the radial force acting on drive gear equals the axial force acting on driven gear.

$$\left.\begin{aligned} F_{x1} &= F_{r2} \\ F_{r1} &= F_{x1} \end{aligned}\right\} \qquad (12.1)$$

(1) Forces in a Straight Bevel Gear Mesh

Figure 12.3 shows how forces act on a straight bevel gear mesh.

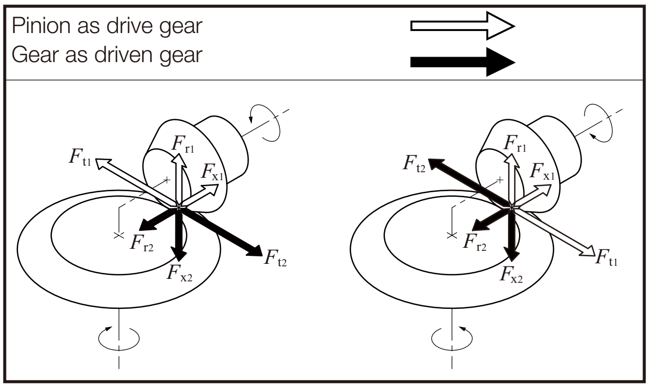


Fig 12.3 Directions of Forces acting on a Straight Bevel Gear Mesh

(2) Forces in a Spiral Bevel Gear Mesh

Spiral bevel gear teeth have convex and concave sides. Depending on which surface the force is acting on, the direction and magnitude changes. They differ depending upon which is driver and which is the driven. Table 12.5 shows the meshing tooth face.

Figure 12.5 shows how the forces act on the teeth of a spiral bevel gear mesh. Negative axial force is the thrust, pushing the two gears together. The bearing must be designed carefully so that it can receive this negative thrust. If there is any axial play in the bearing, it may lead to the undesirable condition of the mesh having no backlash.

Table 12.4 Calculation Examples (Straight Bevel Gear)

No.	Specifications	Symbol	Unit	Formula	Straight Bevel Gear Pinion	Straight Bevel Gear Gear
1	Shaft angle	Σ	Degree	Set Value	90°	
2	Module	m_t	mm		2	
3	Pressure angle	α	Degree		20°	
4	No. of teeth	z	—		20	40
5	Spiral angle	β	Degree		0°	
6	Facewidth	b	mm		15	
7	Input torque	T_1	N·m		1.6646	—
8	Reference diameter	d	mm	zm	40	80
9	Reference cone angle	$\delta_1 \cdot \delta_2$	degree	$\tan^{-1}\left(\frac{z_1}{z_2}\right)$ $\Sigma - \delta_1$	26.56505	63.43495
10	Center reference diameter	d_m	mm	$d - b\sin\delta$	33.292	66.584
11	Tangential force	F_t	N	$\frac{2000T}{d_m}$	100.0	
12	Axial force	F_x		$F_t \tan\alpha \sin\delta$	16.3	32.6
13	Radial force	F_r		$F_t \tan\alpha \cos\delta$	32.6	16.3
14	Output torque	T_2	N·m	$\frac{F_t d_{m2}}{2000}$	—	3.329

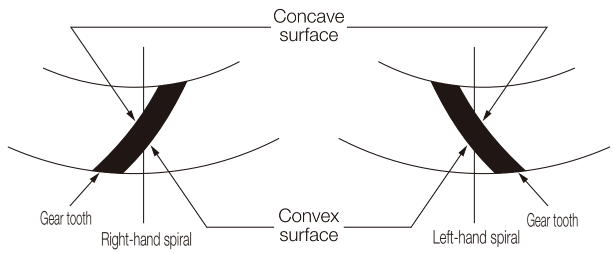
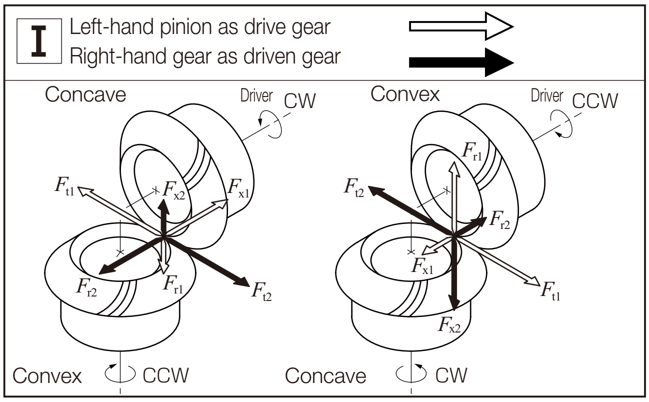


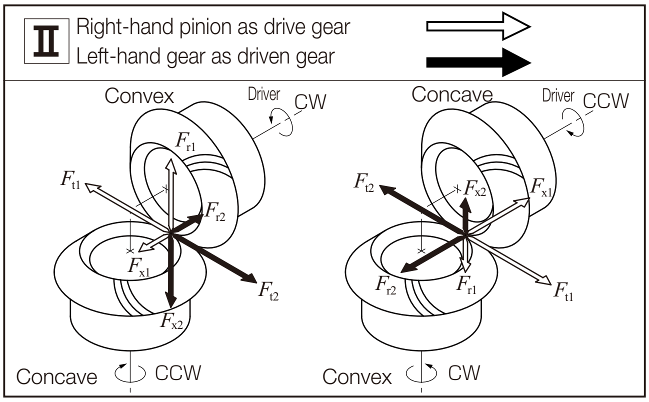
Fig.12.4 Convex surface and concave surface of a spiral bevel gear

Table 12.5 Meshing Tooth Face

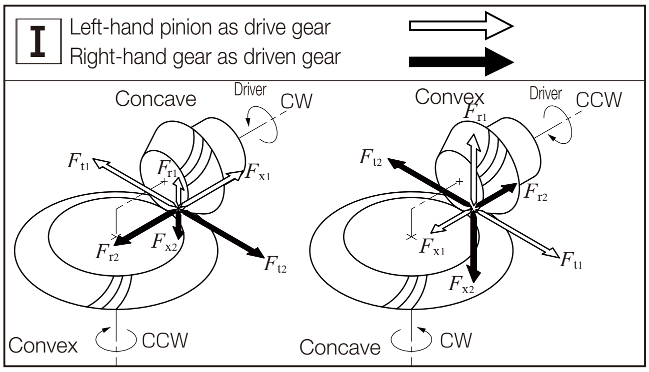
Drive Gear Spiral-hand	Drive Gear Rotational direction	Drive Gear Mashing tooth face	Driven Gear Mashing tooth face	Driven Gear Rotational direction	Driven Gear Spiral-hand
L	CW	Concave	Convex	CCW	R
	CCW	Convex	Concave	CW	
R	CW	Convex	Concave	CCW	L
	CCW	Concave	Convex	CW	

$\Sigma = 90°,\ \alpha_n = 20°,\ \beta_m = 35°,\ z_2/z_1 < 1.57357$





$\Sigma = 90°,\ \alpha_n = 20°,\ \beta_m = 35°,\ z_2/z_1 \geqq 1.57357$



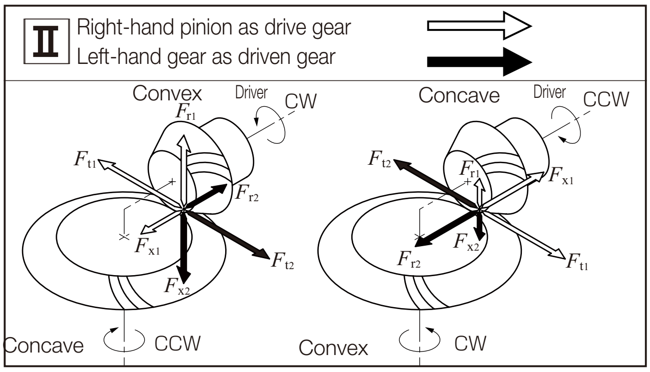


Fig.12.5 The Directions of Forces Carried by Spiral Bevel Gears

Table 12.6 Calculation Examples (Spiral Bevel Gears)

No.	Specifications	Symbol	Unit	Formula		Spiral Bevel Gears Pinion	Spiral Bevel Gears Gear
1	Shaft angle	Σ	Degree	Set Value		90°	
2	Transverse module	m_t	mm			2	
3	Pressure angle	α_n	Degree			20°	
4	No. of teeth	z	—			20	40
5	Spiral angle	β	Degree			35°	
6	Facewidth	b	mm			15	
7	Input torque	T_1	N·m			1.6646	—
8	Reference diameter	d	mm	zm		40	80
9	Reference cone angle	$\delta_1 \cdot \delta_2$	degree	$\tan^{-1}\left(\frac{z_1}{z_2}\right)$	$\Sigma - \delta_1$	26.56505	63.43495
10	Center reference diameter	d_m	mm	$d - b\sin\delta$		33.292	66.584
11	Tangential force	F_t	N	$\frac{2000T}{d_m}$		100.0	
				Contact Face		Convex	Concave
12	Axial force	F_x	N	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n\sin\delta - \sin\beta_m\cos\delta)$	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n\sin\delta + \sin\beta_m\cos\delta)$	– 42.8	71.1
13	Radial force	F_r	N	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n\cos\delta + \sin\beta_m\sin\delta)$	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n\cos\delta - \sin\beta_m\sin\delta)$	71.1	– 42.8
				Contact Face		Concave	Convex
14	Axial force	F_x	N	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n\sin\delta + \sin\beta_m\cos\delta)$	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n\sin\delta - \sin\beta_m\cos\delta)$	82.5	8.4
15	Radial force	F_r	N	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n\cos\delta - \sin\beta_m\sin\delta)$	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n\cos\delta + \sin\beta_m\sin\delta)$	8.4	82.5
16	Output torque	T_2	N·m	——	$\frac{F_t d_{m2}}{2000}$	—	3.329

12.3 Forces in a Nonparallel and Nonintersecting Axes Gear Mesh

(1) Forces in a Worm Gear Pair Mesh

Figure 12.6 shows how the forces act on the teeth of a worm gear pair mesh with a shaft angle $\Sigma = 90°$. Since the power transmission of the worm gear pair mesh has a sliding contact nature, the coefficient of friction on the tooth surface has a great effect on the transmission efficiency η_R and the force acting on the gear mesh.

$$\left.\eta_R = \frac{T_2}{T_1 i} = \frac{\tan\gamma\, F_{t2}}{F_{t1}}\right\} \qquad (12.2)$$

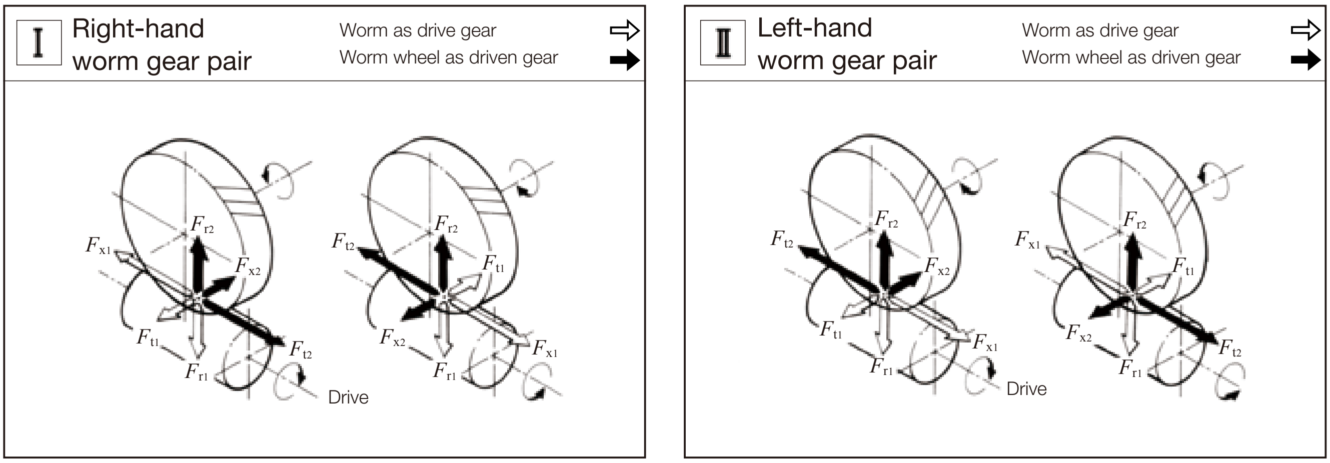


Fig. 12.6 Direction of Forces in a Worm Gear Pair Mesh

Table 12.7 Calculation Examples (Worm Gear Pair)

No.	Specifications	Symbol	Unit	Formula		Worm Gear Pair: Worm	Worm Gear Pair: Wheel
1	Shaft angle	Σ	Degree	Set Value		90°	
2	Axial / transverse module	$m_x \bullet m_t$	mm			2	
3	Normal pressure angle	α_n	Degree			20°	
4	No. of teeth	z	—			1	20
5	Reference diameter (Worm)	d_1	mm			31	—
6	Coefficient of friction	μ	—			0.05	
7	Input torque	T_1	N·m			1.550	—
8	Reference diameter (Wheel)	d_2	mm	—	$z_2 m_t$	—	40
9	Reference cylinder lead angle	γ	Degree	$\tan^{-1}\left(\frac{m_x z_1}{d_1}\right)$		3.69139°	
10	Tangential force	$F_{t1} \bullet F_{t2}$	N	$\frac{2000T_1}{d_1}$	$F_{t1}\frac{\cos\alpha_n\cos\gamma-\mu\sin\gamma}{\cos\alpha_n\sin\gamma+\mu\cos\gamma}$	100.0	846.5
11	Axial force	F_x	N	$F_{t1}\frac{\cos\alpha_n\cos\gamma-\mu\sin\gamma}{\cos\alpha_n\sin\gamma+\mu\cos\gamma}$	F_{t1}	846.5	100.0
12	Radial force	F_r	N	$F_{t1}\frac{\sin\alpha_n}{\cos\alpha_n\sin\gamma+\mu\cos\gamma}$		309.8	
13	Efficiency	η_R	—	$\frac{\tan\gamma\, F_{t2}}{F_{t1}}$		0.546	
14	Output torque	T_2	N·m	——	$\frac{F_{t2}\, d_2}{2000}$	—	16.930

(2) Forces in a Screw Gear Mesh

The forces in a screw gear mesh are similar to those in a worm gear pair mesh.
Figure 12.7 shows the force acts on the teeth of a screw gear mesh with a shaft angle $\Sigma = 90°$, a helix angle = 45°.

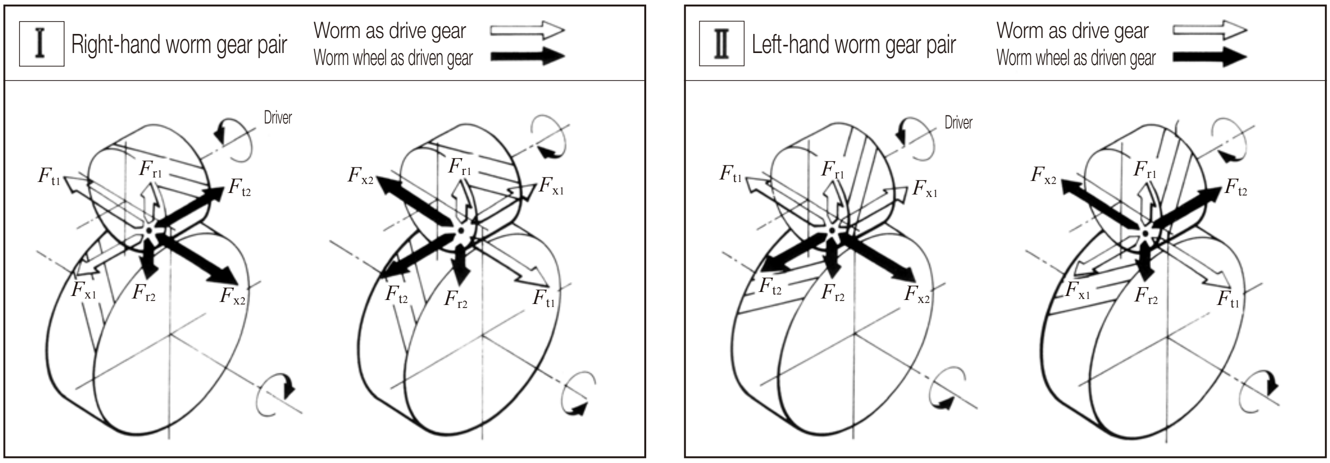


Fig 12.7 Direction of Forces in a Screw Gear Mesh

Table 12.8 Calculation Examples (Screw Gear)

No.	Specifications	Symbol	Unit	Formula		Screw Gear Pinion	Screw Gear Gear
1	Shaft angle	Σ	Degree	Set Value		90°	
2	Normal module	m_n	mm			2	
3	Normal pressure angle	α_n	Degree			20°	
4	No. of teeth	z	—			13	13
5	Spiral angle	β	Degree			45°	
6	Coefficient of friction	μ	—			0.05	
7	Input torque	T_1	N·m			1.838	—
8	Reference diameter	d	mm	$\dfrac{zm_n}{\cos\beta}$		36.770	36.770
9	Tangential force	$F_{t1} \cdot F_{t2}$	N	$\dfrac{2000T_1}{d_1}$	$F_{t1}\dfrac{\cos\alpha_n\sin\beta-\mu\cos\beta}{\cos\alpha_n\cos\beta+\mu\sin\beta}$	100.0	89.9
10	Axial force	F_x	N	$F_{t1}\dfrac{\cos\alpha_n\sin\beta-\mu\cos\beta}{\cos\alpha_n\cos\beta+\mu\sin\beta}$	F_{t1}	89.9	100.0
11	Radial force	F_r	N	$F_{t1}\dfrac{\sin\alpha_n}{\cos\alpha_n\cos\beta+\mu\sin\beta}$		48.9	
12	Efficiency	η	—	$\dfrac{T_2 z_1}{T_1 z_2}$		0.899	
13	Output torque	T_2	N·m	——	$\dfrac{F_{t2} d_2}{2000}$	—	1.653

13 Lubrication of Gears

The purpose of lubricating gears is as follows:

1. Promote sliding between teeth to reduce the coefficient of friction μ.
2. Limit the temperature rise caused by rolling and sliding friction.

To avoid difficulties such as tooth wear and premature failure, the correct lubricant must be chosen.

13.1 Methods of Lubrication

There are three gear lubrication methods in general use:

(1) Grease lubrication.
(2) Splash lubrication (oil bath method).
(3) Forced oil circulation lubrication.

There is no single best lubricant and method. Choice depends upon tangential speed (m/s) and rotating speed (rpm).
At low speed, grease lubrication is a good choice. For medium and high speeds, splash lubrication and forced oil circulation lubrication are more appropriate, but there are exceptions. Sometimes, for maintenance reasons, a grease lubricant is used even with high speed.
Table 13.1 presents lubricants, methods and their applicable ranges of speed.
Grease lubrication can be applied in low speed / low load applications, however, it is important to apply grease periodically, especially for gears of the open-type usage. Since lubricants diminish or become depleted in the long term, periodic checks for oil change or refilling is necessary. Usage of lubricants under improper conditions cause damage to gear teeth. When using gears at high speed / heavy load, or when using easily worn gears such as worms or screw gears, care should be taken in selecting the right type of lubricant; quantity and methods. The proper selection of lubricant is especially important.

Table13.1 —① Ranges of tangential speed (m/s) for spur and bevel gears

No.	Lubrication	Range of tangential speed v (m/s) 0 5 10 15 20 25
1	Grease lubrication	
2	Splash lubrication	
3	Forced oil circultion lubrication	

Table13.1 —② Ranges of sliding speed (m/s) for worm wheels

No.	Lubrication	Range of tangential speed v (m/s) 0 5 10 15 20 25
1	Grease lubrication	
2	Splash lubrication	
3	Forced oil circultion lubrication	

The following is a brief discussion of the three lubrication methods.

(1) Grease Lubrication

Grease lubrication is suitable for any gear system that is open or enclosed, so long as it runs at low speed. There are three major points regarding grease:

◎ Choosing a lubricant with suitable cone penetration.
A lubricant with good fluidity is especially effective in an enclosed system.

◎ Not suitable for use under high load and continuous operation.
The cooling effect of grease is not as good as lubricating oil. So it may become a problem with temperature rise under high load and continuous operating conditions.

◎ Proper quantity of grease
There must be sufficient grease to do the job. However, too much grease can be harmful, particularly in an enclosed system. Excess grease will cause agitation, viscous drag and result in power loss.

(2) Splash Lubrication (Oil Bath Method)

Splash lubrication is used with an enclosed system. The rotating gears splash lubricant onto the gear system and bearings. It needs at least 3m/s tangential speed to be effective. However, splash lubrication has several problems, two of them being oil level and temperature limitation.

① Oil level

There will be excess agitation loss if the oil level is too high. On the other hand, there will not be effective lubrication or ability to cool the gears if the level is too low. Table 13.2 shows guide lines for proper oil level.

Also, the oil level during operation must be monitored, as contrasted with the static level, in that the oil level will drop when the gears are in motion. This problem may be countered by raising the static level of lubricant in an oil pan.

② Temperature limitation

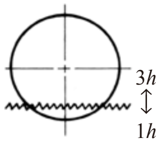
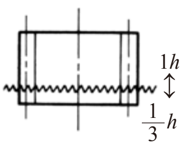
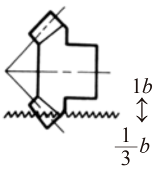
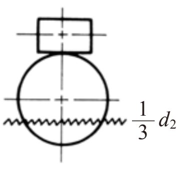
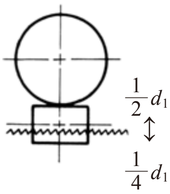
The temperature of a gear system may rise because of friction loss due to gears, bearings and lubricant agitation. Rising temperature may cause one or more of the following problems:

- Lower viscosity of lubricant
- Accelerated degradation of lubricant.
- Deformation of housing, gears and shafts
- Decreased backlash.

New high-performance lubricants can withstand up to 80℃ - 90℃ .

This temperature can be regarded as the limit. If the lubricant's temperature is expected to exceed this limit, cooling fins should be added to the gear box, or a cooling fan incorporated into the system.

Table 13.2 Adequate oil level

Type of gears	Spur gears and helical gears		Bevel gears	Worm gear pair	
Gear orientation	Horizontal shaft	Vertical shaft	(Horizontal shaft)	Worm - above	Worm -below
Oil level Level 0					

0

h = Tooth depth, b = Facewidth, d_2 = Reference diameter of worm wheel, d_1 = Reference diameter of worm

(3) Forced Oil Circulation Lubrication

Forced oil circulation lubrication applies lubricant to the contact portion of the teeth by means of an oil pump. There are drop, spray and oil mist methods of application.

○ Drop Method

An oil pump is used to suck-up the lubricant and then directly drop it on the contact portion of the gears via a delivery pipe.

○ Spray Method

An oil pump is used to spray the lubricant directly on the contact area of the gears.

○ Oil Mist Method

Lubricant is mixed with compressed air to form an oil mist that is sprayed against the contact region of the gears. It is especially suitable for high-speed gearing.

Oil tank, pump, filter, piping and other devices are needed in the forced oil lubrication system. Therefore, it is used only for special high-speed or large gear box applications.

By filtering and cooling the circulating lubricant, the right viscosity and cleanliness can be maintained. This is considered to be the best way to lubricate gears.

13.2 Gear Lubricants

An oil film must be formed at the contact surface of the teeth to minimize friction and to prevent dry metal-to-metal contact. The lubricant should have the properties listed in Table 13.3.

Table 13.3 The properties that lubricant should possess

No.	Properties	Description
1	Correct and proper viscosity	Lubricant should maintain proper viscosity to form a stable oil film at the specified temperature and speed of operation.
2	Antiscoring property	Lubricant should have the property to prevent the scoring failure of tooth surface while under high-pressure of load.
3	Oxidization and heat stability	A good lubricant should not oxidize easily and must perform in moist and high-temperature environment for long duration.
4	Water antiaffinity property	Moisture tends to condense due to temperature change when the gears are stopped. The lubricant should have the property of isolating moisture and water from lubricant
5	Antifoam property	If the lubricant foams under agitation, it will not provide a good oil film. Antifoam property is a vital requirement.
6	Anticorrosion property	Lubrication should be neutral and stable to prevent corrosion from rust that may mix into the oil.

(1) Viscosity of Lubricant

The correct viscosity is the most important consideration in choosing a proper lubricant. The viscosity grade of industrial lubricant is regulated in JIS K 2001. Table 13.4 expresses ISO viscosity grade of industrial lubricants.

Table13.4 ISO viscosity grade of industrial lubricant (JIS K 2001)

ISO Viscosity grade	Kinematic viscosity center value $10^{-6}m^2/s$ (cSt) (40℃)	Kinematic viscosity range $10^{-6}m^2/s$ (cSt) (40℃)
ISO VG 2	2.2	More than 1.98 and less than 2.42
ISO VG 3	3.2	More than 2.88 and less than 3.52
ISO VG 5	4.6	More than 4.14 and less than 5.06
ISO VG 7	6.8	More than 6.12 and less than 7.48
ISO VG 10	10	More than 9.0 and less than 11.0
ISO VG 15	15	More than 13.5 and less than 16.5
ISO VG 22	22	More than 19.8 and less than 24.2
ISO VG 32	32	More than 28.8 and less than 35.2
ISO VG 46	46	More than 41.4 and less than 50.6
ISO VG 68	68	More than 61.2 and less than 74.8
ISO VG 100	100	More than 90.0 and less than 110
ISO VG 150	150	More than 135 and less than 165
ISO VG 220	220	More than 198 and less than 242
ISO VG 320	320	More than 288 and less than 352
ISO VG 460	460	More than 414 and less than 506
ISO VG 680	680	More than 612 and less than 748
ISO VG 1000	1000	More than 900 and less than 1100
ISO VG 1500	1500	More than 1350 and less than 1650
ISO VG 2200	2200	More than 1980 and less than 2420
ISO VG 3200	3200	More than 2880 and less than 3520

(2) Selection of Lubricant

Gear oils are categorized by usage: 2 types for industrial use, 3 types for automobile use, and also classified by viscosity grade. (Table 13.5 – created from the data in JIS K 2219 – 1993: Gear Oils Standards.)

Table 13.5 Types of Gear Oils and the Usage

Type			Usage
For Industrial Usage	1	ISO VG 32 ISO VG 46 ISO VG 68 ISO VG 100 ISO VG 150 ISO VG 220 ISO VG 320 ISO VG 460	Mostly used for lightly loaded enclosed-gears in general type of machines.
	2	ISO VG 68 ISO VG 100 ISO VG 150 ISO VG 220 ISO VG 320 ISO VG 460 ISO VG 680	Mostly used for middle or heavily loaded enclosed-gears in general types of machines or rolling machines.

It is practical to select a lubricant by following the information in a catalog, a technical manual or information from the web site of the oil manufacturer, as well as following the JIS, JGMA and AGMA standards. Table 13.6 shows the proper viscosity for enclosed-gears, recommended by the oil manufacturer.

Table 13.6 Recommended Viscosity for Enclosed Gears

Rotation of Pinion (rpm)	Horsepower (PS)	Reduction Ratio below 10		Reduction Ratio over 10	
		cSt (40℃)	ISO Viscosity Grade	cSt (40℃)	ISO Viscosity Grade
Below 300	Less than 30	5 - 234	150, 220	180 - 279	220
	30 - 100	180 - 279	220	216 - 360	220,320
	More than 100	279 - 378	320	360 - 522	460
300 ' 1,000	Less than 20	81 - 153	100,150	117 - 198	150
	20 - 75	117 - 198	150	180 - 279	220
	More than 75	180 - 279	220	279 - 378	320
1,000 ' 2,000	Less than 10	54 - 117	68,100	59 - 153	68,100,150
	10 - 50	59 - 153	68,100,150	135 - 198	150
	More than 50	135 - 198	150	189 - 342	220,320
2,000 ' 5,000	Less than 5	27 - 36	32	41 - 63	46
	5 - 20	41 - 63	46	59 - 144	68,100
	More than 20	59 - 144	68,100	95 - 153	100,150
More than 5000	Less than1	9 - 31	10,15,22	18 - 32	22,32
	1 - 10	18 - 32	22,32	29 - 63	32,46
	Less than10	29 - 63	32,46	41 - 63	46

NOTE 1. Applicable for spur, helical, bevel and spiral bevel gears where the working temperature (oil temperature) conditions should be between 10 and 50℃ .

NOTE 2. Circulating lubrication or splash lubrication is applied.

After making a decision about which grade of viscosity to select, taking into consideration the usage (for spur gear, worm gear pair etc.) and usage conditions (dimensions of mechanical equipment, ambient temperature etc.), then choose the appropriate lubricant.

Table 13.7 List of a few industrial oils from representative oil manufacturers.

JIS Gear Oils			IDEMITSU	COSMO OIL	JAPAN ENERGY	SHOWA SHELL	ENEOS	MOBIL
For Industrial Usage	1	ISO VG 32	Daphne Super Multi Oil 32	NEW Mighty Super 32 Cosmo Allpus 32	JOMO Lathus 32	Shell Tellus Oil C 32	Super Mulpus DX32	Mobil DTE Oil Light
		ISO VG 68	Daphne Super Multi Oil 68	NEW Mighty Super 68 Cosmo Allpus 68	JOMO Lathus 68	Shell Tellus Oil C 68	Super Mulpus DX68	Mobil DTE Oil Heavy Medium
		ISO VG 100	Daphne Super Multi Oil 100	NEW Mighty Super 100 Cosmo Allpus 100	JOMO Lathus 100	Shell Tellus Oil C 100	Super Mulpus DX100	Mobil DTE Oil Heavy
		ISO VG 150	Daphne Super Multi Oil 150	NEW Mighty Super 150	JOMO Lathus 150	Shell Tellus Oil C 150	Super Mulpus DX150	Mobil Vacuoline 528
	2	ISO VG 100	Daphne Super Multi Oil 100	Cosmo Gear SE100 Cosmo ECO Gear EPS100	JOMO Reductus 100	Shell Omala Oil 100	Bonnoc AX M100 Bonnoc AX AX100	Mobil gear 600 XP 100
		ISO VG 150	Daphne Super Multi Oil 150	Cosmo Gear SE150 Cosmo ECO Gear EPS150	JOMO Reductus 150	Shell Omala Oil 150	Bonnoc AX M150 Bonnoc AX AX150	Mobil gear 600 XP 150
		ISO VG 220	Daphne Super Multi Oil 220	Cosmo Gear SE220 Cosmo ECO Gear EPS220	JOMO Reductus 220	Shell Omala Oil 220	Bonnoc AX M220 Bonnoc AX AX220	Mobil gear 600 XP 220
		ISO VG 320	Daphne Super Gear Oil 320	Cosmo GearSE320 Cosmo ECO Gear EPS320	JOMO Reductus 320	Shell Omala Oil 320	Bonnoc AX M320 Bonnoc AX AX320	Mobil gear 600 XP 320
		ISO VG 460	Daphne Super Gear Oil 460	Cosmo Gear SE460 Cosmo ECO Gear EPS460	JOMO Reductus 460	Shell Omala Oil 460	Bonnoc AX M460 Bonnoc AX AX460	Mobil gear 600 XP 460
		ISO VG 680	Daphne Super Gear Oil 680	Cosmo Gear SE680	JOMO Reductus 680	Shell Omala Oil 680	Bonnoc AX M680 Bonnoc AX AX680	Mobil gear 600 XP 680

(3) Selection of Lubricants for the Worm Gear Pair

After selection of the proper viscosity in accordance with usage (applications of spur gears, worm gears, etc.) and the conditions (size of the device used in, ambient temperature etc) check the brand of lubricants from product information offered by oil manufacturers.

Table 13.8 indicates reference values for proper viscosity recommended in accordance with strength calculations (JGMA405-01(1976)). Table 13.9 lists some of the representative lubricants used for worm gears.

Table 13.8 Reference Viscosity for Worm Gear Lubrication Unit：cSt(37.8℃)

Oil Temperature at Working		Slip Speed m/s		
Max Oil Temperature at Working	Start Oil Temperature	less than 2.5	Over 2.5 less than 5	Over 5
Over 0℃ up to 10℃	Over −10℃ Lower 0℃	110 - 130	110 - 130	110 - 130
	Over 0℃	110 - 150	110 - 150	110 - 150
Over 10℃ up to 30℃	Over 0℃	200 - 245	150 - 200	150 - 200
Over 30℃ up to 55℃	Over 0℃	350 - 510	245 - 350	200 - 245
Over 55℃ up to 80℃	Over 0℃	510 - 780	350 - 510	245 - 350
Over 80℃ up to 100℃	Over 0℃	900 - 1100	510 - 780	350 - 510

Table 13.9 Example of Worm Gear Oils

Viscosity Classification	IDEMITSU	COSMO OIL	JAPAN ENERGY	SHOWA SHELL	ENEOS	MOBIL
ISO VG 150	Daphne Super Multi Oil 150	—	JOMO Reductus 150	Shell Omala Oil 150	Bonnoc AX M150	Mobil Gear 629
ISO VG 220	Daphne Super Multi Oil 220	Cosmo GearW220	JOMO Reductus 220	Shell Omala Oil 220	Bonnoc AX M220	Mobil Gear 630
ISO VG 320	Daphne Super Gear Oil 320	Cosmo GearW320	JOMO Reductus 320	Shell Omala Oil 320	Bonnoc AX M320	Mobil Gear 632
ISO VG 460	Daphne Super Gear Oil 460	Cosmo GearW460	JOMO Reductus 460	Shell Omala Oil 460	Bonnoc AX M460	Mobil Gear 634
ISO VG 680	Daphne Super Gear Oil 680	—	JOMO Reductus 680	Shell Omala Oil 680	Bonnoc AX M680	Mobil Gear 636

(4) Grease Lubrication

Greases are sorted into 7 categories and also segmented by kinds (Components and Properties) and by consistency numbers (Worked Penetration or Viscosity).
Table 13.10 indicates types of grease (4 categories). (Excerpt from JIS K 2220:2003 Lubricating greases).

Table 13.10 Grease Types

Type			Operating Temperature limit ℃	Reference				
				Adequacy of Use				Application Examples
				Load			With water	
Usage	Class	Consistency Number		Low	High	Impact		
General use Grease	1	1, 2, 3, 4	−10 to 60	S	N	N	S	General usage in low loads
	2	2, 3	−10 to 100	S	N	N	N	General usage in medium loads
Grease in Centralized Lubrication	1	00, 0, 1	−10 to 60	S	N	N	S	Centralized Lubricating System for medium loads
	2	0, 1, 2	−10 to 100	S	N	N	S	
	3	0, 1, 2	−10 to 60	S	S	S	S	
	4	0, 1, 2	−10 to 100	S	S	S	S	
High Load Grease	1	0, 1, 2, 3	−10 to 100	S	S	S	S	Use in high/impact loads
Grease for Gear Compound	1	1(1), 2(1), 3(1)	−10 to 100	S	S	S	S	Use for open gears / wire

S: Suitable N: Not Suitable

NOTE (1) Consistency numbers with (1) are classified by viscosity.

REMARKS 1. General use grease in Class 1 consists of base oil and calcium-soap and water-resistance.

2 . General use grease in Class 2 consists of base oil and calcium-soap and heat-resistance.

Table 13.11 lists grease products from representative manufacturers.

Table 13.11 Greases

Consistency Number	IDEMITSU	COSMO OIL	JAPAN ENERGY	SHOWA SHELL	ENEOS
00	—	—	—	Alvania EP Grease R00	—
0	Daphne Eponex Grease SR №0	Cosmo Central Lubrication Grease №0	JOMO LISONIX GREASE EP0	Alvania EP Grease R0	EPNOC GREASE AP(N)0
1	Daphne Eponex Grease SR №1	Cosmo Central Lubrication Grease №1	JOMO LISONIX GREASE EP1	Alvania EP Grease S1	EPNOC GREASE AP(N)1
2	Daphne Eponex Grease SR №2	Cosmo Central Lubrication Grease №2	JOMO LISONIX GREASE EP2	Alvania EP Grease S2	EPNOC GREASE AP(N)2
3	Daphne Eponex Grease SR №3	Cosmo Central Lubrication Grease №3	JOMO LISONIX GREASE EP3	Alvania EP Grease S3	POWERNOC WB3

14 Damage to Gears

Damage to gears is basically categorized by two types; one is the damage to the tooth surface, and the other is breakage of the gear tooth. In addition, there are other specific damages, such as the deterioration of plastic material, the rim or web breakages. Damages occur in various ways, for example, insufficient gear strength, failure in lubrication or mounting and unexpected overloading. Therefore, it is not easy to figure out solutions to causes. Gear damage are defined by the following standards:

- •JGMA 7001-01(1990) Terms of gear tooth failure modes
- •JIS B 0160: 1999 Gears - Wear and damage to gear teeth – Terminology

14.1 Gear Wear and Tooth Surface Fatigue

Wear occurs on tooth surfaces in various ways. Run-in wear is a type of wear with slight asperity occurring on start-up. This wear involves no trouble in operation. Critical wear is the state of gears from which a small quantity of the material is scraped away from the tooth surface. If the wear expands until the tooth profile gets out of the shape, the gear can not be properly meshed anymore.
Tooth surface fatigue occurs when the load is applied on the tooth surface repeatedly, or when the force is applied on the tooth is larger than endurance limit of the material. As the result of the surface fatigue, the material fails and falls off the tooth surface. The surface fatigue includes pitting, case crushing and spalling.
If a critical wear or progressive pitting occurs on the tooth surface, the following phenomena occurs:

- Increase of noise or vibration
- Excessive increase in temperature at the gear device
- Increase of smear by lubricant
- Increase of backlash

By properly removing the causes of these troubles, damage can be avoided.

The following introduces causes of tooth damage and examples of the solution.

(1) When the tooth surface strength is insufficient against the load

Solution ① : Increasing of the strength of the tooth surface

- •Change the material to a stronger material having more hardness.
  S45C → SCM440 / SCM415 etc.
  Refer to the section 9. Gear Material and Heat Treatment (Page 565 – 566).
- •Enlarge the gear size
  Enlarge module and number of teeth.
- •Enlarge the facewidth
- •Exchange the gear to the stronger gear with helical teeth.
  Change from Spur gear to Spiral gear
  Change from Straight Bevel gear to Spiral bevel gear
  (Improvement of overlap ratio)

Solution ② : Decreasing the load

- •Reduce the load by changing driving conditions

(2) Improper tooth contact caused by bad mounting

Solution：Adjusting the tooth contact
Detailed methods for this solution differ with types of gears. For adjustment of bevel and worm gears, refer to the section 8.3 Features of Tooth Contact (Page 562 – 564).

(3) When partial contact occurs due to bad mounting

Solution : Change design of the gear, shaft and bearing to make them stronger.
By increasing stiffness, tooth contact improves.

(4) When lubrication is in a poor condition.

Solution : Provide appropriate conditions for the lubricant; proper type, viscosity, and quantity.
Refer to the section 13 Lubrication of Gears (Page 608 – 611).

14.2 Gear Breakage

There are also several types of gear breakage. Overload breakage occurs if unexpected heavy loads are applied to the tooth. Fatigue breakage occurs if the load is repeatedly added on the tooth surface. The tooth breakage caused by partial contact at the tooth end, occurs on spur or bevel gears.
The following introduces causes of breakage and solution examples.

(1) When the tooth is broken by the impact load

Solution ① Increase bending strength (Gear strength)
Changing the material or enlarging the module is one of the most effective methods.
The method is the same as the method of increasing surface strength.
Solution ② Decrease or eliminate the impact load.
For example, reducing rotating speed is effective.

(2) Fatigue breakage from cyclic Loading

Solution ① Increase gear strength
The detailed method is the same as the way of increasing tooth surface strength.
Solution ② Reducing the load or the rotation

(3) Breakage occurs when the wear progresses and the tooth gets thinner.

In the first place, preventing wear must be performed.

14.3 Types of Damage and Breakage

There are various types of damage and breakage that can occur to gears, this section introduces some of those as defined by the JGMA 7001-01(1990) and the industrial standards set by the Japan Gear Manufacturers Association.

Table 14.1 Damages to Gears

Term No.	Damage	Description
1	Deterioration of tooth surface	
11	Wear (Abrasion)	Gradual loss of material on the tooth surface from various causes.
111	Normal Wear	Not really identified as damage. After initial use, the irregularity of the tooth surfaces is kept in good balance.
1111	Medium Wear	Wear on tooth surface identified by checking tooth contact.
1112	Polishing	The state of the tooth surface becomes smooth like a mirror as the asperity of the surface is removed gradually.
112	Abrasive Wear	Linear scratches run irregularly on the tooth surface in the slipping direction.
113	Excessive Wear	Excessively worn over the lifetime of the product.
114	Interference Wear	Wear of the tooth root, occurred by interference between the corner of the gear and the tooth root of the mating gear.
115	Scratching	Type of abrasive wear. Linear scratches occur on the surface.
116	Scoring	Surface deterioration caused by alternate deposition and tearing of tooth surface.
1161	Medium Scoring	Type of light damage on the tooth surface. Slightly scratched in slipping direction.
1162	Destructive Scoring	Visible scratching and tooth profile destroyed.
1163	Local Scoring	Medium scoring occurred locally.
12	Corrosion	
121	Chemical Corrosion	Brownish-red rust or pitting corrosion occurred on surface.
122	Fretting Corrosion	Surface damage occurs on the part where two of the tooth surfaces are in contact and involve relative reciprocal motion with fine vibration.
123	Scaling	A prominent area of the tooth surface was oxidized when heat treatment was applied. The prominent area gets glossy.
13	Over Heating	Excessive heat temperature on tooth surface. Temper color appears.
14	Cavitation Erosion	Local erosion caused by forced oil-jet lubrication and its impact.
15	Electric Corrosion	Small pitting on tooth surface that occurs due to electric discharge between the meshed gear teeth.
16	Tooth Surface Fatigue	Damages on teeth, involving a fall-off of material.
161	Pitting	Pits occurred on the tooth surface. Pitting often occurs at the pitch line or under.
1611	Initial Pitting	A wearing phenomenon occurs in initial usage. It stops its progression when the tooth surface is broken-in.
1612	Progressive Pitting	A wearing phenomenon occurs and does not stop its progression even when the tooth surface becomes engaged.
1613	Frosting	Slight pitting occurs when only a thin oil film is generated and when heavy loading is applied.
162	Flake Pitting	A kind of spalling. Thin steel pieces fall off from the rather large area of the tooth.
163	Spalling	Material fatigue occurs under the surface, and quite large pieces of steel fall off.
164	Case Crushing	Abrasion occurs on the surface layer. The layer is damaged in broad areas.
17	Permanent Deformation	
171	Indentation	Dent in the tooth occurs by involving an object enmeshed in the teeth while working.
1721	Plastic Deformation	A typical state of permanent deformation. The deformation is not recovered after removing the load.
1721	Rolling	Indented streaks occur around the pitch line.
1722	Deformation by Gear Rattle	Deformation occurs when excessive vibration load is added and the meshed teeth engage with each other.
173	Rippling	Ripples periodically occur on the tooth surface in the rolling and normal direction.
174	Ridging	Ridges or crests occurs from plastic flow of the material right under the tooth surface.
175	Burr	A plastic deformation similar to rolling. The state of the material at the tooth tip or edge is evident.
176	Dent	A small plastic deformation occurs on the tooth surface or at the corner of the tip. This deformation involves concaving and prongs.
18	Crack	A type of fracture. There are two types of cracks, one occurs in the production process, and the other occurs from usage.
181	Quenching Crack	Cracks occurred by quenching.
182	Grinding Crack	Slight cracks occurred when grinding the teeth.
183	Fatigue Crack	Cracks at the tooth root or fillet, occur under reversed alternating stress and variable stress.
2	Tooth Breakage	
21	Overload Breakage	Breakage occurs on the tooth when unexpected heavy loads are applied to the tooth.
22	Breakage on Tooth Ends	Often occurs on spur or spiral gears. This brakeage is caused from partial contacts of meshed teeth in the width-direction.
23	Tooth Shearing	The state of teeth sheared from the body, occurred by a one time excessive load.
24	Smear Breakage	Marked and deformed tooth profile, caused from intolerable heavy loads on the material of the tooth.
25	Fatigue Breakage	A breakage caused from running cracks occurring at the tooth root filet.
3	Rim and Web Breakage	
4	Deterioration of Plastic Gears	
41	Swelling	Volume expansion occurs when solid substance absorbs fluids without changing the structure.

Additional Explanation

(1) Pit:
Tiny holes like pockmarks occur on the tooth surface.

(2) Beach mark fractured surface
Patterns occurred from fatigue breakage, similar to the striping patterns on sandy beaches, which occur from the waves of the sea or ocean.

15 Gear Noise

When gears work, especially at high loads and speeds, the noise and vibration caused by the rotation of the gears is considered a big problem. However, since noise problems tend to happen due to several causes in combination, it is very difficult to identify the cause. The following are ways to reduce noise and these points should be considered in the design stage of gear systems.

(1) Use High-Precision Gears
Reduce the pitch error, tooth profile error, runout error and lead error. Grind teeth to improve the accuracy as well as the surface finish.

(2) Use a Better Surface Finish on Gears
Grinding, lapping and honing the tooth surface, or running in gears in oil for a period of time can also improve the smoothness of tooth surface and reduce the noise.

(3) Ensure a Correct Tooth Contact
Crowning and end relief can prevent edge contact. Proper tooth profile modification is also effective. Eliminate impact on tooth surface.

(4) Have a Proper Amount of Backlash
A smaller backlash will help produce a pulsating transmission. A bigger backlash, in general, causes less problems.

(5) Increase the Transverse Contact Ratio
A bigger contact ratio lowers the noise. Decreasing the pressure angle and/or increasing the tooth depth can produce a larger contact ratio.

(6) Increase the Overlap Ratio
Enlarging the overlap ratio will reduce the noise. Because of this relationship, a helical gear is quieter than the spur gear and a spiral bevel gear is quieter than the straight bevel

(7) Eliminate Interference on the Tooth Profile
Chamfer the corner of the top land, or modify the tooth profile for smooth meshing. Smooth meshing without interfering makes low noise.

(8) Use Gears that have Smaller Teeth
Adopt gears with a smaller module and a larger number of gear teeth.

(9) Use High-Rigidity Gears
Increasing facewidth can give a higher rigidity that will help in reducing noise.
Reinforce housing and shafts to increase rigidity.

(10) Use Resin Materials
Plastic gears will be quiet in light load and low speed operation. Care should be taken to decrease backlash, caused from enlargement by absorption at elevated temperatures.

(11) Use High Vibration Damping Material
Cast iron gears have lower noise than steel gears. Use of gears with the hub made of cast iron is also effective.

(12) Apply Suitable Lubrication
Lubricate gears sufficiently to keep the lubricant film on the surface, under hydrodynamic lubrication. High-viscosity lubricant will have the tendency to reduce the noise.

(13) Lower Load and Speed
Lowering rotational speed and load as far as possible will reduce gear noise.

(14) Use Gears that have No Dents
Gears which have dents on the tooth surface or the tip make cyclic, abnormal sounds.

(15) Avoid too much thinning of the Web
Lightened gears with a thin web thickness make high-frequency noises. Care should be taken.

16 Methods for Determining the Specifications of Gears

16.1 Method for Determining the Specifications of a Spur Gear

Illustrated below are procedural steps to determine specifications of a spur gear.

Steps

① Count how many teeth a sample spur gear has $z =$

② Measure its tip diameter $d_a =$

③ Estimate an approximation of its module, assuming that it has an unshifted standard full depth tooth, using the equation:

$$m = \frac{d_a}{z+2} \qquad m \fallingdotseq$$

④ Measure the span measurement of k and the span number of teeth. Also, measure the $k-1$. Then calculate the difference.

Span number of teeth $k =$ Span measurement $W_k =$

" $k-1 =$ $W_{k-1} =$

The difference $=$

⑤ This difference represents $p_b = \pi m \cos\alpha$

Select module m and pressure angle α from the table on the right.

$m =$

$\alpha =$

⑥ Calculate the profile shift coefficient x based on the above m and pressure angle α and span measurement W.

$x =$

To find the calculation method, please see table 5.10 (No. 2) on Page 542.

Table 16.1 Base Pitch p_b

Module	Pressure angle 20°	Pressure angle 14.5°	Module	Pressure angle 20°	Pressure angle 14.5°
1	2.952	3.042	8	23.619	24.332
1.25	3.690	3.802	9	26.569	27.373
1.5	4.428	4.562	10	29.521	30.415
2	5.904	6.083	11	32.473	33.456
2.5	7.380	7.604	12	35.425	36.498
3	8.856	9.125	14	41.329	42.581
3.5	10.332	10.645	16	47.234	48.664
4	11.808	12.166	18	53.138	54.747
5	14.760	15.208	20	59.042	60.830
6	17.712	18.249	22	64.946	66.913
7	20.664	21.291	25	73.802	76.037

NOTE.
This table deals with the pressure angle 20°and 14.5°only. There maybe cases where the degree of the pressure angle is different. Tooth profiles come with deep tooth depth or shallow tooth depth, other than a full tooth depth.

16.2 Method for Determining the Specifications of a Helical Gear

The Helix angle is what differs helical gears from spur gears and it is necessary that the helix angle is measured accurately. A gear measuring machine can serve this purpose, however, when the machine is unavailable you can use a protractor to obtain an approximation.

Lead p_z of a helical gear can be presented with the equation:

$$p_z = \frac{\pi z m_n}{\sin\beta}$$

Given the lead p_z, number of teeth z normal module m_n, the helix angle β can be found with the equation:

$$\beta = \sin^{-1}\left(\frac{\pi z m_n}{p_z}\right)$$

The number of teeth z and normal module m_n can be obtained using the method for spur gears explained above. In order to obtain p_z, determine d_a by measuring the tip diameter. Then by using a piece of paper and with ink on the outside edge of a helical gear, roll it on the paper pressing it tightly. With a protractor measure the angle of the mark left printed on the paper, β_a. Lead p_z can be obtained with the following equation.

$$p_z = \frac{\pi d_a}{\tan\beta_a}$$

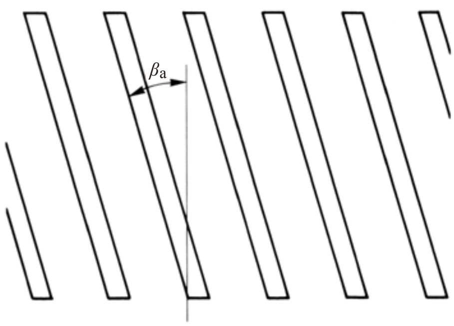


Fig. 16.1 The measurement of a helix angle on tooth tips

17 Gear Systems

This section introduces planetary gear systems, hypocycloid mechanisms, and constrained gear systems, which are special gear systems which offer features such as compact size and high reduction ratio.

17.1 Planetary Gear System

The basic form of a planetary gear system is shown in Figure 17.1. It consists of a sun gear A, planet gears B, internal gear C and carrier D.

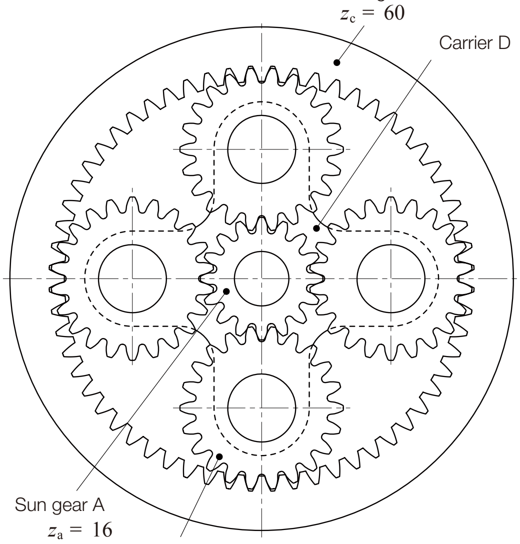


Fig.17.1 An example of a planetary gear system

The input and output axes of a planetary gear system are on a same line. Usually, it uses two or more planet gears to balance the load evenly. It is compact in space, but complex in structure. Planetary gear systems need a high-quality manufacturing process. The load division between planet gears, the interference of the internal gear, the balance and vibration of the rotating carrier, and the hazard of jamming, etc. are inherent problems to be solved.

Figure 17.1 is a so called 2K-H type planetary gear system. The sun gear, internal gear, and the carrier have a common axis.

(1) Relationship Among the Gears in a Planetary Gear System

In order to determine the relationship among the numbers of teeth of the sun gear (z_a), the planet gears B (z_b) and the internal gear C (z_c) and the number of planet gears N in the system, these parameters must satisfy the following three conditions:

Condition No.1

$$z_c = z_a + 2z_b \tag{17.1}$$

This is the condition necessary for the center distances of the gears to match. Since the equation is true only for the standard gear system, it is possible to vary the numbers of teeth by using profile shifted gear designs.

To use profile shifted gears, it is necessary to match the center distance between the sun A and planet B gears, a_1, and the center distance between the planet B and internal C gears, a_2.

$$a_1 = a_2 \tag{17.2}$$

Condition No.2

$$\frac{z_a + z_c}{N} = \text{Integer} \tag{17.3}$$

This is the condition necessary for placing planet gears evenly spaced around the sun gear. If an uneven placement of planet gears is desired, then Equation (17.4) must be satisfied.

$$\frac{(z_a + z_c)\,\theta}{180} = \text{Integer} \tag{17.4}$$

Where θ: half the angle between adjacent planet gears (°)

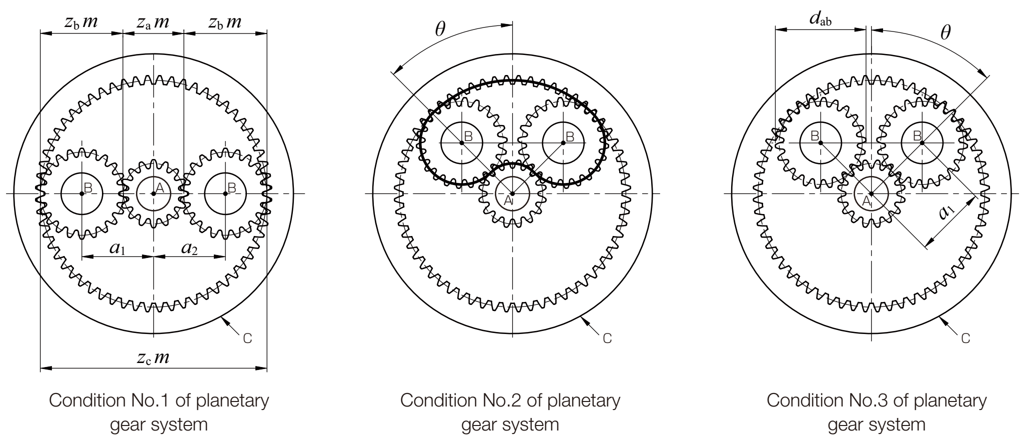


Fig.17.2 Conditions for selecting gears

Condition No.3 $z_b + 2 < (z_a + z_b)\sin\frac{180°}{N}$ (17.5)

Satisfying this condition insures that adjacent planet gears can operate without interfering with each other. This is the condition that must be met for standard gear design with equal placement of planet gears. For other conditions, the system must satisfy the relationship:

$$d_{ab} < 2a_1 \sin\theta \qquad (17.6)$$

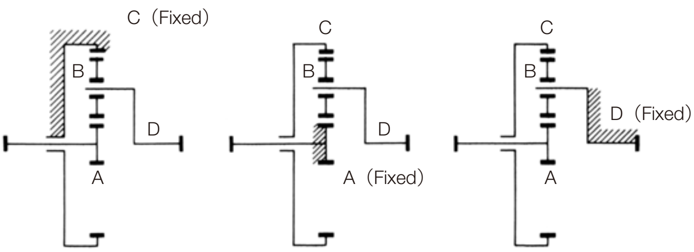
Where: d_{ab} : Tip diameter of the planet gears

a_1 : Center distance between the sun and planet gears

Besides the above three basic conditions, there can be an interference problem between the internal gear C and the planet gears B. See Section 4.2 Internal Gears (Page 611 to 613).

(2) Transmission Ratio of Planetary Gear System

In a planetary gear system, the transmission ratio and the direction of rotation would be changed according to which member is fixed. Figure 17.3 contain three typical types of planetary gear mechanisms,



(a) Planetary type (b) Solar type (c) Star type

Fig.17.3 Planetary gear mechanism

(a) Planetary Type

In this type, the internal gear is fixed. The input is the sun gear and the output is carrier D. The transmission ratio is calculated as in Table 17.1.

Table17.1 Equations of transmission ratio for a planetary type

No.	Description	Sun gear A z_a	Planet gear B z_b	Internal gear C z_c	Carrier D
1	Rotate sun gear once while holding carrier	$+1$	$-\frac{z_a}{z_b}$	$-\frac{z_a}{z_c}$	0
2	System is fixed as a whole while rotating	$+\frac{z_a}{z_c}$	$+\frac{z_a}{z_c}$	$+\frac{z_a}{z_c}$	$+\frac{z_a}{z_c}$
3	Sum of 1 and 2	$1+\frac{z_a}{z_c}$	$\frac{z_a}{z_c}-\frac{z_a}{z_b}$	0 (fixed)	$+\frac{z_a}{z_c}$

$$\text{Transmission ratio} = \frac{1+\frac{z_a}{z_c}}{\frac{z_a}{z_c}} = \frac{z_c}{z_a}+1 \qquad (17.7)$$

Note that the direction of rotation of input and output axes are the same. Example: $z_a = 16$, $z_b = 16$, $z_c = 48$, then transmission ratio = 4.

(b) Solar Type

In this type, the sun gear is fixed. The internal gear C is the input, and carrier D axis is the output. The speed ratio is calculated as in Table 17.2.

Table 17.2 Equations of transmission ratio for a solar type

No.	Description	Sun gear A z_a	Planet gearB z_b	Internal gearC z_c	Carrier D
1	Rotate sun gear once while holding carrier	$+1$	$-\frac{z_a}{z_b}$	$-\frac{z_a}{z_c}$	0
2	System is fixed as a whole while rotating	-1	-1	-1	-1
3	Sum of 1 and 2	0 (fixed)	$-\frac{z_a}{z_b}-1$	$-\frac{z_a}{z_c}-1$	-1

$$\text{Transmission ratio} = \frac{-\frac{z_a}{z_c}-1}{-1} = \frac{z_a}{z_c}+1 \qquad (17.8)$$

Note that the directions of rotation of input and output axes are the same.

Example: $z_a = 16$, $z_b = 16$, $z_c = 48$,

then the transmission ratio = 1.33333

(c) Star Type

This is the type in which Carrier D is fixed. The planet gears B rotate only on fixed axes. In a strict definition, this train loses the features of a planetary system and it becomes an ordinary gear train. The sun gear is an input axis and the internal gear is the output. The transmission ratio is :

$$\text{Transmission Ratio} = -\frac{z_c}{z_a} \qquad (17.9)$$

Referring to Figure 2.3(c), the planet gears are merely idlers. Input and output axes have opposite rotations.

Example: $z_a = 16$, $z_b = 16$, $z_c = 48$,

then transmission ratio = -3 .

17.2 Hypocycloid Mechanism

In the meshing of an internal gear and an external gear, if the difference in numbers of teeth of two gears is quite small, a profile shifted gear could prevent the interference. Table 17.3 is an example of how to prevent interference under the conditions of z_2 = 50 and the difference of numbers of teeth of two gears ranges from 1 to 8.

Table17.3 The meshing of internal and external gears of small difference of numbers of teeth （$m = 1$、$\alpha = 20°$）

z_1	49	48	47	46	45	44	43	42
x_1	0							
z_2	50							
x_2	1.00	0.60	0.40	0.30	0.20	0.11	0.06	0.01
α_b	61.0605°	46.0324°	37.4155°	32.4521°	28.2019°	24.5356°	22.3755°	20.3854°
a	0.971	1.354	1.775	2.227	2.666	3.099	3.557	4.010
ε	1.105	1.512	1.726	1.835	1.933	2.014	2.053	2.088

All combinations above will not cause involute interference or trochoid interference, but trimming interference is still present. In order to assemble successfully, the external gear should be assembled by inserting it in the axial direction. A profile shifted internal gear and external gear, in which the difference of numbers of teeth is small, belong to the field of hypocyclic mechanisms, which can produce a large reduction ratio in single step, such as 1/100.

$$\text{Transmission ratio} = -\frac{z_1}{z_2 - z_1} \qquad (17.10)$$

In Figure 17.4 the gear train has a difference of numbers of teeth of only 1; z_1 = 30 and z_2 = 31. This results in a transmission ratio of 30.

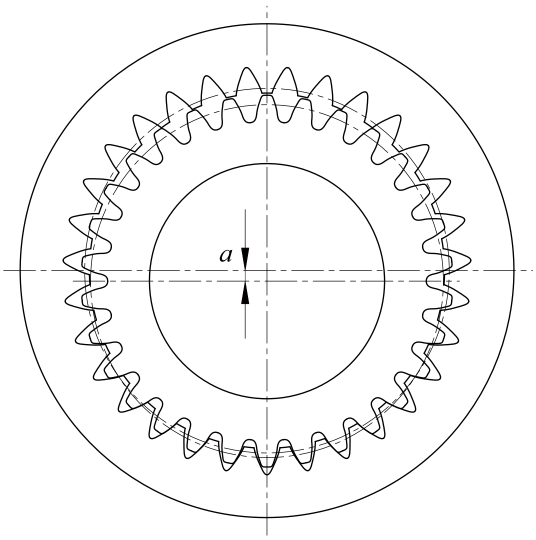


Fig.17.4 The meshing of internal gear and external gear in which the numbers of teeth difference is 1

17.3 Constrained Gear System

A planetary gear system which has four gears is an example of a constrained gear system. It is a closed loop system in which the power is transmitted from the driving gear through other gears and eventually to the driven gear. A closed loop gear system will not work if the gears do not meet specific conditions.

Let z_1, z_2 and z_3 be the numbers of gear teeth, as in Figure 17.5. Meshing cannot function if the length of the heavy line (belt) does not divide evenly by pitch. Equation（17.11）defines this condition.

$$\frac{z_1\theta_1}{180} + \frac{z_2\,(180+\theta_1+\theta_2)}{180} + \frac{z_3\theta_2}{180} = \text{integer} \qquad (17.11)$$

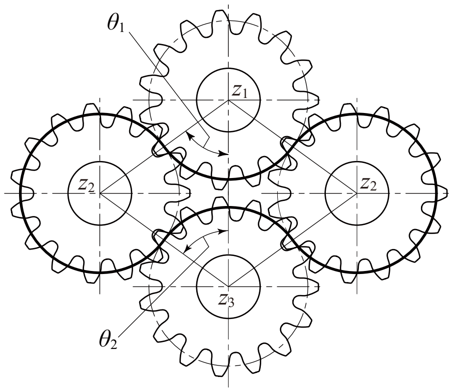


Fig.17.5 Constrained gear system

Figure 17.6 shows a constrained gear system in which a rack is meshed. The heavy line in Figure 17.6 corresponds to the belt in Figure 17.5. If the length of the belt cannot be evenly divided by pitch then the system does not work. It is described by Equation（17.12）.

$$\frac{z_1\theta_1}{180} + \frac{z_2\,(180+\theta_1)}{180} + \frac{a}{\pi m} = \text{integer} \qquad (17.12)$$

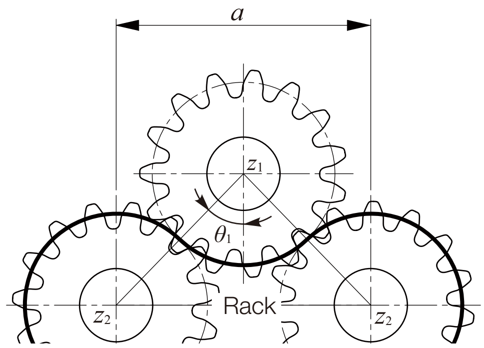


Fig.17.6 Constrained gear system containing a rack

< JIS Japan Industrial Standards for Gearing >

1 Precision Standard for Spur and Helical Gears Excerpted from JIS B 1702-1：1998、JIS B 1702-2：1998

JIS B 1702-01: 1998 and JIS B1702-02: 1998 cancel and replace the former JIS B 1702: 1976 (Accuracy for spur and helical gears). This revision was made to conform to International Standard Organization (ISO) standards.

New standards for gear accuracy are: JIS B 1702−1：1998（Cylindrical gears − ISO system of accuracy - Part 1 Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth）and JIS B 1702−2：1998（Cylindrical gears − ISO system of accuracy - Part 2 : Definitions and allowable values of deviations relevant to radial composite deviations and runout information).

Shown here are tables, extracted from JIS B 1702-01 and 1702-02, concerning the gear accuracy.

To distinguish new standards from old ones, each of the grades under the new standards has the prefix "N".

< Discrepancies between new and old standards >

As some change in the classification has been made of module and reference diameter (reference pitch diameter, in old standards), there may be some difficulties to find out what old grade in question corresponds to new grade.

As a rough measure new accuracy grades are said to be equal to old accuracy grades plus 4. In certain cases, however, this formula does not apply.

As the need arises, please refer to "JGMA/TR 0001 (2000): Comparison table between new and old accuracy grades".

Table 1 Single pitch deviation $\pm f_{pt}$

Reference diameter	Module	Accuracy grades								
		N4	N5	N6	N7	N8	N9	N10	N11	N12
d mm	m mm	$\pm f_{pt}$ μm								
$5 \leqq d \leqq 20$	$0.5 \leqq m \leqq 2$	3.3	4.7	6.5	9.5	13.0	19.0	26.0	37.0	53.0
	$2 < m \leqq 3.5$	3.7	5.0	7.5	10.0	15.0	21.0	29.0	41.0	59.0
$20 < d \leqq 50$	$0.5 \leqq m \leqq 2$	3.5	5.0	7.0	10.0	14.0	20.0	28.0	40.0	56.0
	$2 < m \leqq 3.5$	3.9	5.5	7.5	11.0	15.0	22.0	31.0	44.0	62.0
	$3.5 < m \leqq 6$	4.3	6.0	8.5	12.0	17.0	24.0	34.0	48.0	68.0
	$6 < m \leqq 10$	4.9	7.0	10.0	14.0	20.0	28.0	40.0	56.0	79.0
$50 < d \leqq 125$	$0.5 \leqq m \leqq 2$	3.8	5.5	7.5	11.0	15.0	21.0	30.0	43.0	61.0
	$2 < m \leqq 3.5$	4.1	6.0	8.5	12.0	17.0	23.0	33.0	47.0	66.0
	$3.5 < m \leqq 6$	4.6	6.5	9.0	13.0	18.0	26.0	36.0	52.0	73.0
	$6 < m \leqq 10$	5.0	7.5	10.0	15.0	21.0	30.0	42.0	59.0	84.0
	$10 < m \leqq 16$	6.5	9.0	13.0	18.0	25.0	35.0	50.0	71.0	100.0
	$16 < m \leqq 25$	8.0	11.0	16.0	22.0	31.0	44.0	63.0	89.0	125.0
$125 < d \leqq 280$	$0.5 \leqq m \leqq 2$	4.2	6.0	8.5	12.0	17.0	24.0	34.0	48.0	67.0
	$2 < m \leqq 3.5$	4.6	6.5	9.0	13.0	18.0	26.0	36.0	51.0	73.0
	$3.5 < m \leqq 6$	5.0	7.0	10.0	14.0	20.0	28.0	40.0	56.0	79.0
	$6 < m \leqq 10$	5.5	8.0	11.0	16.0	23.0	32.0	45.0	64.0	90.0
	$10 < m \leqq 16$	6.5	9.5	13.0	19.0	27.0	38.0	53.0	75.0	107.0
	$16 < m \leqq 25$	8.0	12.0	16.0	23.0	33.0	47.0	66.0	93.0	132.0
$280 < d \leqq 560$	$0.5 \leqq m \leqq 2$	4.7	6.5	9.5	13.0	19.0	27.0	38.0	54.0	76.0
	$2 < m \leqq 3.5$	5.0	7.0	10.0	14.0	20.0	29.0	41.0	57.0	81.0
	$3.5 < m \leqq 6$	5.5	8.0	11.0	16.0	22.0	31.0	44.0	62.0	88.0
	$6 < m \leqq 10$	6.0	8.5	12.0	17.0	25.0	35.0	49.0	70.0	99.0
	$10 < m \leqq 16$	7.0	10.0	14.0	20.0	29.0	41.0	58.0	81.0	115.0
	$16 < m \leqq 25$	9.0	12.0	18.0	25.0	35.0	50.0	70.0	99.0	140.0

Table 2 Total cumulative pitch deviation F_p

Reference diameter d mm	Module m mm	Accuracy grades N4	N5	N6	N7	N8	N9	N10	N11	N12
		F_p μm								
$5 \leqq d \leqq 20$	$0.5 \leqq m \leqq 2$	8.0	11.0	16.0	23.0	32.0	45.0	64.0	90.0	127.0
	$2 < m \leqq 3.5$	8.5	12.0	17.0	23.0	33.0	47.0	66.0	94.0	133.0
$20 < d \leqq 50$	$0.5 \leqq m \leqq 2$	10.0	14.0	20.0	29.0	41.0	57.0	81.0	115.0	162.0
	$2 < m \leqq 3.5$	10.0	15.0	21.0	30.0	42.0	59.0	84.0	119.0	168.0
	$3.5 < m \leqq 6$	11.0	15.0	22.0	31.0	44.0	62.0	87.0	123.0	174.0
	$6 < m \leqq 10$	12.0	16.0	23.0	33.0	46.0	65.0	93.0	131.0	185.0
$50 < d \leqq 125$	$0.5 \leqq m \leqq 2$	13.0	18.0	26.0	37.0	52.0	74.0	104.0	147.0	208.0
	$2 < m \leqq 3.5$	13.0	19.0	27.0	38.0	53.0	76.0	107.0	151.0	214.0
	$3.5 < m \leqq 6$	14.0	19.0	28.0	39.0	55.0	78.0	110.0	156.0	220.0
	$6 < m \leqq 10$	14.0	20.0	29.0	41.0	58.0	82.0	116.0	164.0	231.0
	$10 < m \leqq 16$	15.0	22.0	31.0	44.0	62.0	88.0	124.0	175.0	248.0
	$16 < m \leqq 25$	17.0	24.0	34.0	48.0	68.0	96.0	136.0	193.0	273.0
$125 < d \leqq 280$	$0.5 \leqq m \leqq 2$	17.0	24.0	35.0	49.0	69.0	98.0	138.0	195.0	276.0
	$2 < m \leqq 3.5$	18.0	25.0	35.0	50.0	70.0	100.0	141.0	199.0	282.0
	$3.5 < m \leqq 6$	18.0	25.0	36.0	51.0	72.0	102.0	144.0	204.0	288.0
	$6 < m \leqq 10$	19.0	26.0	37.0	53.0	75.0	106.0	149.0	211.0	299.0
	$10 < m \leqq 16$	20.0	28.0	39.0	56.0	79.0	112.0	158.0	223.0	316.0
	$16 < m \leqq 25$	21.0	30.0	43.0	60.0	85.0	120.0	170.0	241.0	341.0
$280 < d \leqq 560$	$0.5 \leqq m \leqq 2$	23.0	32.0	46.0	64.0	91.0	129.0	182.0	257.0	364.0
	$2 < m \leqq 3.5$	23.0	33.0	46.0	65.0	92.0	131.0	185.0	261.0	370.0
	$3.5 < m \leqq 6$	24.0	33.0	47.0	66.0	94.0	133.0	188.0	266.0	376.0
	$6 < m \leqq 10$	24.0	34.0	48.0	68.0	97.0	137.0	193.0	274.0	387.0
	$10 < m \leqq 16$	25.0	36.0	50.0	71.0	101.0	143.0	202.0	285.0	404.0
	$16 < m \leqq 25$	27.0	38.0	54.0	76.0	107.0	151.0	214.0	303.0	428.0

Table 3 Total profile deviation F_α

Reference diameter d mm	Module m mm	Accuracy grades F_α μm N4	N5	N6	N7	N8	N9	N10	N11	N12
$5 \leqq d \leqq 20$	$0.5 \leqq m \leqq 2$	3.2	4.6	6.5	9.0	13.0	18.0	26.0	37.0	52.0
	$2 < m \leqq 3.5$	4.7	6.5	9.5	13.0	19.0	26.0	37.0	53.0	75.0
$20 < d \leqq 50$	$0.5 \leqq m \leqq 2$	3.6	5.0	7.5	10.0	15.0	21.0	29.0	41.0	58.0
	$2 < m \leqq 3.5$	5.0	7.0	10.0	14.0	20.0	29.0	40.0	57.0	81.0
	$3.5 < m \leqq 6$	6.0	9.0	12.0	18.0	25.0	35.0	50.0	70.0	99.0
	$6 < m \leqq 10$	7.5	11.0	15.0	22.0	31.0	43.0	61.0	87.0	123.0
$50 < d \leqq 125$	$0.5 \leqq m \leqq 2$	4.1	6.0	8.5	12.0	17.0	23.0	33.0	47.0	66.0
	$2 < m \leqq 3.5$	5.5	8.0	11.0	16.0	22.0	31.0	44.0	63.0	89.0
	$3.5 < m \leqq 6$	6.5	9.5	13.0	19.0	27.0	38.0	54.0	76.0	108.0
	$6 < m \leqq 10$	8.0	12.0	16.0	23.0	33.0	46.0	65.0	92.0	131.0
	$10 < m \leqq 16$	10.0	14.0	20.0	28.0	40.0	56.0	79.0	112.0	159.0
	$16 < m \leqq 25$	12.0	17.0	24.0	34.0	48.0	68.0	96.0	136.0	192.0
$125 < d \leqq 280$	$0.5 \leqq m \leqq 2$	4.9	7.0	10.0	14.0	20.0	28.0	39.0	55.0	78.0
	$2 < m \leqq 3.5$	6.5	9.0	13.0	18.0	25.0	36.0	50.0	71.0	101.0
	$3.5 < m \leqq 6$	7.5	11.0	15.0	21.0	30.0	42.0	60.0	84.0	119.0
	$6 < m \leqq 10$	9.0	13.0	18.0	25.0	36.0	50.0	71.0	101.0	143.0
	$10 < m \leqq 16$	11.0	15.0	21.0	30.0	43.0	60.0	85.0	121.0	171.0
	$16 < m \leqq 25$	13.0	18.0	25.0	36.0	51.0	72.0	102.0	144.0	204.0
$280 < d \leqq 560$	$0.5 \leqq m \leqq 2$	6.0	8.5	12.0	17.0	23.0	33.0	47.0	66.0	94.0
	$2 < m \leqq 3.5$	7.5	10.0	15.0	21.0	29.0	41.0	58.0	82.0	116.0
	$3.5 < m \leqq 6$	8.5	12.0	17.0	24.0	34.0	48.0	67.0	95.0	135.0
	$6 < m \leqq 10$	10.0	14.0	20.0	28.0	40.0	56.0	79.0	112.0	158.0
	$10 < m \leqq 16$	12.0	16.0	23.0	33.0	47.0	66.0	93.0	132.0	186.0
	$16 < m \leqq 25$	14.0	19.0	27.0	39.0	55.0	78.0	110.0	155.0	219.0

Table 4 Total helix deviation F_β

Reference diameter d mm	Facewidth b mm	Accuracy grades N4	N5	N6	N7	N8	N9	N10	N11	N12
		F_β μm								
$5 \leqq d \leqq 20$	$4 \leqq b \leqq 10$	4.3	6.0	8.5	12.0	17.0	24.0	35.0	49.0	69.0
	$10 < b \leqq 20$	4.9	7.0	9.5	14.0	19.0	28.0	39.0	55.0	78.0
	$20 < b \leqq 40$	5.5	8.0	11.0	16.0	22.0	31.0	45.0	63.0	89.0
	$40 < b \leqq 80$	6.5	9.5	13.0	19.0	26.0	37.0	52.0	74.0	105.0
$20 < d \leqq 50$	$4 \leqq b \leqq 10$	4.5	6.5	9.0	13.0	18.0	25.0	36.0	51.0	72.0
	$10 < b \leqq 20$	5.0	7.0	10.0	14.0	20.0	29.0	40.0	57.0	81.0
	$20 < b \leqq 40$	5.5	8.0	11.0	16.0	23.0	32.0	46.0	65.0	92.0
	$40 < b \leqq 80$	6.5	9.5	13.0	19.0	27.0	38.0	54.0	76.0	107.0
	$80 < b \leqq 160$	8.0	11.0	16.0	23.0	32.0	46.0	65.0	92.0	130.0
$50 < d \leqq 125$	$4 \leqq b \leqq 10$	4.7	6.5	9.5	13.0	19.0	27.0	38.0	53.0	76.0
	$10 < b \leqq 20$	5.5	7.5	11.0	15.0	21.0	30.0	42.0	60.0	84.0
	$20 < b \leqq 40$	6.0	8.5	12.0	17.0	24.0	34.0	48.0	68.0	95.0
	$40 < b \leqq 80$	7.0	10.0	14.0	20.0	28.0	39.0	56.0	79.0	111.0
	$80 < b \leqq 160$	8.5	12.0	17.0	24.0	33.0	47.0	67.0	94.0	133.0
	$160 < b \leqq 250$	10.0	14.0	20.0	28.0	40.0	56.0	79.0	112.0	158.0
	$250 < b \leqq 400$	12.0	16.0	23.0	33.0	46.0	65.0	92.0	130.0	184.0
$125 < d \leqq 280$	$4 \leqq b \leqq 10$	5.0	7.0	10.0	14.0	20.0	29.0	40.0	57.0	81.0
	$10 < b \leqq 20$	5.5	8.0	11.0	16.0	22.0	32.0	45.0	63.0	90.0
	$20 < b \leqq 40$	6.5	9.0	13.0	18.0	25.0	36.0	50.0	71.0	101.0
	$40 < b \leqq 80$	7.5	10.0	15.0	21.0	29.0	41.0	58.0	82.0	117.0
	$80 < b \leqq 160$	8.5	12.0	17.0	25.0	35.0	49.0	69.0	98.0	139.0
	$160 < b \leqq 250$	10.0	14.0	20.0	29.0	41.0	58.0	82.0	116.0	164.0
	$250 < b \leqq 400$	12.0	17.0	24.0	34.0	47.0	67.0	95.0	134.0	190.0
$280 < d \leqq 560$	$10 \leqq b \leqq 20$	6.0	8.5	12.0	17.0	24.0	34.0	48.0	68.0	97.0
	$20 < b \leqq 40$	6.5	9.5	13.0	19.0	27.0	38.0	54.0	76.0	108.0
	$40 < b \leqq 80$	7.5	11.0	15.0	22.0	31.0	44.0	62.0	87.0	124.0
	$80 < b \leqq 160$	9.0	13.0	18.0	26.0	36.0	52.0	73.0	103.0	146.0
	$160 < b \leqq 250$	11.0	15.0	21.0	30.0	43.0	60.0	85.0	121.0	171.0
	$250 < b \leqq 400$	12.0	17.0	25.0	35.0	49.0	70.0	98.0	139.0	197.0

Table 5 Total radial composite deviation F_i'' JIS B 1702-2:1998

Reference diameter	Normal module	Accuracy grades								
		N4	N5	N6	N7	N8	N9	N10	N11	N12
d mm	m_n mm	F_i'' μm								
$5 \leqq d \leqq 20$	$0.2 \leqq m_n \leqq 0.5$	7.5	11	15	21	30	42	60	85	120
	$0.5 < m_n \leqq 0.8$	8.0	12	16	23	33	46	66	93	131
	$0.8 < m_n \leqq 1.0$	9.0	12	18	25	35	50	70	100	141
	$1.0 < m_n \leqq 1.5$	10	14	19	27	38	54	76	108	153
	$1.5 < m_n \leqq 2.5$	11	16	22	32	45	63	89	126	179
	$2.5 < m_n \leqq 4.0$	14	20	28	39	56	79	112	158	223
$20 < d \leqq 50$	$0.2 \leqq m_n \leqq 0.5$	9.0	13	19	26	37	52	74	105	148
	$0.5 < m_n \leqq 0.8$	10	14	20	28	40	56	80	113	160
	$0.8 < m_n \leqq 1.0$	11	15	21	30	42	60	85	120	169
	$1.0 < m_n \leqq 1.5$	11	16	23	32	45	64	91	128	181
	$1.5 < m_n \leqq 2.5$	13	18	26	37	52	73	103	146	207
	$2.5 < m_n \leqq 4.0$	16	22	31	44	63	89	126	178	251
	$4.0 < m_n \leqq 6.0$	20	28	39	56	79	111	157	222	314
	$6.0 < m_n \leqq 10$	26	37	52	74	104	147	209	295	417
$50 < d \leqq 125$	$0.2 \leqq m_n \leqq 0.5$	12	16	23	33	46	66	93	131	185
	$0.5 < m_n \leqq 0.8$	12	17	25	35	49	70	98	139	197
	$0.8 < m_n \leqq 1.0$	13	18	26	36	52	73	103	146	206
	$1.0 < m_n \leqq 1.5$	14	19	27	39	55	77	109	154	218
	$1.5 < m_n \leqq 2.5$	15	22	31	43	61	86	122	173	244
	$2.5 < m_n \leqq 4.0$	18	25	36	51	72	102	144	204	288
	$4.0 < m_n \leqq 6.0$	22	31	44	62	88	124	176	248	351
	$6.0 < m_n \leqq 10$	28	40	57	80	144	161	227	321	454
$125 < d \leqq 280$	$0.2 \leqq m_n \leqq 0.5$	15	21	30	42	60	85	120	170	240
	$0.5 < m_n \leqq 0.8$	16	22	31	44	63	89	126	178	252
	$0.8 < m_n \leqq 1.0$	16	23	33	46	65	92	131	185	261
	$1.0 < m_n \leqq 1.5$	17	24	34	48	68	97	137	193	273
	$1.5 < m_n \leqq 2.5$	19	26	37	53	75	106	149	211	299
	$2.5 < m_n \leqq 4.0$	21	30	43	61	86	121	172	243	343
	$4.0 < m_n \leqq 6.0$	25	36	51	72	102	144	203	287	406
	$6.0 < m_n \leqq 10$	32	45	64	90	127	180	255	360	509
$280 < d \leqq 560$	$0.2 \leqq m_n \leqq 0.5$	19	28	39	55	78	110	156	220	311
	$0.5 < m_n \leqq 0.8$	20	29	40	57	81	114	161	228	323
	$0.8 < m_n \leqq 1.0$	21	29	42	59	83	117	166	235	332
	$1.0 < m_n \leqq 1.5$	22	30	43	61	86	122	172	243	344
	$1.5 < m_n \leqq 2.5$	23	33	46	65	92	131	185	262	370
	$2.5 < m_n \leqq 4.0$	26	37	52	73	104	146	207	293	414
	$4.0 < m_n \leqq 6.0$	30	42	60	84	119	169	239	337	477
	$6.0 < m_n \leqq 10$	36	51	73	103	145	205	290	410	580
$560 < d \leqq 1000$	$0.2 \leqq m_n \leqq 0.5$	25	35	50	70	99	140	198	280	396
	$0.5 < m_n \leqq 0.8$	25	36	51	72	102	144	204	288	408
	$0.8 < m_n \leqq 1.0$	26	37	52	74	104	148	209	295	417
	$1.0 < m_n \leqq 1.5$	27	38	54	76	107	152	215	304	429
	$1.5 < m_n \leqq 2.5$	28	40	57	80	114	161	228	322	455
	$2.5 < m_n \leqq 4.0$	31	44	62	88	125	177	250	353	499
	$4.0 < m_n \leqq 6.0$	35	50	70	99	141	199	281	398	562
	$6.0 < m_n \leqq 10$	42	59	83	118	166	235	333	471	665

Table 6 shows the allowable runout appearing in JIS B 1702-2：1998 Appendix B (Reference material)

Table 6 Runout F_r (μm)

Reference diameter d mm	Normal module m_n mm	Accuracy grades F_r μm								
		N4	N5	N6	N7	N8	N9	N10	N11	N12
$5 \leqq d \leqq 20$	$0.5 \leqq m_n \leqq 2.0$	6.5	9.0	13	18	25	36	51	72	102
	$2.0 < m_n \leqq 3.5$	6.5	9.5	13	19	27	38	53	75	106
$20 < d \leqq 50$	$0.5 \leqq m_n \leqq 2.0$	8.0	11	16	23	32	46	65	92	130
	$2.0 < m_n \leqq 3.5$	8.5	12	17	24	34	47	67	95	134
	$3.5 < m_n \leqq 6.0$	8.5	12	17	25	35	49	70	99	139
	$6.0 < m_n \leqq 10$	9.5	13	19	26	37	52	74	105	148
$50 < d \leqq 125$	$0.5 \leqq m_n \leqq 2.0$	10	15	21	29	42	59	83	118	167
	$2.0 < m_n \leqq 3.5$	11	15	21	30	43	61	86	121	171
	$3.5 < m_n \leqq 6.0$	11	16	22	31	44	62	88	125	176
	$6.0 < m_n \leqq 10$	12	16	23	33	46	65	92	131	185
	$10 < m_n \leqq 16$	12	18	25	35	50	70	99	140	198
	$16 < m_n \leqq 25$	14	19	27	39	55	77	109	154	218
$125 < d \leqq 280$	$0.5 \leqq m_n \leqq 2.0$	14	20	28	39	55	78	110	156	221
	$2.0 < m_n \leqq 3.5$	14	20	28	40	56	80	113	159	225
	$3.5 < m_n \leqq 6.0$	14	20	29	41	58	82	115	163	231
	$6.0 < m_n \leqq 10$	15	21	30	42	60	85	120	169	239
	$10 < m_n \leqq 16$	16	22	32	45	63	89	126	179	252
	$16 < m_n \leqq 25$	17	24	34	48	68	96	136	193	272
$280 < d \leqq 560$	$0.5 \leqq m_n \leqq 2.0$	18	26	36	51	73	103	146	206	291
	$2.0 < m_n \leqq 3.5$	18	26	37	52	74	105	148	209	296
	$3.5 < m_n \leqq 6.0$	19	27	38	53	75	106	150	213	301
	$6.0 < m_n \leqq 10$	19	27	39	55	77	109	155	219	310
	$10 < m_n \leqq 16$	20	29	40	57	81	114	161	228	323
	$16 < m_n \leqq 25$	21	30	43	61	86	121	171	242	343

2 Precision Standard for Bevel Gears

Excerpted from JIS B 1704：1978

Gear Tolerances

Accuracy grades	Error	Transverse Module																				
		0.6 to 1							1 to 1.6							1.6 to 2.5						
		Reference diameter（mm）																				
		3 to 6 incl.	6 to 12 incl.	12 to 25 incl.	25 to 50 incl.	50 to 100 incl.	100 to 200 incl.		6 to 12 incl.	12 to 25 incl.	25 to 50 incl.	50 to 100 incl.	100 to 200 incl.	200 to 400 incl.		12 to 25 incl.	25 to 50 incl.	50 to 100 incl.	100 to 200 incl.	200 to 400 incl.	400 to 800 incl.	
0	Single pitch error (±)	4	4	4	4	5	5		4	4	4	5	5	6		4	4	5	5	6	6	
	Pitch variation	5	5	5	5	6	6		5	5	6	6	7	7		5	6	6	7	8	8	
	C pitch error (±)	14	15	16	17	18	20		15	16	17	19	20	22		17	18	19	21	23	26	
	Runout	5	7	10	14	20	28		7	10	14	20	28	40		10	14	20	28	40	56	
1	Single pitch error (±)	6	7	7	7	8	9		7	7	8	8	9	10		7	8	8	9	10	11	
	Pitch variation	8	9	9	10	10	11		9	9	10	11	11	13		10	10	11	12	13	14	
	Accumulative pitch error (±)	25	26	28	30	32	34		27	29	30	32	35	39		30	32	34	36	40	44	
	Runout	7	10	15	21	30	43		10	15	21	30	43	60		15	21	30	43	60	86	
2	Single pitch error (±)	12	12	13	13	14	15		12	13	14	14	16	17		13	14	15	16	17	19	
	Pitch variation	15	16	16	17	18	20		16	17	18	19	20	22		17	18	19	21	23	25	
	Accumulative pitch error (±)	46	48	50	53	57	61		49	52	54	58	62	68		54	56	60	64	69	76	
	Runout	11	15	22	31	45	63		15	22	31	45	63	89		22	31	45	63	89	125	
3	Single pitch error (±)								23	23	25	26	28	30		24	25	27	28	31	33	
	Pitch variation								29	30	32	34	36	39		31	33	35	37	40	43	
	Accumulative pitch error (±)								90	94	98	105	110	120		97	100	105	115	120	135	
	Runout	16	24	33	48	67	95		24	33	48	67	95	135		33	48	67	95	135	190	
4	Single pitch error (±)								41	42	44	46	49	52		43	45	47	50	55	57	
	Pitch variation								53	55	57	60	63	68		56	58	61	65	69	75	
	Accumulative pitch error (±)								165	170	175	185	195	210		170	180	190	200	210	230	
	Runout	25	35	50	71	100	145		35	50	71	100	145	200		50	71	100	145	200	290	
5	Pitch variation															110	115	120	125	132	150	
	Runout	37	52	75	105	150	210		52	75	105	150	210	300		75	105	150	210	300	430	
6	Pitch variation															210	220	240	250	270	290	
	Runout	56	79	110	160	230	320		79	110	160	230	320	450		110	160	230	320	450	640	

Gear Tolerances

Accuracy grades	Error	Transverse Module: 2.5 to 4							Transverse Module: 4 to 6							Transverse Module: 6 to 10						
		Reference diameter (mm)																				
		12 to 25 incl.	25 to 50 incl.	50 to 100 incl.	100 to 200 incl.	200 to 400 incl.	400 to 800 incl.	800 to 1600 incl.		25 to 50 incl.	50 to 100 incl.	100 to 200 incl.	200 to 400 incl.	400 to 800 incl.	800 to 1600 incl.		25 to 50 incl.	50 to 100 incl.	100 to 200 incl.	200 to 400 incl.	400 to 800 incl.	800 to 1600 incl.
0	Single pitch error (±)	5	5	5	6	6	7	8		5	6	6	7	7	8		6	6	7	7	8	9
	Pitch variation	6	6	7	7	8	9	10		7	7	8	9	9	11		8	8	9	9	10	11
	Accumulative pitch error (±)	18	19	21	22	24	27	31		21	22	24	26	29	32		24	25	27	29	32	35
	Runout	10	14	20	28	40	56	79		14	20	28	40	56	79		14	20	28	40	56	79
1	Single pitch error (±)	8	8	9	10	10	12	13		9	10	10	11	12	14		10	11	11	12	13	15
	Pitch variation	10	11	12	12	14	15	17		12	12	13	14	16	18		13	14	15	16	17	19
	Accumulative pitch error (±)	32	33	36	38	42	46	51		36	38	41	45	49	54		41	43	46	49	54	59
	Runout	15	21	30	43	60	86	120		21	30	43	60	86	120		21	30	43	60	86	120
2	Single pitch error (±)	14	15	16	17	18	20	22		16	17	18	19	21	23		18	19	20	21	23	25
	Pitch variation	18	19	20	22	24	26	29		21	22	23	25	27	30		23	24	26	27	30	32
	Accumulative pitch error (±)	57	59	63	67	72	79	88		64	67	72	77	84	92		71	75	79	84	91	100
	Runout	22	31	45	63	89	125	180		31	45	63	89	125	180		31	45	63	89	125	180
3	Single pitch error (±)	25	27	28	30	32	35	38		28	30	31	34	36	40		31	33	34	37	39	43
	Pitch variation	33	34	36	39	41	45	49		37	39	41	44	47	52		41	42	45	48	51	56
	Accumulative pitch error (±)	100	105	110	120	130	140	150		115	120	125	135	145	160		125	130	140	145	155	170
	Runout	33	48	67	95	135	190	270		48	67	95	135	190	270		48	67	95	135	190	270
4	Single pitch error (±)	45	47	50	52	55	59	65		50	52	54	58	62	68		54	56	59	62	67	72
	Pitch variation	59	61	65	67	72	77	84		65	67	71	75	81	88		71	73	77	81	87	100
	Accumulative pitch error (±)	180	185	200	210	220	240	260		200	210	220	230	250	270		220	230	240	250	270	290
	Runout	50	71	100	145	200	290	400		71	100	145	200	290	400		71	100	145	200	290	400
5	Pitch variation	115	120	125	130	135	155	170		125	130	135	150	165	175		135	140	155	165	175	185
	Runout	75	105	150	210	300	430	600		105	150	210	300	430	600		105	150	210	300	430	600
6	Pitch variation	220	240	250	260	280	290	310		250	260	270	290	300	330		270	280	290	310	320	340
	Runout	110	160	230	320	450	640	900		160	230	320	450	640	900		160	230	320	450	640	900
7	Pitch variation	250	360	500	720	1000	1450	2000		360	500	720	1000	1450	2000		360	500	720	1000	1450	2000

3 Backlash Standard for Spur and Helical Gears

Excerpted from the superseded standard, JIS B 1703 : 1976

Table of values for calculation of BACKLASH（JIS Grade 0 and Grade 5）

Unit μm

Transverse module (mm)	Value for calculation of backlash	Reference diameter (mm): 1.5 to 3 incl.		Over 3 to 6 incl.		Over 6 to 12 incl.		1Over 12 to 25 incl.		Over 25 to 50 incl.		Over 50 to 100 incl.		Over 100 to 200 incl.		Over 200 to 400 incl.		Over 400 to 800 incl.		Over 800 to 1600 incl.		Over 1600 to 3200 incl.	
	Gear accuracy grades (JIS)	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5
0.5	Min. value	15		20		25		30		35		45		60		70							
	Max. value	40	70	50	90	60	110	70	130	90	170	110	200	140	250	180	320						
1	Min. value			25		25		35		40		50		60		70		90					
	Max. value			60	100	70	120	80	150	100	180	120	220	150	270	180	330	230	410				
1.5	Min. value					30		35		45		50		60		80		90					
	Max. value					80	140	90	160	110	190	130	230	160	280	190	350	240	420				
2	Min. value							40		50		60		70		80		100		120			
	Max. value							100	180	120	210	140	250	170	300	200	360	240	440	300	540		
2.5	Min. value							45		50		60		70		80		100		120			
	Max. value							110	190	120	220	150	260	170	310	210	380	250	450	310	550		
3	Min. value									50		60		70		90		100		130		150	
	Max. value									130	240	150	280	180	330	220	390	260	470	310	570	380	690
3.5	Min. value									60		60		80		90		110		130		160	
	Max. value									140	250	160	290	190	340	220	400	270	480	320	580	390	700
4	Min. value									60		70		80		90		110		130		160	
	Max. value									150	270	170	310	200	360	230	420	280	500	330	590	400	720
5	Min. value									70		70		90		100		120		140		170	
	Max. value									160	300	190	340	210	390	250	450	290	530	350	620	420	750
6	Min. value									70		80		90		110		120		150		170	
	Max. value									180	330	200	370	230	410	260	480	310	560	360	650	430	780
7	Min. value									80		90		100		110		130		150		180	
	Max. value									200	360	220	390	240	440	280	510	320	580	380	680	450	810
8	Min. value									90		90		110		120		140		160		190	
	Max. value									210	380	240	420	260	470	300	540	340	610	400	710	460	840
10	Min. value											110		120		130		150		170		200	
	Max. value											270	480	300	530	330	590	370	670	430	770	500	900
12	Min. value											120		130		140		160		180		210	
	Max. value											300	540	330	590	360	650	410	730	460	830	530	950
14	Min. value											130		140		160		180		200		220	
	Max. value											330	600	360	650	390	710	440	790	490	890	560	1010
16	Min. value													160		170		190		210		240	
	Max. value													390	710	420	760	470	850	530	950	590	1070
18	Min. value													170		180		200		220		250	
	Max. value													430	770	460	830	500	910	560	1000	630	1130
20	Min. value													180		200		210		240		260	
	Max. value													460	820	490	890	540	960	590	1060	660	1190
22	Min. value															210		230		250		280	
	Max. value															520	950	570	1020	620	1120	690	1250
25	Min. value															230		250		270		300	
	Max. value															570	1030	620	1110	670	1210	740	1330

Calculation of Backlash
Accuracy grade JIS 0

Normal module: 3
Number of teeth: 25 / 50
Helix angle: 35°
Transverse module: $\frac{3}{\cos 35°} = 3.66$
Reference diameter of pinion: 91.5
Reference diameter of gear : 183.0
Pinion
Min. value 70μm , Max. value 170μm
Gear
Min. value 80μm , Max. value 200μm
Backlash
Min. value 1 70 + 080 = 150μm
Max. value 170 + 200 = 370μm

Depending on usage of the gear, the backlash can be set with a value designated for the gear and with a different accuracy grade.

W Unit of tolerance

$W = \sqrt[3]{d_0} + 0.65 m_s$ (μm)

where d_0 : Reference diameter (mm)

m_s : Transverse module (mm)

NOTE 1.
The minimum value to be applied in the case of high-speed operation is $12.5W$.

Equation of Backlash

JIS	Min. value	Max. value
0	$10W$	$25W$
1 NOTE1		$28W$
2 NOTE1		$31.5W$
3		$35.5W$
4		$40W$
5		$45W$
6		$50W$
7		$63W$
8		$90W$

4 Backlash Standard for Bevel Gears

Excerpted from JIS B 1705：1973

Table of values for calculation of BACKLASH (JIS Grade 0 and Grade 4)

Unit μm

Transverse module (mm)	Value for calculation of backlash	Reference diameter (mm): Over 3 to 6 incl.		Over 6 to 12 incl.		Over 12 to 25 incl.		Over 25 to 50 incl.		Over 50 to 100 incl.		Over 100 to 200 incl.		Over 200 to 400 incl.		Over 400 to 800 incl.		Over 800 to 1600 incl.	
	Gear accuracy grades (JIS)	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4
0.5	Min. value	20		25		30		35		45		60							
	Max. value	50	100	60	120	70	150	90	180	110	230	140	280						
1	Min. value	25		25		35		40		50		60							
	Max. value	60	120	70	140	80	160	100	200	120	240	150	300						
1.5	Min. value			30		35		45		50		60		80					
	Max. value			80	150	90	180	110	220	130	260	160	310	190	380				
2	Min. value					40		50		60		70		80		100			
	Max. value					100	200	120	230	140	280	170	330	200	400	240	490		
2.5	Min. value					45		50		60		70		80		100		120	
	Max. value					110	210	120	250	150	290	170	350	210	420	250	500	310	610
3	Min. value					45		50		60		70		90		100		130	
	Max. value					110	230	130	270	150	310	180	360	220	430	260	520	310	630
3.5	Min. value					50		60		60		80		90		110		130	
	Max. value					120	240	140	280	160	320	190	380	220	450	270	540	320	640
4	Min. value					50		60		70		80		90		110		130	
	Max. value					130	260	150	300	170	340	200	400	230	470	280	550	330	660
5	Min. value							70		70		90		100		120		140	
	Max. value							160	330	190	370	210	430	250	500	290	580	350	690
6	Min. value							70		80		90		110		120		150	
	Max. value							180	360	200	410	230	460	260	530	310	620	360	730
7	Min. value							80		90		100		110		130		150	
	Max. value							200	400	220	440	240	490	280	560	320	650	380	760
8	Min. value							90		90		110		120		140		160	
	Max. value							210	430	240	470	260	530	300	600	340	680	400	790
10	Min. value							100		110		120		130		150		170	
	Max. value							240	490	270	540	300	590	330	660	370	750	430	860
12	Min. value									120		130		140		160		180	
	Max. value									300	600	330	660	360	730	410	810	460	920
14	Min. value									130		140		160		180		200	
	Max. value									330	670	360	720	390	790	440	880	490	990
16	Min. value									150		160		170		190		210	
	Max. value									360	730	390	790	420	850	470	940	530	1050
18	Min. value											170		180		200		220	
	Max. value											430	850	460	920	500	1010	560	1120
20	Min. value											180		200		210		240	
	Max. value											460	920	490	990	540	1070	590	1180
22	Min. value											200		210		230		250	
	Max. value											490	980	520	1050	570	1140	620	1250
25	Min. value											210		230		250		270	
	Max. value											540	1070	570	1150	620	1230	670	1340

Calculation of Backlash
Accuracy grade JIS 0

Module: 3
Number of teeth: 25 / 50
Reference diameter of pinion: 75mm
Reference diameter of gear: 150mm
Pinion
Min. value 60μm
Max. value 150μm
Gear
Min. value 70μm
Max. value 180μm
Backlash
Min. value 160 + 070 = 130μm
Max. value 150 + 180 = 330μm

Depending on usage of the gear, the backlash can be set with a value designated for the gear and with a different accuracy grade.

W Unit of tolerance

$W = \sqrt[3]{d_0} + 0.65 m_s$ (μm)

where d_0 : Reference diameter (mm)

m_s : Transverse module (mm)

NOTE 1.

The minimum value to be applied in the case of high-speed operation is $12.5W$.

Equation of Backlash

JIS	Min. value	Max. value
0	$10W$	$25W$
1		$30W$
2		$35.5W$
3		$42.5W$
4		$50W$
5		$60W$
6		$71W$

5 Common Deviations of Hole Dimensions

Excerpted from JIS B 0401– 2:1998

Unit μm

Size Range (mm)		B	C		D			E			F			G		H					
over	up to	B10	C9	C10	D8	D9	D10	E7	E8	E9	F6	F7	F8	G6	G7	H5	H6	H7	H8	H9	H10
—	3	+ 180 + 140	+ 85 + 60	+ 100	+ 34	+ 45 + 20	+ 60	+ 24	+ 28 + 14	+ 39	+ 12	+ 16 + 6	+ 20	+ 8	+ 12 + 2	+ 4	+ 6	+ 10 0	+ 14	+ 25	+ 40
3	6	+ 188 + 140	+ 100 + 70	+ 118	+ 48	+ 60 + 30	+ 78	+ 32	+ 38 + 20	+ 50	+ 18	+ 22 + 10	+ 28	+ 12	+ 16 + 4	+ 5	+ 8	+ 12 0	+ 18	+ 30	+ 48
6	10	+ 208 + 150	+ 116 + 80	+ 138	+ 62	+ 76 + 40	+ 98	+ 40	+ 47 + 25	+ 61	+ 22	+ 28 + 13	+ 35	+ 14	+ 20 + 5	+ 6	+ 9	+ 15 0	+ 22	+ 36	+ 58
10 14	14 18	+ 220 + 150	+ 138 + 95	+ 165	+ 77	+ 93 + 50	+ 120	+ 50	+ 59 + 32	+ 75	+ 27	+ 34 + 16	+ 43	+ 17	+ 24 + 6	+ 8	+ 11	+ 18 0	+ 27	+ 43	+ 70
18 24	24 30	+ 244 + 160	+ 162 + 110	+ 194	+ 98	+ 117 + 65	+ 149	+ 61	+ 73 + 40	+ 92	+ 33	+ 41 + 20	+ 53	+ 20	+ 28 + 7	+ 9	+ 13	+ 21 0	+ 33	+ 52	+ 84
30	40	+ 270 + 170	+ 182 + 120	+ 220	+ 119	+ 142 + 80	+ 180	+ 75	+ 89 + 50	+ 112	+ 41	+ 50 + 25	+ 64	+ 25	+ 34 + 9	+ 11	+ 16	+ 25 0	+ 39	+ 62	+ 100
40	50	+ 280 + 180	+ 192 + 130	+ 230																	
50	65	+ 310 + 190	+ 214 + 140	+ 260	+ 146	+ 174 + 100	+ 220	+ 90	+ 106 + 60	+ 134	+ 49	+ 60 + 30	+ 76	+ 29	+ 40 + 10	+ 13	+ 19	+ 30 0	+ 46	+ 74	+ 120
65	80	+ 320 + 200	+ 224 + 150	+ 270																	
80	100	+ 360 + 220	+ 257 + 170	+ 310	+ 174	+ 207 + 120	+ 260	+ 107	+ 126 + 72	+ 159	+ 58	+ 71 + 36	+ 90	+ 34	+ 47 + 12	+ 15	+ 22	+ 35 0	+ 54	+ 87	+ 140
100	120	+ 380 + 240	+ 267 + 180	+ 320																	
120	140	+ 420 + 260	+ 300 + 200	+ 360	+ 208	+ 245 + 145	+ 305	+ 125	+ 148 + 85	+ 185	+ 68	+ 83 + 43	+ 106	+ 39	+ 54 + 14	+ 18	+ 25	+ 40 0	+ 63	+ 100	+ 160
140	160	+ 440 + 280	+ 310 + 210	+ 370																	
160	180	+ 470 + 310	+ 330 + 230	+ 390																	
180	200	+ 525 + 340	+ 355 + 240	+ 425	+ 242	+ 285 + 170	+ 355	+ 146	+ 172 + 100	+ 215	+ 79	+ 96 + 50	+ 122	+ 44	+ 61 + 15	+ 20	+ 29	+ 46 0	+ 72	+ 115	+ 185
200	225	+ 565 + 380	+ 375 + 260	+ 445																	
225	250	+ 605 + 420	+ 395 + 280	+ 465																	
250	280	+ 690 + 480	+ 430 + 300	+ 510	+ 271	+ 320 + 190	+ 400	+ 162	+ 191 + 110	+ 240	+ 88	+ 108 + 56	+ 137	+ 49	+ 69 + 17	+ 23	+ 32	+ 52 0	+ 81	+ 130	+ 210
280	315	+ 750 + 540	+ 460 + 330	+ 540																	
315	355	+ 830 + 600	+ 500 + 360	+ 590	+ 299	+ 350 + 210	+ 440	+ 182	+ 214 + 125	+ 265	+ 98	+ 119 + 62	+ 151	+ 54	+ 75 + 18	+ 25	+ 36	+ 57 0	+ 89	+ 140	+ 230
355	400	+ 910 + 680	+ 540 + 400	+ 630																	
400	450	+ 1010 + 760	+ 595 + 440	+ 690	+ 327	+ 385 + 230	+ 480	+ 198	+ 232 + 135	+ 290	+ 108	+ 131 + 68	+ 165	+ 60	+ 83 + 20	+ 27	+ 40	+ 63 0	+ 97	+ 155	+ 250
450	500	+ 1090 + 840	+ 635 + 480	+ 730																	

REMARK: For each range shown in the Table, the numerical values given in the upper line indicate upper deviations and those given in the lower line indicate lower deviations.

Unit μm

Size Range (mm)		JS			K			M			N		P		R	S	T	U	X
over	up to	JS5	JS6	JS7	K5	K6	K7	M5	M6	M7	N6	N7	P6	P7	R7	S7	T7	U7	X7
—	3	± 2	± 3	± 5	0 − 4	0 − 6	0 − 10	− 2 − 6	− 2 − 8	− 2 − 12	− 4 − 10	− 4 − 14	− 6 − 12	− 6 − 16	− 10 − 20	− 14 − 24	—	− 18 − 28	− 20 − 30
3	6	± 2.5	± 4	± 6	0 − 5	+ 2 − 6	+ 3 − 9	− 3 − 8	− 1 − 9	0 − 12	− 5 − 13	− 4 − 16	− 9 − 17	− 8 − 20	− 11 − 23	− 15 − 27	—	− 19 − 31	− 24 − 36
6	10	± 3	± 4.5	± 7.5	+ 1 − 5	+ 2 − 7	+ 5 − 10	− 4 − 10	− 3 − 12	0 − 15	− 7 − 16	− 4 − 19	− 12 − 21	− 9 − 24	− 13 − 28	− 17 − 32	—	− 22 − 37	− 28 − 43
10	14	± 4	± 5.5	± 9	+ 2 − 6	+ 2 − 9	+ 6 − 12	− 4 − 12	− 4 − 15	0 − 18	− 9 − 20	− 5 − 23	− 15 − 26	− 11 − 29	− 16 − 34	− 21 − 39	—	− 26 − 44	− 33 − 51
14	18																		− 38 − 56
18	24	± 4.5	± 6.5	±10.5	+ 1 − 8	+ 2 − 11	+ 6 − 15	− 5 − 14	− 4 − 17	0 − 21	− 11 − 24	− 7 − 28	− 18 − 31	− 14 − 35	− 20 − 41	− 27 − 48	—	− 33 − 54	− 46 − 67
24	30																− 33 − 54	− 40 − 61	− 56 − 77
30	40	± 5.5	± 8	±12.5	+ 2 − 9	+ 3 − 13	+ 7 − 18	− 5 − 16	− 4 − 20	0 − 25	− 12 − 28	− 8 − 33	− 21 − 37	− 17 − 42	− 25 − 50	− 34 − 59	− 39 − 64	− 51 − 76	—
40	50																− 45 − 70	− 61 − 86	
50	65	± 6.5	± 9.5	±15	+ 3 − 10	+ 4 − 15	+ 9 − 21	− 6 − 19	− 5 − 24	0 − 30	− 14 − 33	− 9 − 39	− 26 − 45	− 21 − 51	− 30 − 60	− 42 − 72	− 55 − 85	− 76 − 106	—
65	80														− 32 − 62	− 48 − 78	− 64 − 94	− 91 − 121	
80	100	± 7.5	±11	±17.5	+ 2 − 13	+ 4 − 18	+ 10 − 25	− 8 − 23	− 6 − 28	0 − 35	− 16 − 38	− 10 − 45	− 30 − 52	− 24 − 59	− 38 − 73	− 58 − 93	− 78 − 113	− 111 − 146	—
100	120														− 41 − 76	− 66 − 101	− 91 − 126	− 131 − 166	
120	140	± 9	±12.5	±20	+ 3 − 15	+ 4 − 21	+ 12 − 28	− 9 − 27	− 8 − 33	0 − 40	− 20 − 45	− 12 − 52	− 36 − 61	− 28 − 68	− 48 − 88	− 77 − 117	− 107 − 147	—	—
140	160														− 50 − 90	− 85 − 125	− 119 − 159		
160	180														− 53 − 93	− 93 − 133	− 131 − 171		
180	200	±10	±14.5	±23	+ 2 − 18	+ 5 − 24	+ 13 − 33	− 11 − 31	− 8 − 37	0 − 46	− 22 − 51	− 14 − 60	− 41 − 70	− 33 − 79	− 60 − 106	− 105 − 151	—	—	—
200	225														− 63 − 109	− 113 − 159			
225	250														− 67 − 113	− 123 − 169			
250	280	±11.5	±16	±26	+ 3 − 20	+ 5 − 27	+ 16 − 36	− 13 − 36	− 9 − 41	0 − 52	− 25 − 57	− 14 − 66	− 47 − 79	− 36 − 88	− 74 − 126	—	—	—	—
280	315														− 78 − 130				
315	355	±12.5	±18	±28.5	+ 3 − 22	+ 7 − 29	+ 17 − 40	− 14 − 39	− 10 − 46	0 − 57	− 26 − 62	− 16 − 73	− 51 − 87	− 41 − 98	− 87 − 144	—	—	—	—
355	400														− 93 − 150				
400	450	±13.5	±20	±31.5	+ 2 − 25	+ 8 − 32	+ 18 − 45	− 16 − 43	− 10 − 50	0 − 63	− 27 − 67	− 17 − 80	− 55 − 95	− 45 − 108	− 103 − 166	—	—	—	—
450	500														− 109 − 172				

REMARK: For each range shown in the Table, the numerical values given in the upper line indicate upper deviations and those given in the lower line indicate lower deviations.

6 Common Deviations of Shaft Dimensions

Excerpted from JIS B 0401-2:1998

Unit μm

<table>
<tr><th colspan="2">Size Range (mm)</th><th>b</th><th>c</th><th colspan="2">d</th><th colspan="3">e</th><th colspan="3">f</th><th colspan="3">g</th><th colspan="6">h</th></tr>
<tr><th>over</th><th>up to</th><th>b9</th><th>c9</th><th>d8</th><th>d9</th><th>e7</th><th>e8</th><th>e9</th><th>f6</th><th>f7</th><th>f8</th><th>g4</th><th>g5</th><th>g6</th><th>h4</th><th>h5</th><th>h6</th><th>h7</th><th>h8</th><th>h9</th></tr>
<tr><td rowspan="2">—</td><td rowspan="2">3</td><td>−140</td><td>−60</td><td colspan="2">−20</td><td colspan="3">−14</td><td colspan="3">−6</td><td colspan="3">−2</td><td colspan="6">0</td></tr>
<tr><td>−165</td><td>−85</td><td>−34</td><td>−45</td><td>−24</td><td>−28</td><td>−39</td><td>−12</td><td>−16</td><td>−20</td><td>−5</td><td>−6</td><td>−8</td><td>−3</td><td>−4</td><td>−6</td><td>−10</td><td>−14</td><td>−25</td></tr>
<tr><td rowspan="2">3</td><td rowspan="2">6</td><td>−140</td><td>−70</td><td colspan="2">−30</td><td colspan="3">−20</td><td colspan="3">−10</td><td colspan="3">−4</td><td colspan="6">0</td></tr>
<tr><td>−170</td><td>−100</td><td>−48</td><td>−60</td><td>−32</td><td>−38</td><td>−50</td><td>−18</td><td>−22</td><td>−28</td><td>−8</td><td>−9</td><td>−12</td><td>−4</td><td>−5</td><td>−8</td><td>−12</td><td>−18</td><td>−30</td></tr>
<tr><td rowspan="2">6</td><td rowspan="2">10</td><td>−150</td><td>−80</td><td colspan="2">−40</td><td colspan="3">−25</td><td colspan="3">−13</td><td colspan="3">−5</td><td colspan="6">0</td></tr>
<tr><td>−186</td><td>−116</td><td>−62</td><td>−76</td><td>−40</td><td>−47</td><td>−61</td><td>−22</td><td>−28</td><td>−35</td><td>−9</td><td>−11</td><td>−14</td><td>−4</td><td>−6</td><td>−9</td><td>−15</td><td>−22</td><td>−36</td></tr>
<tr><td rowspan="2">10</td><td rowspan="2">14</td><td rowspan="2">−150</td><td rowspan="2">−95</td><td colspan="2" rowspan="2">−50</td><td colspan="3" rowspan="2">−32</td><td colspan="3" rowspan="2">−16</td><td colspan="3" rowspan="2">−6</td><td colspan="6" rowspan="2">0</td></tr>
<tr></tr>
<tr><td rowspan="2">14</td><td rowspan="2">18</td><td rowspan="2">−193</td><td rowspan="2">−138</td><td rowspan="2">−77</td><td rowspan="2">−93</td><td rowspan="2">−50</td><td rowspan="2">−59</td><td rowspan="2">−75</td><td rowspan="2">−27</td><td rowspan="2">−34</td><td rowspan="2">−43</td><td rowspan="2">−11</td><td rowspan="2">−14</td><td rowspan="2">−17</td><td rowspan="2">−5</td><td rowspan="2">−8</td><td rowspan="2">−11</td><td rowspan="2">−18</td><td rowspan="2">−27</td><td rowspan="2">−43</td></tr>
<tr></tr>
<tr><td rowspan="2">18</td><td rowspan="2">24</td><td rowspan="2">−160</td><td rowspan="2">−110</td><td colspan="2" rowspan="2">−65</td><td colspan="3" rowspan="2">−40</td><td colspan="3" rowspan="2">−20</td><td colspan="3" rowspan="2">−7</td><td colspan="6" rowspan="2">0</td></tr>
<tr></tr>
<tr><td rowspan="2">24</td><td rowspan="2">30</td><td rowspan="2">−212</td><td rowspan="2">−162</td><td rowspan="2">−98</td><td rowspan="2">−117</td><td rowspan="2">−61</td><td rowspan="2">−73</td><td rowspan="2">−92</td><td rowspan="2">−33</td><td rowspan="2">−41</td><td rowspan="2">−53</td><td rowspan="2">−13</td><td rowspan="2">−16</td><td rowspan="2">−20</td><td rowspan="2">−6</td><td rowspan="2">−9</td><td rowspan="2">−13</td><td rowspan="2">−21</td><td rowspan="2">−33</td><td rowspan="2">−52</td></tr>
<tr></tr>
<tr><td rowspan="2">30</td><td rowspan="2">40</td><td>−170</td><td>−120</td><td colspan="2" rowspan="2">−80</td><td colspan="3" rowspan="2">−50</td><td colspan="3" rowspan="2">−25</td><td colspan="3" rowspan="2">−9</td><td colspan="6" rowspan="2">0</td></tr>
<tr><td>−232</td><td>−182</td></tr>
<tr><td rowspan="2">40</td><td rowspan="2">50</td><td>−180</td><td>−130</td><td rowspan="2">−119</td><td rowspan="2">−142</td><td rowspan="2">−75</td><td rowspan="2">−89</td><td rowspan="2">−112</td><td rowspan="2">−41</td><td rowspan="2">−50</td><td rowspan="2">−64</td><td rowspan="2">−16</td><td rowspan="2">−20</td><td rowspan="2">−25</td><td rowspan="2">−7</td><td rowspan="2">−11</td><td rowspan="2">−16</td><td rowspan="2">−25</td><td rowspan="2">−39</td><td rowspan="2">−62</td></tr>
<tr><td>−242</td><td>−192</td></tr>
<tr><td rowspan="2">50</td><td rowspan="2">65</td><td>−190</td><td>−140</td><td colspan="2" rowspan="2">−100</td><td colspan="3" rowspan="2">−60</td><td colspan="3" rowspan="2">−30</td><td colspan="3" rowspan="2">−10</td><td colspan="6" rowspan="2">0</td></tr>
<tr><td>−264</td><td>−214</td></tr>
<tr><td rowspan="2">65</td><td rowspan="2">80</td><td>−200</td><td>−150</td><td rowspan="2">−146</td><td rowspan="2">−174</td><td rowspan="2">−90</td><td rowspan="2">−106</td><td rowspan="2">−134</td><td rowspan="2">−49</td><td rowspan="2">−60</td><td rowspan="2">−76</td><td rowspan="2">−18</td><td rowspan="2">−23</td><td rowspan="2">−29</td><td rowspan="2">−8</td><td rowspan="2">−13</td><td rowspan="2">−19</td><td rowspan="2">−30</td><td rowspan="2">−46</td><td rowspan="2">−74</td></tr>
<tr><td>−274</td><td>−224</td></tr>
<tr><td rowspan="2">80</td><td rowspan="2">100</td><td>−220</td><td>−170</td><td colspan="2" rowspan="2">−120</td><td colspan="3" rowspan="2">−72</td><td colspan="3" rowspan="2">−36</td><td colspan="3" rowspan="2">−12</td><td colspan="6" rowspan="2">0</td></tr>
<tr><td>−307</td><td>−257</td></tr>
<tr><td rowspan="2">100</td><td rowspan="2">120</td><td>−240</td><td>−180</td><td rowspan="2">−174</td><td rowspan="2">−207</td><td rowspan="2">−107</td><td rowspan="2">−126</td><td rowspan="2">−159</td><td rowspan="2">−58</td><td rowspan="2">−71</td><td rowspan="2">−90</td><td rowspan="2">−22</td><td rowspan="2">−27</td><td rowspan="2">−34</td><td rowspan="2">−10</td><td rowspan="2">−15</td><td rowspan="2">−22</td><td rowspan="2">−35</td><td rowspan="2">−54</td><td rowspan="2">−87</td></tr>
<tr><td>−327</td><td>−267</td></tr>
<tr><td rowspan="2">120</td><td rowspan="2">140</td><td>−260</td><td>−200</td><td colspan="2" rowspan="3">−145</td><td colspan="3" rowspan="3">−85</td><td colspan="3" rowspan="3">−43</td><td colspan="3" rowspan="3">−14</td><td colspan="6" rowspan="3">0</td></tr>
<tr><td>−360</td><td>−300</td></tr>
<tr><td rowspan="2">140</td><td rowspan="2">160</td><td>−280</td><td>−210</td></tr>
<tr><td>−380</td><td>−310</td><td rowspan="3">−208</td><td rowspan="3">−245</td><td rowspan="3">−125</td><td rowspan="3">−148</td><td rowspan="3">−185</td><td rowspan="3">−68</td><td rowspan="3">−83</td><td rowspan="3">−106</td><td rowspan="3">−26</td><td rowspan="3">−32</td><td rowspan="3">−39</td><td rowspan="3">−12</td><td rowspan="3">−18</td><td rowspan="3">−25</td><td rowspan="3">−40</td><td rowspan="3">−63</td><td rowspan="3">−100</td></tr>
<tr><td rowspan="2">160</td><td rowspan="2">180</td><td>−310</td><td>−230</td></tr>
<tr><td>−410</td><td>−330</td></tr>
<tr><td rowspan="2">180</td><td rowspan="2">200</td><td>−340</td><td>−240</td><td colspan="2" rowspan="3">−170</td><td colspan="3" rowspan="3">−100</td><td colspan="3" rowspan="3">−50</td><td colspan="3" rowspan="3">−15</td><td colspan="6" rowspan="3">0</td></tr>
<tr><td>−455</td><td>−355</td></tr>
<tr><td rowspan="2">200</td><td rowspan="2">225</td><td>−380</td><td>−260</td></tr>
<tr><td>−495</td><td>−375</td><td rowspan="3">−242</td><td rowspan="3">−285</td><td rowspan="3">−146</td><td rowspan="3">−172</td><td rowspan="3">−215</td><td rowspan="3">−79</td><td rowspan="3">−96</td><td rowspan="3">−122</td><td rowspan="3">−29</td><td rowspan="3">−35</td><td rowspan="3">−44</td><td rowspan="3">−14</td><td rowspan="3">−20</td><td rowspan="3">−29</td><td rowspan="3">−46</td><td rowspan="3">−72</td><td rowspan="3">−115</td></tr>
<tr><td rowspan="2">225</td><td rowspan="2">250</td><td>−420</td><td>−280</td></tr>
<tr><td>−535</td><td>−395</td></tr>
<tr><td rowspan="2">250</td><td rowspan="2">280</td><td>−480</td><td>−300</td><td colspan="2" rowspan="2">−190</td><td colspan="3" rowspan="2">−110</td><td colspan="3" rowspan="2">−56</td><td colspan="3" rowspan="2">−17</td><td colspan="6" rowspan="2">0</td></tr>
<tr><td>−610</td><td>−430</td></tr>
<tr><td rowspan="2">280</td><td rowspan="2">315</td><td>−540</td><td>−330</td><td rowspan="2">−271</td><td rowspan="2">−320</td><td rowspan="2">−162</td><td rowspan="2">−191</td><td rowspan="2">−240</td><td rowspan="2">−88</td><td rowspan="2">−108</td><td rowspan="2">−137</td><td rowspan="2">−33</td><td rowspan="2">−40</td><td rowspan="2">−49</td><td rowspan="2">−16</td><td rowspan="2">−23</td><td rowspan="2">−32</td><td rowspan="2">−52</td><td rowspan="2">−81</td><td rowspan="2">−130</td></tr>
<tr><td>−670</td><td>−460</td></tr>
<tr><td rowspan="2">315</td><td rowspan="2">355</td><td>−600</td><td>−360</td><td colspan="2" rowspan="2">−210</td><td colspan="3" rowspan="2">−125</td><td colspan="3" rowspan="2">−62</td><td colspan="3" rowspan="2">−18</td><td colspan="6" rowspan="2">0</td></tr>
<tr><td>−740</td><td>−500</td></tr>
<tr><td rowspan="2">355</td><td rowspan="2">400</td><td>−680</td><td>−400</td><td rowspan="2">−299</td><td rowspan="2">−350</td><td rowspan="2">−182</td><td rowspan="2">−214</td><td rowspan="2">−265</td><td rowspan="2">−98</td><td rowspan="2">−119</td><td rowspan="2">−151</td><td rowspan="2">−36</td><td rowspan="2">−43</td><td rowspan="2">−54</td><td rowspan="2">−18</td><td rowspan="2">−25</td><td rowspan="2">−36</td><td rowspan="2">−57</td><td rowspan="2">−89</td><td rowspan="2">−140</td></tr>
<tr><td>−820</td><td>−540</td></tr>
<tr><td rowspan="2">400</td><td rowspan="2">450</td><td>−760</td><td>−440</td><td colspan="2" rowspan="2">−230</td><td colspan="3" rowspan="2">−135</td><td colspan="3" rowspan="2">−68</td><td colspan="3" rowspan="2">−20</td><td colspan="6" rowspan="2">0</td></tr>
<tr><td>−915</td><td>−595</td></tr>
<tr><td rowspan="2">450</td><td rowspan="2">500</td><td>−840</td><td>−480</td><td rowspan="2">−327</td><td rowspan="2">−385</td><td rowspan="2">−198</td><td rowspan="2">−232</td><td rowspan="2">−290</td><td rowspan="2">−108</td><td rowspan="2">−131</td><td rowspan="2">−165</td><td rowspan="2">−40</td><td rowspan="2">−47</td><td rowspan="2">−60</td><td rowspan="2">−20</td><td rowspan="2">−27</td><td rowspan="2">−40</td><td rowspan="2">−63</td><td rowspan="2">−97</td><td rowspan="2">−155</td></tr>
<tr><td>−995</td><td>−635</td></tr>
</table>

REMARK: For each range shown in the Table, the numerical values given in the upper line indicate upper deviations and those given in the lower line indicate lower deviations.

Unit μm

Size Range (mm)		js				k			m			n	p	r	s	t	u	x
over	up to	js4	js5	js6	js7	k4	k5	k6	m4	m5	m6	n6	p6	r6	s6	t6	u6	x6
—	3	± 1.5	± 2	± 3	± 5	+ 3	+ 4 0	+ 6	+ 5	+ 6 + 2	+ 8	+ 10 + 4	+ 12 + 6	+ 16 + 10	+ 20 + 14	—	+ 24 + 18	+ 26 + 20
3	6	± 2	± 2.5	± 4	± 6	+ 5	+ 6 + 1	+ 9	+ 8	+ 9 + 4	+ 12	+ 16 + 8	+ 20 + 12	+ 23 + 15	+ 27 + 19	—	+ 31 + 23	+ 36 + 28
6	10	± 2	± 3	± 4.5	± 7.5	+ 5	+ 7 + 1	+ 10	+ 10	+ 12 + 6	+ 15	+ 19 + 10	+ 24 + 15	+ 28 + 19	+ 32 + 23	—	+ 37 + 28	+ 43 + 34
10	14	± 2.5	± 4	± 5.5	± 9	+ 6	+ 9 + 1	+ 12	+ 12	+ 15 + 7	+ 18	+ 23 + 12	+ 29 + 18	+ 34 + 23	+ 39 + 28	—	+ 44 + 33	+ 51 + 40
14	18																	+ 56 + 45
18	24	± 3	± 4.5	± 6.5	±10.5	+ 8	+ 11 + 2	+ 15	+ 14	+ 17 + 8	+ 21	+ 28 + 15	+ 35 + 22	+ 41 + 28	+ 48 + 35	—	+ 54 + 41	+ 67 + 54
24	30															+ 54 + 41	+ 61 + 48	+ 77 + 64
30	40	± 3.5	± 5.5	± 8	±12.5	+ 9	+ 13 + 2	+ 18	+ 16	+ 20 + 9	+ 25	+ 33 + 17	+ 42 + 26	+ 50 + 34	+ 59 + 43	+ 64 + 48	+ 76 + 60	—
40	50															+ 70 + 54	+ 86 + 70	
50	65	± 4	± 6.5	± 9.5	±15	+ 10	+ 15 + 2	+ 21	+ 19	+ 24 + 11	+ 30	+ 39 + 20	+ 51 + 32	+ 60 + 41	+ 72 + 53	+ 85 + 66	+ 106 + 87	—
65	80													+ 62 + 43	+ 78 + 59	+ 94 + 75	+ 121 + 102	
80	100	± 5	± 7.5	±11	±17.5	+ 13	+ 18 + 3	+ 25	+ 23	+ 28 + 13	+ 35	+ 45 + 23	+ 59 + 37	+ 73 + 51	+ 93 + 71	+ 113 + 91	+ 146 + 124	—
100	120													+ 76 + 54	+ 101 + 79	+ 126 + 104	+ 166 + 144	
120	140	± 6	± 9	±12.5	±20	+ 15	+ 21 + 3	+ 28	+ 27	+ 33 + 15	+ 40	+ 52 + 27	+ 68 + 43	+ 88 + 63	+ 117 + 92	+ 147 + 122	—	—
140	160													+ 90 + 65	+ 125 + 100	+ 159 + 134		
160	180													+ 93 + 68	+ 133 + 108	+ 171 + 146		
180	200	± 7	±10	±14.5	±23	+ 18	+ 24 + 4	+ 33	+ 31	+ 37 + 17	+ 46	+ 60 + 31	+ 79 + 50	+ 106 + 77	+ 151 + 122	—	—	—
200	225													+ 109 + 80	+ 159 + 130			
225	250													+ 113 + 84	+ 169 + 140			
250	280	± 8	±11.5	±16	±26	+ 20	+ 27 + 4	+ 36	+ 36	+ 43 + 20	+ 52	+ 66 + 34	+ 88 + 56	+ 126 + 94	—	—	—	—
280	315													+ 130 + 98				
315	355	± 9	±12.5	±18	±28.5	+ 22	+ 29 + 4	+ 40	+ 39	+ 46 + 21	+ 57	+ 73 + 37	+ 98 + 62	+ 144 + 108	—	—	—	—
355	400													+ 150 + 114				
400	450	±10	±13.5	±20	±31.5	+ 25	+ 32 + 5	+ 45	+ 43	+ 50 + 23	+ 63	+ 80 + 40	+ 108 + 68	− 166 − 126	—	—	—	—
450	500													− 172 − 132				

REMARK: For each range shown in the Table, the numerical values given in the upper line indicate upper deviations and those given in the lower line indicate lower deviations.

7 Centre Holes

Excerpted from JIS B 1011:1987

Types of Centre Holes

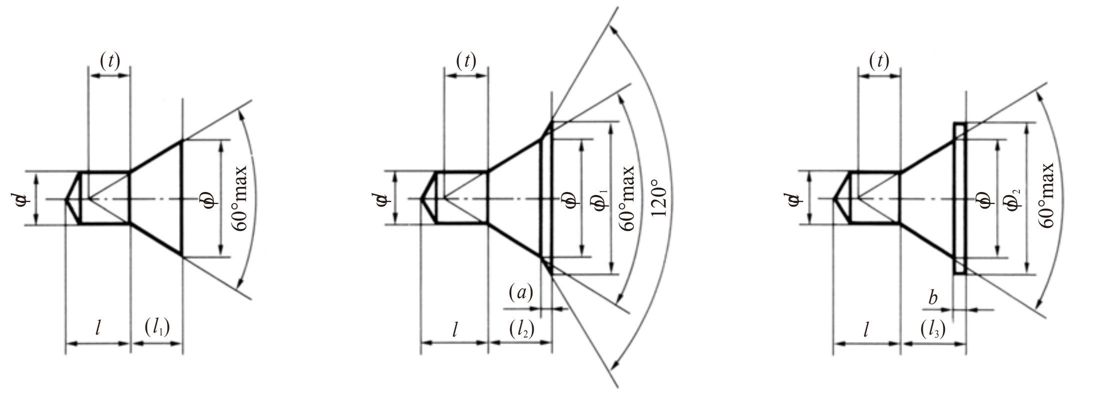
Angle			Form
60-degree	75-degree	90-degree	A
			B
			C
	—		R

NOTE 1. Angles indicated here denote the angles of applicable center holes

2. Use of the 75-degree center hole should be avoided.

60-degree Centre Holes

Type A　　Type B　　Type C



Unit mm

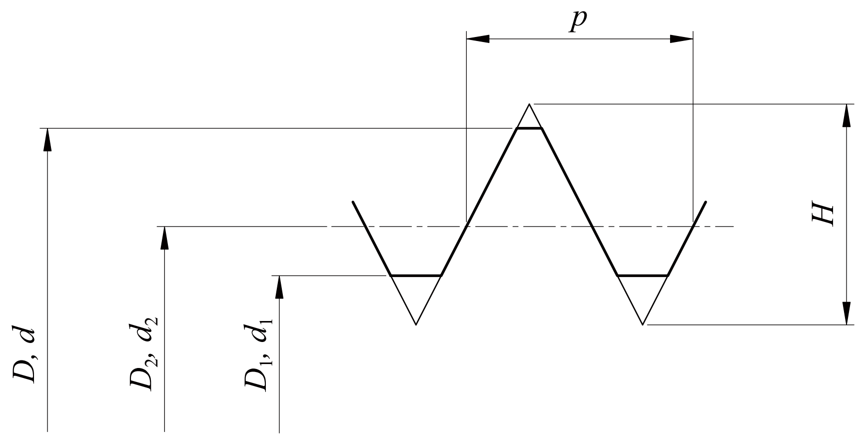
d Nominal diameter	D	D_1	D_2 (Min.)	l NOTE 1 (Max.)	b (Approx.)	Informative note				
						l_1	l_2	l_3	t	a
(0.5)	1.06	1.6	1.6	1	0.2	0.48	0.64	0.68	0.5	0.16
(0.63)	1.32	2	2	1.2	0.3	0.6	0.8	0.9	0.6	0.2
(0.8)	1.7	2.5	2.5	1.5	0.3	0.78	1.01	1.08	0.7	0.23
1	2.12	3.15	3.15	1.9	0.4	0.97	1.27	1.37	0.9	0.3
(1.25)	2.65	4	4	2.2	0.6	1.21	1.6	1.81	1.1	0.39
1.6	3.35	5	5	2.8	0.6	1.52	1.99	2.12	1.4	0.47
2	4.25	6.3	6.3	3.3	0.8	1.95	2.54	2.75	1.8	0.59
2.5	5.3	8	8	4.1	0.9	2.42	3.2	3.32	2.2	0.78
3.15	6.7	10	10	4.9	1	3.07	4.03	4.07	2.8	0.96
4	8.5	12.5	12.5	6.2	1.3	3.9	5.05	5.2	3.5	1.15
(5)	10.6	16	16	7.5	1.6	4.85	6.41	6.45	4.4	1.56
6.3	13.2	18	18	9.2	1.8	5.98	7.36	7.78	5.5	1.38
(8)	17	22.4	22.4	11.5	2	7.79	9.35	9.79	7	1.56
10	21.2	28	28	14.2	2.2	9.7	11.66	11.9	8.7	1.96

NOTE 1: The value l shall not be less than the value t .

REMARK: The in nominal diameters in parentheses should be avoided if possible.

8 Metric Coarse Screw Threads – Minor Diameter

Excerpted from JIS B 0205-4 : 2001, JIS B 0209-2 : 2001



Fit Quality : Medium quality
Length of Fit : Medium grade
Tolerance Class : 6H

Basic Dimensions

Nominal Diameter (Major Diameter of External Thread)	Pitch	Minor Diameter of Internal Thread		
d	p	Reference Value D_1	Maximum	Minimum
M1.6	0.35	1.221	1.321	1.221
(M1.8)	0.35	1.421	1.521	1.421
M2	0.4	1.567	1.679	1.567
(M2.2)	0.45	1.713	1.838	1.713
M2.5	0.45	2.013	2.138	2.013
M3	0.5	2.459	2.599	2.459
(M3.5)	0.6	2.851	3.011	2.85
M4	0.7	3.242	3.422	3.242
(M4.5)	0.75	3.688	3.878	3.688
M5	0.8	4.134	4.334	4.134
M6	1	4.918	5.154	4.917
(M7)	1	5.918	6.154	5.917
M8	1.25	6.647	6.912	6.647
M10	1.5	8.376	8.676	8.376
M12	1.75	10.106	10.441	10.106
(M14)	2	11.835	12.21	11.835
M16	2	13.835	14.21	13.835
(M18)	2.5	15.294	15.744	15.294
M20	2.5	17.294	17.744	17.294
(M22)	2.5	19.294	19.744	19.294
M24	3	20.753	21.253	20.752
(M27)	3	23.753	24.253	23.752
M30	3.5	26.211	26.771	26.211

d : Basic dimension of major diameter of external thread

p : Pitch

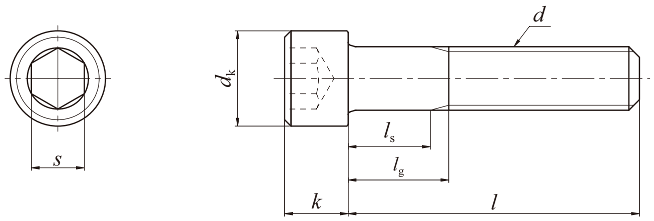
D_1 : Reference dimension of minor diameter of internal thread

$D_1 = D - 1.0825p$

D : Reference dimension of major diameter of internal thread

9 Dimensions of Hexagon Socket Head Cap Screws

Excerpted from JIS B 1176 : 2006



Nominal designation of threads (d)			M3		M4		M5		M6		M8		M10		M12		(M14)(4)		M16		M20	
Pitch (p)			0.5		0.7		0.8		1		1.25		1.5		1.75		2		2		2.5	
d_k	Maximum (1)		5.5		7		8.5		10		13		16		18		21		24		30	
	Maximum (2)		5.68		7.22		8.72		10.22		13.27		16.27		18.27		21.33		24.33		30.33	
	Minimum		5.32		6.78		8.28		9.78		12.73		15.73		17.73		20.67		23.67		29.67	
k	Maximum		3.00		4.00		5.00		6.00		8.00		10.00		12.00		14.00		16.00		20.00	
	Minimum		2.86		3.82		4.82		5.7		7.64		9.64		11.57		13.57		15.57		19.48	
s (3)	Nominal		2.5		3		4		5		6		8		10		12		14		17	
	Maximum		2.58		3.08		4.095		5.14		6.14		8.175		10.175		12.212		14.212		17.23	
	Minimum		2.52		3.02		4.02		5.02		6.02		8.025		10.025		12.032		14.032		17.05	
l (5)			l_s or l_g																			
			l_s	l_g	l_s	l_g	l_s	l_g	l_s	l_g	l_s	l_g	l_s	l_g	l_s	l_g	ls	l_g	l_s	l_g	l_s	l_g
Nominal length	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
5	4.76	5.24																				
6	5.76	6.24																				
8	7.71	8.29																				
10	9.71	10.29																				
12	11.65	12.35																				
16	15.65	16.35																				
20	19.58	20.42	4.5	7																		
25	24.58	25.42	9.5	12	6.5	10	4	8														
30	29.58	30.42			11.5	15	9	13	6	11												
35	34.5	35.5			16.5	20	14	18	11	16												
40	39.5	40.5					19	23	16	21	5.75	12										
45	44.5	45.5					24	28	21	26	10.75	17	5.5	13								
50	49.5	50.5							26	31	15.75	22	10.5	18								
55	54.4	55.6							31	36	20.75	27	15.5	23	10.25	19						
60	59.4	60.6									25.75	32	20.5	28	15.25	24	10	20				
65	64.4	65.6									30.75	37	25.5	33	20.25	29	15	25	11	21		
70	69.4	70.6									35.75	42	30.5	38	25.25	34	20	30	16	26		
80	79.4	80.6									45.75	52	40.5	48	35.25	44	30	40	26	36	15.5	28
90	89.3	90.7											50.5	58	45.25	54	40	50	36	46	25.5	38
100	99.3	100.7											60.5	68	55.25	64	50	60	46	56	35.5	48
110	109.3	110.7													65.25	74	60	70	56	66	45.5	58
120	119.3	120.7													75.25	84	70	80	66	76	55.5	68
130	129.2	130.8															80	90	76	86	65.5	78
140	139.2	140.8															90	100	86	96	75.5	88
150	149.2	150.8																	96	106	85.5	98
160	159.2	160.8																	106	116	95.5	108
180	179.2	180.8																			115.5	128
200	199.1	200.9																			135.5	148

NOTE 1 Applied to the head without knurls

NOTE 2 Applied to the head with knurls

NOTE 3 For more information on hexagon hole dimension s and gauge inspection e, refer to JIS B 1016.

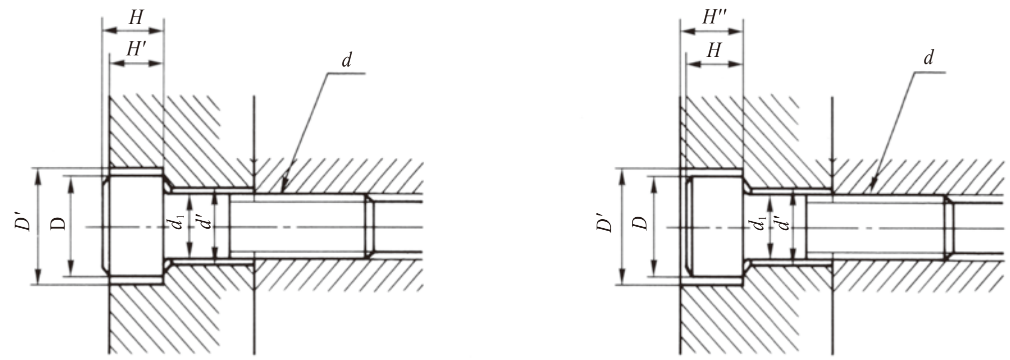
NOTE 4 Nominal designation of threads, marked with parentheses ( ), should not be used where practically possible.

NOTE 5 The range of popular nominal lengths is the area marked with the bold lines in the table. The shaded area shows complete threads, and the length of incomplete threads under the head is to be within $3p$. The values not in the shaded area denote the value l_s and l_g , where the equation is:

$$l_{gMax.} = l_{Nominal} - b \quad l_{sMin.} = l_{gMax.} - 5p$$

10 Dimensions of Counterbores and Bolt Holes for Hexagon Socket Head Cap Screws   Excerpted from the superseded standard, JIS B 1176 : 1974

Unit mm



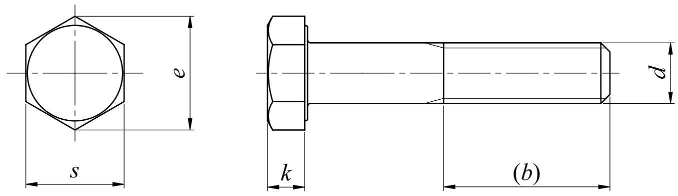
Designation of threads(d)	M3	M4	M5	M6	M8	M10	M12	M14	M16	M18	M20	M22	M24	M27	M30
d_1	3	4	5	6	8	10	12	13.5	15.5	17.5	20	22	24	27	30
d'	3.4	4.5	5.5	6.6	9	11	14 (13.5)	16 (15.5)	18 (17.5)	20	22	24	26	30	33
D	5.5	7	8.5	10	13	16	18	21	24	27	30	33	36	40	45
(D)	6.5	8	9.5	11	14	17.5	20	23	26	29	32	35	39	43	48
H	3	4	5	6	8	10	12	14	16	18	20	22	24	27	30
(H)	2.7	3.6	4.6	5.5	7.4	9.2	11	12.8	14.5	16.5	18.5	20.5	22.5	25	28
(H')	3.3	4.4	5.4	6.5	8.6	10.8	13	15.2	17.5	21.5	19.5	23.5	25.5	29	32

REMARK: The diameter (d') of bolt hole in the table conforms to bolt hole diameter Grade 2 specified in JIS B 1001–1968 (Diameter of bolt hole and counterbore.
For diameters of M12, M14 and M16, the values marked with parentheses ( ), are of the Grade 2 specified in JIS B 1001-1985.

11 Dimension for Hexagon Head Bolt with Nominal Diameter Body - Coarse Threads – (Grade A First Choice)

Excerpted from JIS B 1180 : 2004

Unit : mm

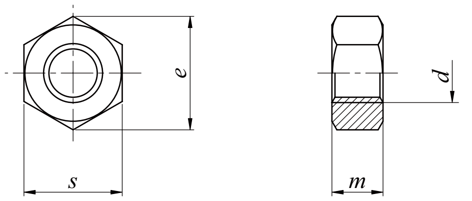


Nominal designation of threads (d)		M3	M4	M5	M6	M8	M10	M12	M16	M20
b (Reference)	NOTE (1)	12	14	16	18	22	26	30	38	46
e	Min.	6.01	7.66	8.79	11.05	14.38	17.77	20.03	26.75	33.53
k	Reference Dimension	2	2.8	3.5	4	5.3	6.4	7.5	10	12.5
	Max.	2.125	2.925	3.65	4.15	5.45	6.58	7.68	10.18	12.715
	Min.	1.875	2.675	3.35	3.85	5.15	6.22	7.32	9.82	12.285
s	Max. (Reference Dimension)	5.50	7.00	8.00	10.00	13.00	16.00	18.00	24.00	30.00
	Min.	5.32	6.78	7.78	9.78	12.73	15.73	17.73	23.67	29.67

NOTE(1) $l_{\text{Nominal diameter}} \leqq 125$mm

12 Hexagon Nuts - Style1 - Coarse Threads (First Choice)

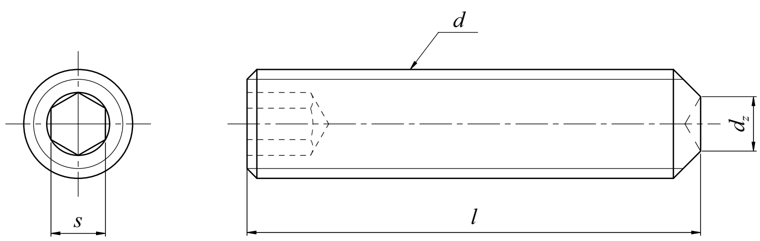
Excerpted from JIS B 1181 : 2004



Nominal designation of threads (*d*)		M3	M4	M5	M6	M8	M10	M12	M16	M20
e	Min.	6.01	7.66	8.79	11.05	14.38	17.77	20.03	26.75	32.95
m	Max.	2.40	3.2	4.7	5.2	6.80	8.40	10.80	14.8	18.0
	Min.	2.15	2.9	4.4	4.9	6.44	8.04	10.37	14.1	16.9
s	Max. (Reference Dimension)	5.50	7.00	8.00	10.00	13.00	16.00	18.00	24.00	30.00
	Min.	5.32	6.78	7.78	9.78	12.73	15.73	17.73	23.67	29.16

13 Dimensions of Hexagon Socket Set Screws – Cup Point

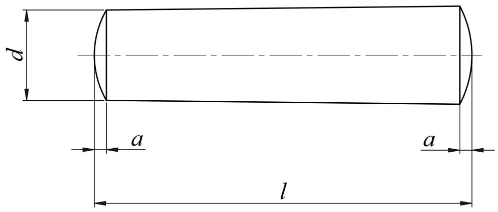
Excerpted from JIS B 1177 : 2007



Nominal designation of threads(*d*)			M3	M4	M5	M6	M8	M10
Pitch (*p*)			0.5	0.7	0.8	1	1.25	1.5
d_z	Max.		1.40	2.00	2.50	3.00	5.00	6.00
	Min.		1.15	1.75	2.25	2.75	4.70	5.70
s (1)	Nominal diameter		1.5	2	2.5	3	4	5
	Max.		1.545	2.045	2.560	3.071	4.084	5.084
	Min.		1.520	2.020	2.520	3.020	4.020	5.020
l			(Reference) Approximate weight per 1000 pieces Unit: kg (Density: 7.85 kg/dm3)					
Nominal length	Max.	Min.						
2.5	2.3	2.7						
3	2.8	3.2	0.1					
4	3.76	4.24	0.14	0.23				
5	4.76	5.24	0.18	0.305	0.42			
6	5.76	6.24	0.22	0.38	0.54	0.74		
8	7.71	8.29	0.3	0.53	0.78	1.09	1.88	
10	9.71	10.29	0.38	0.68	1.02	1.44	2.51	3.72
12	11.65	12.35	0.46	0.83	1.26	1.79	3.14	4.73
16	15.65	16.35	0.62	1.13	1.74	2.49	4.4	6.73
20	19.58	20.42		1.4	2.22	3.19	5.66	8.72
25	24.58	25.42			2.82	4.07	7.24	11.2
30	29.58	30.42				4.94	8.81	13.7
35	34.5	35.5					10.4	16.2
40	39.5	40.5					12	18.7
45	44.5	45.5						21.2
50	49.5	50.5						23.6

14 Dimensions of Taper Pins

Excerpted from JIS B 1352 : 1988

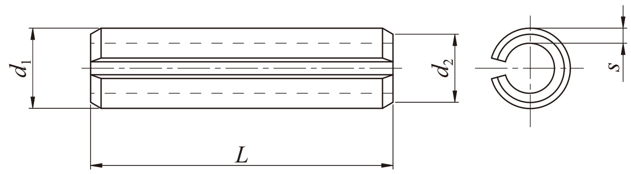


Nominal diameter			1.2	1.5	2	2.5	3	4	5	6	8	10
d	Basic Dimension		1.2	1.5	2	2.5	3	4	5	6	8	10
	Tolerance (h10)		0 -0.040					0 -0.048			0 -0.058	
a	Approximate		0.16	0.2	0.25	0.3	0.4	0.5	0.63	0.8	1	1.2
l												
Nominal length	Minimum	Maximum										
5	4.75	5.25										
6	5.75	6.25										
8	7.75	8.25										
10	9.75	10.3										
12	11.5	12.5										
14	13.5	14.5										
16	15.5	16.5										
18	17.5	18.5										
20	19.5	20.5										
22	21.5	22.5										
24	23.5	24.5										
26	25.5	26.5										
28	27.5	28.5										
30	29.5	30.5										
32	31.5	32.5										
35	34.5	35.5										
40	39.5	40.5										
45	44.5	45.5										
50	49.5	50.5										
55	54.5	55.5										
60	59.5	60.5										
65	64.5	65.5										
70	69.5	70.5										
75	74.5	75.5										

NOTE : Recommendable lengths (*l*) for the nominal diameter of the pins, are marked with bold lines in the table.

15 Spring-type Straight Pins – Slotted

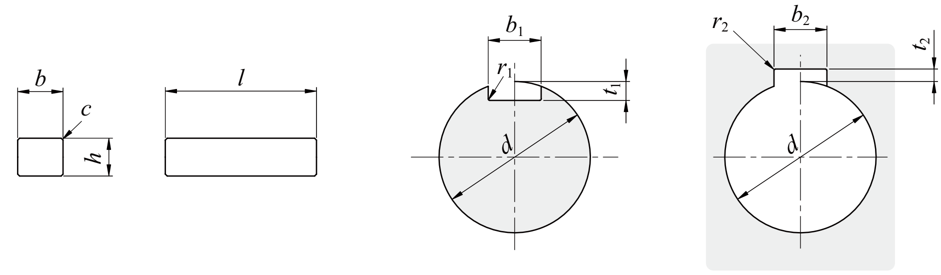
Excerpted from JIS B 2808 : 2005



Nominal diameter		1	1.2	1.4	1.5	1.6	2	2.5	3	4	5	6
Base diameter d_1	Max.	1.2	1.4	1.6	1.7	1.8	2.25	2.75	3.25	4.4	5.4	6.4
	Min.	1.1	1.3	1.5	1.6	1.7	2.15	2.65	3.15	4.2	5.2	6.2
Chamfer diameter d_2	Max.	0.9	1.1	1.3	1.4	1.5	1.9	2.4	2.9	3.9	4.8	5.8
Shear load kN (Min.)		0.69	1.02	1.35	1.55	1.68	2.76	4.31	6.2	10.8	17.25	24.83
Applicable pins (Reference)	Diameter	1	1.2	1.4	1.5	1.6	2	2.5	3	4	5	6
	Tolerance	+ 0.08 0					+ 0.09 0			+ 0.12 0		
Length L	Tolerance											
4	+ 0.5 0											
5												
6												
8												
10												
12	+ 1 0											
14												
16												
18												
20												
22												
25												
28												
32												
36												
40												
45												
50												
56	+ 1.5 0											
63												

NOTE：Recommendable lengths are marked with bold lines in the table.

16 Keys and Keyways **Excerpted from JIS B 1301:1996**



Keyway of shaft Keyway of hole

Unit mm

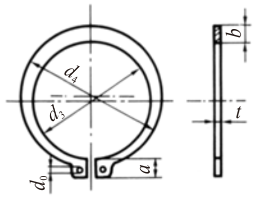
Nominal size of key $b \times h$	Dimension of Parallel Key: b Basic dimension	b Tolerance (h9)	h Basic dimension	h Tolerance		c	l	Dimension of Parallel Keyway: Basic dimension of b_1 and b_2	Tight-fit b_1 and b_2 Tolerance (P9)	Normal type b_1 Tolerance (N9)	Normal type b_2 Tolerance (J_S9)	r_1 and r_2	Basic dimension of t_1	Basic dimension of t_2	Basic dimension of t_1 and t_2	Note: Applicable shaft dia. d
2× 2	2	0 − 0.025	2	0 − 0.025	h 9	0.16 ~ 0.25	6 ~ 20	2	− 0.006 − 0.031	− 0.004 − 0.029	±0.0125	0.08 ~ 0.16	1.2	1.0	+ 0.1 0	6 ~ 8
3× 3	3		3				6 ~ 36	3					1.8	1.4		8 ~ 10
4× 4	4	0 − 0.030	4	0 − 0.030			8 ~ 45	4	− 0.012 − 0.042	0 − 0.030	±0.0150		2.5	1.8		10 ~ 12
5× 5	5		5			0.25 ~ 0.40	10 ~ 56	5				0.16 ~ 0.25	3.0	2.3		12 ~ 17
6× 6	6		6				14 ~ 70	6					3.5	2.8		17 ~ 22
(7× 7)	7	0 − 0.036	7	0 − 0.036			16 ~ 80	7	− 0.015 − 0.051	0 − 0.036	±0.0180		4.0	3.3	+ 0.2 0	20 ~ 25
8× 7	8		7	0 − 0.090	h11		18 ~ 90	8					4.0	3.3		22 ~ 30
10× 8	10		8			0.40 ~ 0.60	22 ~ 110	10				0.25 ~ 0.40	5.0	3.3		30 ~ 38
12× 8	12	0 − 0.043	8				28 ~ 140	12	− 0.018 − 0.061	0 − 0.043	±0.0215		5.0	3.3		38 ~ 44
14× 9	14		9				36 ~ 160	14					5.5	3.8		44 ~ 50
(15×10)	15		10				40 ~ 180	15					5.0	5.3		50 ~ 55
16×10	16		10				45 ~ 180	16					6.0	4.3		50 ~ 58
18×11	18		11	0 − 0.110			50 ~ 200	18					7.0	4.4		58 ~ 65
20×12	20	0 − 0.052	12			0.60 ~ 0.80	56 ~ 220	20	− 0.022 − 0.074	0 − 0.052	±0.0260	0.40 ~ 0.60	7.5	4.9		65 ~ 75
22×14	22		14				63 ~ 250	22					9.0	5.4		75 ~ 85
(24×16)	24		16				70 ~ 280	24					8.0	8.4		80 ~ 90
25×14	25		14				70 ~ 280	25					9.0	5.4		85 ~ 95
28×16	28		16				80 ~ 320	28					10.0	6.4		95 ~ 110
32×18	32	0 − 0.062	18				90 ~ 360	32	− 0.026 − 0.088	0 − 0.062	±0.0310		11.0	7.4		110 ~ 130
(35×22)	35		22	0 − 0.130		1.00 ~ 1.20	100 ~ 400	35				0.70 ~ 1.00	11.0	11.4	+ 0.3 0	125 ~ 140
36×20	36		20				—	36					12.0	8.4		130 ~ 150
(38×24)	38		24				—	38					12.0	12.4		140 ~ 160
40×22	40		22				—	40					13.0	9.4		150 ~ 170
(42×26)	42		26				—	42					13.0	13.4		160 ~ 180
45×25	45		25				—	45					15.0	10.4		170 ~ 200
50×28	50		28				—	50					17.0	11.4		200 ~ 230
56×32	56	0 − 0.074	32	0 − 0.160		1.60 ~ 2.00	—	56	− 0.032 − 0.106	0 − 0.074	±0.0370	1.20 ~ 1.60	20.0	12.4		230 ~ 260
63×32	63		32				—	63					20.0	12.4		260 ~ 290
70×36	70		36				—	70					22.0	14.4		290 ~ 330
80×40	80		40			2.50 ~ 3.00	—	80				2.00 ~ 2.50	25.0	15.4		330 ~ 380
90×45	90	0 − 0.087	45				—	90	− 0.037 − 0.124	0 − 0.087	±0.0435		28.0	17.4		380 ~ 440
100×50	100		50				—	100					31.0	19.5		440 ~ 500

NOTE (1) Applicable shaft diameter in the table is determined from the troupe corresponding to the key strength, and shown as a guide for general-purpose use0. If key size is adequate for transmitted torque, the shaft which is thicker than applicable shaft diameter, can be used. In that case, it is better to modify t_1 and t_2 so that the key surface can equally contact with the shaft and the bore. The shaft which is thicker than the applicable shaft diameter, should not be used.

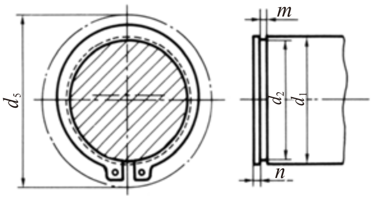
REMARK The nominal diameters marked with parentheses ( ), are not used for new designs as they are not defined by the corresponding international standards.

17 Retaining Rings Excerpted from JIS B 2804:2001

17.1 C-type Retaining Ring (Shaft Use)



When the retaining ring is set on an applicable shaft, the position of the hole with diameter d_0 shall not sink in the groove.



The dimension d_5 is the maximum diameter of outer periphery when the retaining ring is set on the shaft.

NOTE (1) Designation 1 should be given priority. As the need arises apply designation 2 and 3 in that order. The abolition of designation 3 is scheduled in the future.

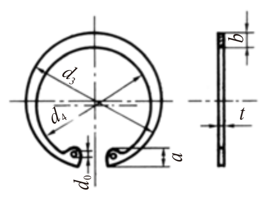
(2) Thickness value (t) 1.5 mm can be applied instead of (t) 1.6 mm for the time being. In that case, m should be 1.65 mm.

REMARKS 1. The smallest width of the retaining ring should not be shorter than basic dimension t.

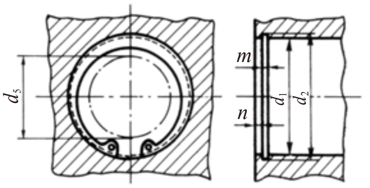
2. The dimensions applied are the recommended dimensions.

Designation (1)		Retaining Rings							Applicable shaft (For reference)						
		d_3		t		b	a	d_0	d_5	d_1	d_2		m		n
1	2	Basic dimension	Tolerance	Basic dimension	Tolerance	Approx.	Approx.	Min.			Basic dimension	Tolerance	Basic dimension	Tolerance	Min.
10		9.3	±0.15	1	±0.05	1.6	3.0	1.2	17	10	9.6	0 −0.09	1.15	+ 0.14 0	1.5
	11	10.2				1.8	3.1		18	11	10.5	0 − 0.11			
12		11.1	±0.18			1.8	3.2	1.5	19	12	11.5				
14		12.9				2.0	3.4	1.7	22	14	13.4				
15		13.8				2.1	3.5		23	15	14.3				
16		14.7				2.2	3.6		24	16	15.2				
17		15.7				2.2	3.7		25	17	16.2				
18		16.5		1.2	±0.06	2.6	3.8		26	18	17.0		1.35		
	19	17.5				2.7	3.8	2	27	19	18.0				
20		18.5	±0.20			2.7	3.9		28	20	19.0	0 − 0.21			
22		20.5				2.7	4.1		31	22	21.0				
	24	22.2				3.1	4.2		33	24	22.9				
25		23.2				3.1	4.3		34	25	23.9				
	26	24.2				3.1	4.4		35	26	24.9				
28		25.9		(2) 1.6		3.1	4.6		38	28	26.6		(2) 1.75		
30		27.9				3.5	4.8		40	30	28.6				
32		29.6				3.5	5.0	2.5	43	32	30.3	0 − 0.25			
35		32.2	±0.25			4.0	5.4		46	35	33.0				
	36	33.2		1.8	±0.07	4.0	5.4		47	36	34.0		1.95		2
	38	35.2				4.5	5.6		50	38	36.0				
40		37.0	±0.40			4.5	5.8		53	40	38.0				
	42	38.5				4.5	6.2		55	42	39.5				
45		41.5				4.8	6.3		58	45	42.5				
	48	44.5				4.8	6.5		62	48	45.5				
50		45.8		2		5.0	6.7		64	50	47.0		2.2		
55		50.8	±0.45			5.0	7.0		70	55	52.0	0			
	56	51.8				5.0	7.0		71	56	53.0				
60		55.8				5.5	7.2		75	60	57.0				
65		60.8		2.5	±0.08	6.4	7.4		81	65	62.0		2.7		2.5
70		65.5				6.4	7.8		86	70	67.0				
75		70.5				7.0	7.9		92	75	72.0				
80		74.5				7.4	8.2		97	80	76.5				
85		79.5		3	±0.09	8.0	8.4	3	103	85	81.5	0 − 0.35	3.2	+ 0.18 0	3
90		84.5	±0.55			8.0	8.7		108	90	86.5				
95		89.5				8.6	9.1		114	95	91.5				
100		94.5				9.0	9.5		119	100	96.5				
	105	98.0		4		9.5	9.8		125	105	101.0	0 − 0.54	4.2		4
110		103.0				9.5	10.0		131	110	106.0				
120		113.0				10.3	10.9		143	120	116.0				

17.2 C-type Retaining Ring (Hole Use)



When the retaining ring is set on an applicable hole, the position of the hole with diameter d_0 shall not sink in the groove.



The dimension d_5 is the minimum diameter of inner periphery when the retaining ring is set in the hole.

Designation (1)		Retaining ring							Applicable shaft (For reference)						
		d_3		t		b	a	d_0	d_5	d_1	d_2		m		n
1	2	Basic dimension	Tolerance	Basic dimension	Tolerance	Approx.	Approx.	Min.			Basic dimension	Tolerance	Basic dimension	Tolerance	Min.
10		10.7	±0.18	1	±0.05	1.8	3.1	1.2	3	10	10.4	+ 0.11 0	1.15	+ 0.14 0	1.5
11		11.8				1.8	3.2		4	11	11.4				
12		13.0				1.8	3.3	1.5	5	12	12.5				
	13	14.1				1.8	3.5		6	13	13.6				
14		15.1				2.0	3.6	1.7	7	14	14.6				
	15	16.2				2.0	3.6		8	15	15.7				
16		17.3				2.0	3.7		8	16	16.8				
	17	18.3				2.0	3.8		9	17	17.8				
18		19.5	±0.20			2.5	4.0		10	18	19.0	+ 0.21 0			
19		20.5				2.5	4.0	2	11	19	20.0				
20		21.5				2.5	4.0		12	20	21.0				
22		23.5				2.5	4.1		13	22	23.0				
	24	25.9		1.2	±0.06	2.5	4.3		15	24	25.2		1.35		
25		26.9				3.0	4.4		16	25	26.2				
	26	27.9				3.0	4.6		16	26	27.2				
28		30.1	±0.25			3.0	4.6		18	28	29.4				
30		32.1				3.0	4.7		20	30	31.4	+ 0.25 0			
32		34.4				3.5	5.2	2.5	21	32	33.7				

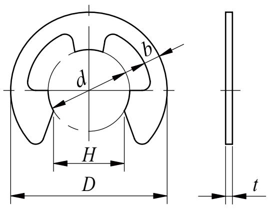
NOTE (1)Designation 1 should be given priority. As the need arises apply designation 2 in that order.
(2)Thickness value (t) 1.5 mm can be applied instead of (t) 1.6 mm for the time being. In that case, m should be 1.65 mm.

REMARKS
1. The smallest width of the retaining ring should not be shorter than basic dimension t .
2. The dimensions applied are the recommended dimensions.
3. The desirable dimension for d_4 is: $d_4 = d_3 - (1.4 \sim 1.5)\,b$

Designation (1)		Retaining Rings							Applicable shaft (For reference)						
		d_3		t		b	a	d_0	d_5	d_1	d_2		m		n
1	2	Basic dimension	Tolerance	Basic dimension	Tolerance	Approx.	Approx.	Min.			Basic dimension	Tolerance	Basic dimension	Tolerance	Min.
35		37.8	±0.25	(2) 1.6	±0.06	3.5	5.2	2.5	24	35	37.0	+ 0.25 0	(2) 1.75	+ 0.14 0	2
	36	38.8				3.5	5.2		25	36	38.0				
37		39.8				3.5	5.2		26	37	39.0				
	38	40.8				4.0	5.3		27	38	40.0				
40		43.5	±0.40	1.8	±0.07	4.0	5.7		28	40	42.5		1.95		
42		45.5				4.0	5.8		30	42	44.5				
45		48.5				4.5	5.9		33	45	47.5				
47		50.5	±0.45			4.5	6.1		34	47	49.5				
	48	51.5				4.5	6.2		35	48	50.5	+ 0.30 0			
50		54.2		2		4.5	6.5		37	50	53.0		2.2		
52		56.2				5.1	6.5		39	52	55.0				
55		59.2				5.1	6.5		41	55	58.0				
	56	60.2				5.1	6.6		42	56	59.0				
60		64.2				5.5	6.8		46	60	63.0				
62		66.2				5.5	6.9		48	62	65.0				
	63	67.2				5.5	6.9		49	63	66.0				
	65	69.2		2.5	±0.08	5.5	7.0		50	65	68.0		2.7		2.5
68		72.5				6.0	7.4		53	68	71.0				
	70	74.5				6.0	7.4		55	70	73.0				
72		76.5				6.6	7.4		57	72	75.0				
75		79.5				6.6	7.8		60	75	78.0				
80		85.5	±0.55			7.0	8.0		64	80	83.5	+ 0.35 0			
85		90.5		3	±0.09	7.0	8.0	3	69	85	88.5		3.2		3
90		95.5				7.6	8.3		73	90	93.5				
95		100.5				8.0	8.5		77	95	98.5				
100		105.5				8.3	8.8		82	100	103.5				
	105	112.0				8.9	9.1		86	105	109.0	+ 0.54 0	4.2	+ 0.18 0	4
110		117.0				8.9	10.2		89	110	114.0				
	112	119.0		4		8.9	10.2		90	112	116.0				
	115	122.0	±0.65			9.5	10.2		94	115	119.0				
120		127.0				9.5	10.7		98	120	124.0	+ 0.63 0			
125		132.0				10.0	10.7	3.5	103	125	129.0				

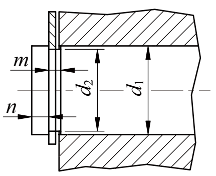
17.3 E-type Retaining Ring

Free State



The shape is only an example.

Usage State



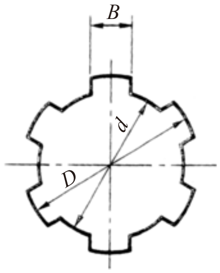
Designation	Retaining Rings									Applicable shaft (For reference)						
	d (1)		D		H		t		b	d_1		d_2		m		n
	Basic dimension	Tolerance	Basic dimension	Tolerance	Basic dimension	Tolerance	Min.	Tolerance	Approx.	Over	Below	Basic dimension	Tolerance	Basic dimension	Tolerance	Min.
0.8	0.8	0 − 0.08	2	±0.1	0.7	0 −0.25	0.2	±0.02	0.3	1.0	1.4	0.8	+ 0.05 0	0.30	+ 0.05 0	0.4
1.2	1.2	0 −0.09	3		1.0		0.3	±0.025	0.4	1.4	2.0	1.2	+ 0.06 0	0.40		0.6
1.5	1.5		4	±0.2	1.3		0.4	±0.03	0.6	2.0	2.5	1.5		0.50		0.8
2	2.0		5		1.7		0.4		0.7	2.5	3.2	2.0				1.0
2.5	2.5		6		2.1		0.4		0.8	3.2	4.0	2.5				
3	3.0		7		2.6		0.6	±0.04	0.9	4.0	5.0	3.0		0.70	+ 0.10 0	
4	4.0	0 −0.12	9		3.5	0 −0.30	0.6		1.1	5.0	7.0	4.0	+ 0.075 0			1.2
5	5.0		11		4.3		0.6		1.2	6.0	8.0	5.0				
6	6.0		12		5.2		0.8		1.4	7.0	9.0	6.0		0.90		
7	7.0	0 −0.15	14		6.1	0 −0.35	0.8		1.6	8.0	11.0	7.0	+ 0.09 0			1.5
8	8.0		16		6.9		0.8		1.8	9.0	12.0	8.0				1.8
9	9.0		18		7.8		0.8		2.0	10.0	14.0	9.0				2.0
10	10.0	0 −0.18	20	±0.3	8.7	0 −0.45	1.0	±0.05	2.2	11.0	15.0	10.0	+ 0.11 0	1.15		
12	12.0		23		10.4		1.0		2.4	13.0	18.0	12.0				2.5
15	15.0		29		13.0		(2) 1.6	±0.06	2.8	16.0	24.0	15.0		(2) 1.75	+ 0.14 0	3.0
19	19.0	0 −0.21	37		16.5		(2) 1.6		4.0	20.0	31.0	19.0	+ 0.13 0			3.5
24	24.0		44		20.8	0 −0.50	2.0	±0.07	5.0	25.0	38.0	24.0		2.20		4.0

NOTE (1) Cylindrical gauge is used for measurement d .
(2) Thickness value (t) 1.5 mm can be applied instead of (t) 1.6 mm for the time being. In that case m should be 1.65 mm.

REMARK: Applicable shaft diameters in the table are recommended values, shown only as reference.

18 Straight-sided Splines

Excerpted from JIS B 1601:1996



N : No. of grooves
d : Small diameter
D : Large diameter
B : Spline width

Basic Dimensions of Hole and Shaft

d mm	For light loads				For medium loads			
	Designation *N* × *d* × *D*	*N* No. of grooves	*D* mm	*B* mm	Designation *N* × *d* × *D*	*N* No. of grooves	*D* mm	*B* mm
11	—	—	—	—	6 × 11 × 14	6	14	3
13	—	—	—	—	6 × 13 × 16	6	16	3.5
16	—	—	—	—	6 × 16 × 20	6	20	4
18	—	—	—	—	6 × 18 × 22	6	22	5
21	—	—	—	—	6 × 21 × 25	6	25	5
23	6 × 23 × 26	6	26	6	6 × 23× 28	6	28	6
26	6 × 26 × 30	6	30	6	6 × 26 × 32	6	32	6
28	6 × 28 × 32	6	32	7	6 × 28 × 34	6	34	7
32	8 × 32 × 36	8	36	6	8 × 32 × 38	8	38	6
36	8 × 36 × 40	8	40	7	8 × 36 × 42	8	42	7
42	8 × 42 × 46	8	46	8	8 × 42 × 48	8	48	8
46	8 × 46 × 50	8	50	9	8 × 46 × 54	8	54	9
52	8 × 52 × 58	8	58	10	8 × 52 × 60	8	60	10
56	8 × 56 × 62	8	62	10	8 × 56 × 65	8	65	10
62	8 × 62 × 68	8	68	12	8 × 62 × 72	8	72	12
72	10 × 72 × 78	10	78	12	10 × 72 × 82	10	82	12
82	10 × 82 × 88	10	88	12	10 × 82 × 92	10	92	12
92	10 × 92 × 98	10	98	14	10 × 92 ×102	10	102	14
102	10 ×102 ×108	10	108	16	10 ×102 ×112	10	112	16
112	10 ×112 ×120	10	120	18	10 ×112 ×125	10	125	18

Dimension Tolerance for the Hole and Shaft

Hole Tolerance						Shaft Tolerance			Coupling Form
No treatment applied after broaching			Heat treatment applied after broaching						
B	*D*	*d*	*B*	*D*	*d*	*B*	*D*	*d*	
H9	H10	H7	H11	H10	H7	d10	a11	f7	Free
						f9	a11	g7	Sliding
						h10	a11	h7	Fixed

Tolerance and Fits

		Width *B*		Minor dia. *d*	Major dia. *D*	Reference
		When hub is not hardened	When hub is hardened			Basis for selection of fit
Hole	Sliding and fixing	D9	F10	H7	H11	
Shaft	For sliding	f 9	d 9	e8	a11	For general applications, where fitting length is approximately twice the minor dia. or more (long fit). Where precision fit is not required.
		h8	e8	f 7		For general applications, where fitting length is not less than approximately twice the minor dia. (short fit). Where precision fitting is required.
		js7(1) or k7(2)	f7	g6		Where precision fit is particularly required.
	For fixing	n7	h7	js7	a11	For general applications.
		p6	h6	js6		For precision fit.
		s6	js6	k6		
		s6(1) or u6(2)	js6(1) or k6(2)	m6		For firm fixing.
		u6	m6	n6		For applications in which no removing is performed.

The data in this table is applicable for Straight-sided Splines defined by the former JIS B1601.

NOTES (1) Applicable where the width is 6 mm and under.
(2) Applicable where the width is over 6 mm.

REMARK: While dimension tolerances for the width (B) and the small diameter (d) are related to each other, dimension symbols must be selected from the fields in the same line. For example, if f7 is selected for a small diameter, h8 should be selected for the width of the hole, with no quenching treatment.

19 Permissible Deviations in Dimensions without Tolerance Indication for Injection Molded Products

Excerpted from JIS B 0405:1991

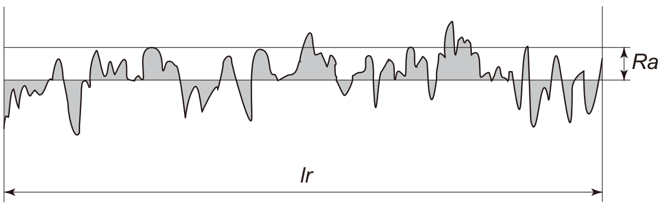
Unit mm

Div. of dimensions \ Grade	Fine Grade	Medium Grade	Coarse Grade
From 0.5 up to 3	±0.05	±0.1	±0.2
Over 3 up to 6			±0.3
Over 6 up to 30	±0.1	±0.2	±0.5
Over 30 up to 120	±0.15	±0.3	±0.8
Over 120 up to 400	±0.2	±0.5	±1.2
Over 400 up to 1000	±0.3	±0.8	±2
Over 1000 up to 2000	±0.5	±1.2	±3

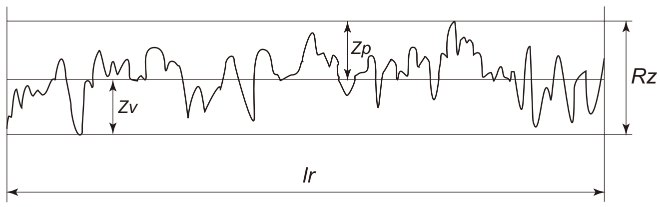
20 Surface Roughness Excerpted from JIS B 0601: 2001

Surface Roughness

① Arithmetic Mean Deviation of the Profile (*Ra*)
Arithmetic mean deviation of the profile denotes the average absolute value of the height $Z(x)$ of the roughness profile on the reference length (lr).



② Maximum Height of the Profile (*Rz*)
The maximum height of the profile is the distance between the maximum peak height (Zp) and the maximum valley depth (Zv) of the roughness profile on the reference length (lr).



NOTE : In JIS B 0601: 1994, the symbol R_z is used to indicate the "Ten-Point Height of Irregularities".

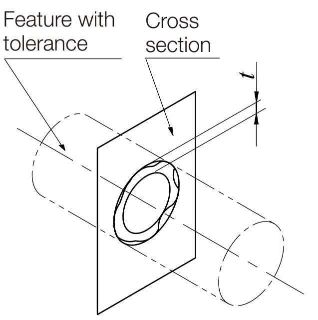
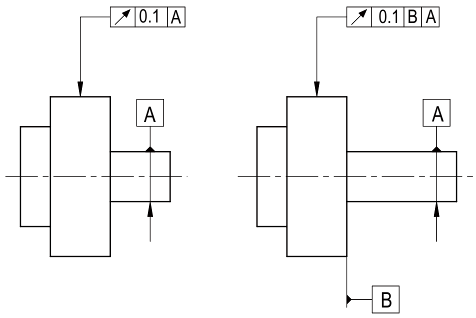
③ Maximum Height of the Cross Section (*Pt*)
The maximum height of the cross section is the distance between the maximum peak height (Zp) and the maximum valley depth (Zv) of cross-sectional profile on the evaluation length (ln).

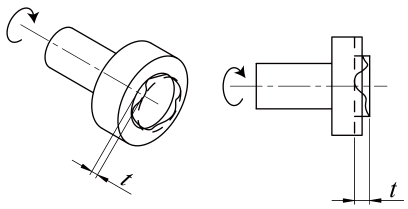
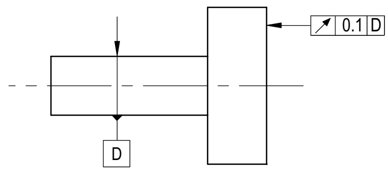
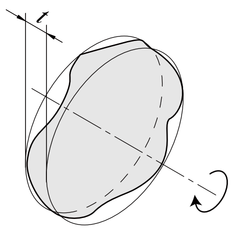
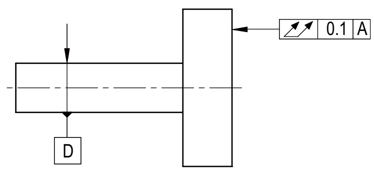
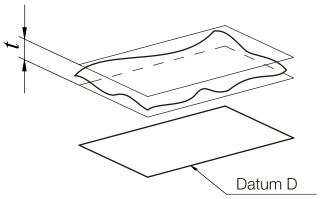
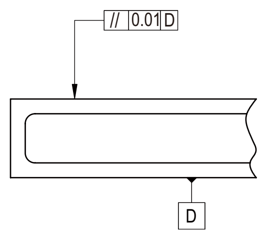
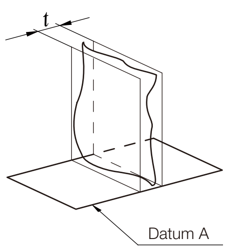
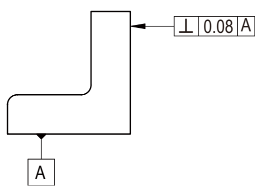
Table 1 Points of Revised Standards

Parameter (Roughness)	Cross-sectional	JIS B0601 1970	1982	1994	2001
Maximum height	Roughness Profile	*R*max	*R*max	—	*Pt*
Ten- point height of irregulari-		*R*z	*R*z	—	—
Centerline average roughness	Roughness Profile	*R*a	*R*a	(*R*a75) Appendix	(*Ra*75) Reference
Arithmetic Mean Deviation of the		—	—	*R*a	*Ra*
Maximum Height of the Profile		—	—		*Rz*
Ten-Point Height of Irregularities		—	—	*R*z	(*Rz*JIS) Reference

【Description】
JIS B 0601-1994 (Surface Roughness – Definitions and designation) was revised in 2001 and was replaced by JIS B 0601-2001 (Geometrical Product Specifications (GPS)– Surface texture : Profile method– Terms : definitions and surface texture parameters)
The title of the standard has changed, and the contents have also changed.
Centerline average roughness (Ra75) and Ten-point height of irregularities (Rz JIS) are described in the appendix as a reference.

21 Geometrical Symbols for Gear Design Excerpted from JIS B 0021:1998

Symbol	Tolerance Zone	Description
↗	Circular Runout Tolerance – Radial Runout Circular runout tolerance zone is controlled within the arbitrary cross section, which is right-angled to the axis line of two circles corresponding to datum axis line, and the radius disengages at distance *t*.  Generally, the runout is applicable for complete rotation around the shaft, however, it can be controlled to apply partially on one revolution.	Actual circular runout tolerance, in rotational direction, must be less than 0.1 on the arbitrary cross section, at the point of when it rotates around datum axis line A and contacts with datum plane B. at the same time. 

<table>
<tr><th>Symbol</th><th>Tolerance Zone</th><th>Description</th></tr>
<tr><td>↗</td><td>Circular Runout Tolerance – Axial Runout
The tolerance zone is controlled at the position in the arbitrary radial direction, by two circles apart at distance t, which are in the cylindrical section, with the axis line corresponding to the datum.

</td><td>For the cylindrical shaft, corresponding to the datum axis line D, the actual line in the axial direction must be positioned between two circles, apart at the distance 0.1.

</td></tr>
<tr><td>⌰</td><td>Total Runout Tolerance
The tolerance zone disengages at the distance t, and is controlled by two parallel planes, which are at right angles to the datum.

</td><td>Actual surface must disengage at the distance 0.1, and must be positioned between two the parallel planes, which are right-angled to the datum axis line D.

</td></tr>
<tr><td>//</td><td>Parallelism Tolerance
The tolerance zone disengages at the distance t, and is controlled by two parallel planes, which are parallel to the datum plane.

</td><td>Actual surface must disengage at the distance 0.01, and must be positioned between two parallel planes, which are parallel to the datum plane D.

</td></tr>
<tr><td>⊥</td><td>Squareness Tolerance
The tolerance zone disengages at the distance t, and is controlled by two parallel planes, which are at right angles to the datum.

</td><td>Actual surface must disengage at the distance 0.08, and must be positioned between two parallel planes, which are at right angles to the datum plane A.

</td></tr>
</table>

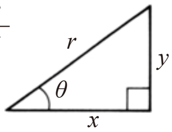
< NUMERICAL EXPRESSION, UNIT AND OTHER DATA >

1 Mathematical formulas

① Trigonometric functions sin, cos, tan

$\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$, $\tan\theta = \frac{y}{x}$

$\sin^2\theta + \cos^2\theta = 1$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$



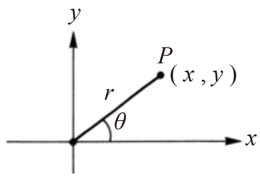
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta$

② The relationship between rectangular coordinates (x, y) and polar coordinates (r, θ)

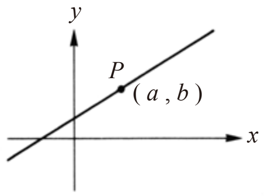
$x = r\cos\theta$, $y = r\sin\theta$, t: $\frac{y}{x}$

$r^2 = x^2 + y^2$



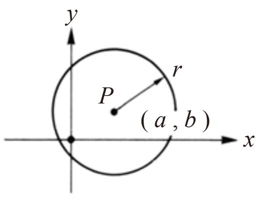
③ The equation for the straight line which passes the point (a, b) with an incantation

$(y - b) = m(x - a)$



④ The equation for the circle, the center of which is the point (a, b), with the radius r

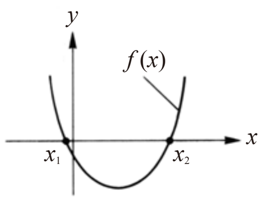
$(x - a)^2 + (y - b)^2 = r^2$



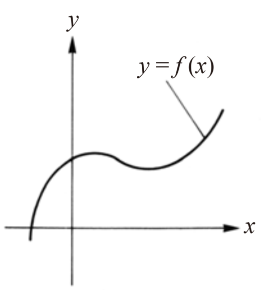
⑤ The root of the quadratic

$y = ax^2 + bx + c = 0 \ (a \neq 0)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



⑥ Function $y = f(x)$ and derivative $y' = f'(x)$



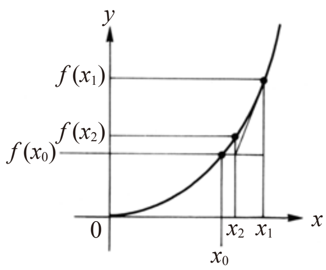
Function $y = f(x)$ and derivative $y' = f'(x)$

$y=f(x)$	$y'=f'(x)$	$y=f(x)$	$y'=f'(x)$
$\cos x$	$-\sin x$	$\frac{1}{\cos x}$	$\frac{\tan x}{\cos x}$
$\sin x$	$\cos x$	$\frac{1}{\sin x}$	$-\frac{1}{\sin x \tan x}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\frac{1}{\tan x}$	$-\frac{1}{\sin^2 x}$
$\tan x - x$ NOTE1.	$\tan^2 x$	$\sin^2 x$	$2\sin x\cos x$
		$\sin^3 x$	$3\sin^2 x\cos x$

NOTE 1. Involute function.

An example of the application of the derivative : Newton's law

Introduced here is the method to obtain an approximate value of x_0 when the value of $f(x_0)$ is given in the involute function



$$f(x) = \tan x - x$$

First, with an initial value x_1 optionally chosen, obtain an approximate value x_2 from:

$$x_2 = x_1 - \frac{f(x_1) - f(x_0)}{\tan^2 x_1}$$

where $f'(x_1) = \tan^2 x_1$

Then, when the difference between the approximate value $f(x_2)$ and the given $f(x_0)$ is large, obtain an approximate value x_3 using the same method.

$$x_3 = x_2 - \frac{f(x_2) - f(x_0)}{\tan^2 x_2}$$

The more accurate value of x_0 can be obtained by the repeated calculation using this method.

The involute function, inv α, is frequently used in the calculation of gearing. Therefore, the Newton's law is really useful.

2 International System of Units (SI)

Base unit Excerpted from JIS Z 8202-0:2000

SI Base Unit

Base Quantity	Base unit	
	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Current	ampere	A
Temperature	kelvin	k
Amount of substance	mole	mol
Luminous intensity	candela	cd

Derived Units with Special Names Excerpted from JIS Z 8202-0:2000

Derived Quantity	SI Derived Unit		
	Name	Symbol	SI Base Units and Derived Units
Angle	radian	rad	1rad = 1m/m = 1
Solid angle	steradian	sr	$1sr = 1m^2 / m^2 = 1$
Frequency	hertz	Hz	$1Hz = 1s^{-1}$
Force, Weight	newton	N	$1N = 1kg \bullet m/s^2$
Pressure, Stress	pascal	Pa	$1Pa = 1N/m^2$
Energy, Work, Heat	joule	J	1J = 1N • m
Power, Radiant flux	watt	W	1W = 1J/s
Electric charge, Electric flux	coulomb	C	1C = 1A • s
Voltage, Electrical potential difference, Electromotive force	volt	V	1V = 1W/A
Electric capacitance	farad	F	1F = 1C/V
Electric resistance	ohm	Ω	1Ω=1V/A
Celsius Temperature	Celsius *	˚C	1˚C= 1K

*Celsius is the unit used to indicate the value of Celsius temperature, substituted for the unit Kelvin.
(Referred from JIS Z 8202-4:2000 4-1,a and 4-2,a)

Conversation Factors for SI (International System of Units)

Name	Equivalents
Force	1N = 0.101972 kgf 1kgf = 9.80665N
Stress	$1MPa = 1N/mm^2 = 0.101972\ kgf/mm^2$ $1kgf/mm^2 = 9.80665MPa = 9.80665N/mm^2$
Work Energy	$1J = 2.77778 \times 10^{-7} kW{\cdot}h = 0.101972\ kgf{\cdot}m$ $1kgf{\cdot}m = 9.80665J,\ 1kW{\cdot}h = 3.600 \times 10^6 J$
Work Power	$1kW = 101.972\ kgf{\cdot}m/s$ $1kgf{\cdot}m/s = 9.80665 \times 10^{-3} kW$

Prefix

Prefix (Symbol)	Factor	Prefix (Symbol)	Factor
yotta (Y)	10^{24}	desi (d)	10^{-1}
zetta (Z)	10^{21}	centi (c)	10^{-2}
exa (E)	10^{18}	milli (m)	10^{-3}
peta (P)	10^{15}	micro (μ)	10^{-6}
tera (T)	10^{12}	nano (n)	10^{-9}
giga (G)	10^{9}	pico (p)	10^{-12}
mega (M)	10^{6}	femto (f)	10^{-15}
kilo (k)	10^{3}	atto (a)	10^{-18}
hecto (h)	10^{2}	zepto (z)	10^{-21}
deca (da)	10^{1}	yocto (y)	10^{-24}

Conversion of Units: Angles

Conversion of Degree to Minute and Second (if θ = 20.5445°)				
Terms	Symbol	Unit	Formula	Example
Degree	D	˚	Discard the figures after the decimal point.	20˚
Minute	M		$(\theta - D)\times 60$ Discard the figures after the decimal point	32
Second	S		$\{(\theta - D) \times 60 - M\}\times 60$	40.2

Conversion of Minute, Second to Degree (If D˚M'S"= 25˚30' 40")				
Term	Symbol	Unit	Formula	Example
Degree	θ	˚	$\theta^\circ = D^\circ + \frac{M'}{60} + \frac{S''}{3600}$	25.5111˚

Conversion of Degree to Radian (If θ = 25°)				
Term	Symbol	Unit	Formula	Example
Radian	θ	rad	$\theta \text{rad} = \frac{\theta^\circ \times \pi}{180}$	0.4363

Conversion of Radian to Degree (if θ = 1.5 rad)				
Term	Symbol	Unit	Formula	Example
Degree	θ	˚	$\theta^\circ = \frac{\theta \text{rad} \times 180}{\pi}$	85.9437˚

3 Dynamic Conversion Formulas

Shown are formulas enabling easy calculation for the conversion of torque or power. Not only the international system of units, the gravitational system of units are also introduced, as they are used concurrently.

To Know	Unit	Know		Unit	Conversion factors
Torque T	N・m	Force	F	N	$T = \dfrac{F \times r}{1000}$
		Radius	r	mm	
		Power	P	kW	$T = \dfrac{9549P}{n}$
		Number of revolutions	n	min^{-1}	
Dynamics P (Power)	kW	Torque	T	N・m	$P = \dfrac{T \times n}{9549}$
		Number of revolutions	n	min^{-1}	
		Force	F	N	$P = \dfrac{F \times v}{1000}$
		Velocity	v	m/s	

To Know	Unit	Know		Unit	Conversion factors
Torque T	kgf・m	Force	F	kgf	$T = \dfrac{F \times r}{1000}$
		Radius	r	mm	
		Power	P	kW	$T = \dfrac{974 \times P}{n}$
		Number of revolutions	n	min^{-1}	
Dynamics P (Power)	kW	Torque	T	kgf・m	$P = \dfrac{T \times n}{974}$
		Number of revolutions	n	min^{-1}	
		Force	F	kgf	$P = \dfrac{F \times v}{102}$
		Velocity	v	m/s	

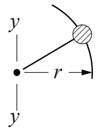
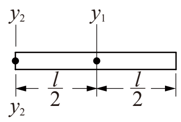
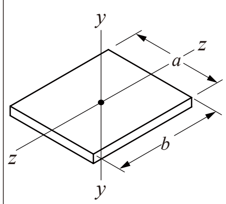
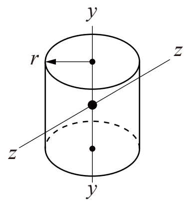
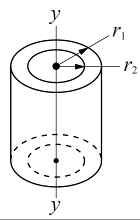
NOTE : Number of revolutions 1 min^{-1} =1 rpm

REMARK 1 : Metric horsepower 1PS ≒ 735W = 0.735kW

REMARK 2 : Horsepower 1HP ≒ 746W = 0.746kW

To Know	Unit	Know		Unit	Conversion factor
Velocity v	m/s	Diameter	d	mm	$v = \dfrac{\pi \times d \times n}{60000}$
		Number of revolutions	n	min^{-1}	

Moment of inertia mk^2 ($kg \cdot m^2$) $\dfrac{W}{g}k^2$ ($kgf \cdot ms^2$)

Denomination	Shape	Rotation axis	Moment of inertia · Unit	
			SI $kg \cdot m^2$	Gravitational system of units $kgf \cdot ms^2$
Mass point		$y — y$	$m r^2$	$\dfrac{W}{g} r^2$
Pole		$y_1 — y_1$	$m \dfrac{l^2}{12}$	$\dfrac{W}{g} \dfrac{l^2}{12}$
		$y_2 — y_2$	$m \dfrac{l^2}{3}$	$\dfrac{W}{g} \dfrac{l^2}{3}$
Plate		$y — y$	$m \dfrac{a^2+b^2}{12}$	$\dfrac{W}{g} \dfrac{a^2+b^2}{12}$
		$z — z$	$m \dfrac{a^2}{12}$	$\dfrac{W}{g} \dfrac{a^2}{12}$
Cylinder		$y — y$	$m \dfrac{r^2}{2}$	$\dfrac{W}{g} \dfrac{r^2}{2}$
		$z — z$	$m \dfrac{r^2}{4}$	$\dfrac{W}{g} \dfrac{r^2}{4}$
Hollow cylinder		$y — y$	$m \dfrac{r_1^2+r_2^2}{2}$	$\dfrac{W}{g} \dfrac{r_1^2+r_2^2}{2}$

NOTE 1. m : Mass W : Weight g : Acceleration of gravity = 9.80665m/s²

NOTE 2. $GD^2 = 4gI$ ($kgf \cdot m^2$)

4 Table for Weight of Steel Bar

Section	Dimension (mm)	Weight of steel bar for 1m (kgf/m)
Round	○ d	$0.00616d^2$
Square	□ s	$0.00785s^2$
Hexagon	⬡ h	$0.00680h^2$

Weight of round steel bar (kgf/m)

Diameter	0	1	2	3	4	5	6	7	8	9
00	0	0.01	0.02	0.06	0.10	0.15	0.22	0.30	0.39	0.50
10	0.62	0.75	0.89	1.04	1.21	1.39	1.58	1.78	2.00	2.22
20	2.46	2.72	2.98	3.26	3.55	3.85	4.16	4.49	4.83	5.18
30	5.54	5.92	6.31	6.71	7.12	7.55	7.98	8.43	8.90	9.37
40	9.86	10.36	10.87	11.39	11.93	12.47	13.03	13.61	14.19	14.79
50	15.40	16.02	16.66	17.30	17.96	18.63	19.32	20.01	20.72	21.44
60	22.18	22.92	23.68	24.45	25.23	26.03	26.83	27.65	28.48	29.33
70	30.18	31.05	31.93	32.83	33.73	34.65	35.58	36.52	37.48	38.44
80	39.42	40.42	41.42	42.44	43.47	44.51	45.56	46.63	47.70	48.79
90	49.90	51.01	52.14	53.28	54.43	55.59	56.77	57.96	59.16	60.37
100	61.60	62.84	64.09	65.35	66.63	67.91	69.21	70.53	71.85	73.19
110	74.54	75.90	77.27	78.66	80.06	81.47	82.89	84.32	85.77	87.23
120	88.70	90.19	91.69	93.19	94.72	96.25	97.80	99.35	100.93	102.51
130	104.10	105.71	107.33	108.96	110.61	112.27	113.94	115.62	117.31	119.02
140	120.74	122.47	124.21	125.97	127.73	129.51	131.31	133.11	134.93	136.76
150	138.60	140.45	142.32	144.20	146.09	147.99	149.91	151.84	153.78	155.73
160	157.70	159.67	161.66	163.67	165.68	167.71	169.75	171.80	173.86	175.94
170	178.02	180.13	182.24	184.36	186.50	188.65	190.81	192.99	195.17	197.37
180	199.58	201.81	204.04	206.29	208.55	210.83	213.11	215.41	217.72	220.04
190	222.38	224.72	227.08	229.45	231.84	234.23	236.64	239.06	241.50	243.94
200	246.40	248.87	251.35	253.85	256.36	258.87	261.41	263.95	266.51	269.08
210	271.66	274.25	276.86	279.47	282.10	284.75	287.40	290.07	292.75	295.44
220	298.14	300.86	303.59	306.33	309.08	311.85	314.63	317.42	320.22	323.04
230	325.86	328.70	331.56	334.42	337.30	340.19	343.09	346.00	348.93	351.87
240	354.82	357.78	360.75	363.74	366.74	369.75	372.78	375.82	378.87	381.93
250	385.00	388.09	391.19	394.30	397.42	400.55	403.70	406.86	410.03	413.22
260	416.42	419.63	422.85	426.08	429.33	432.59	435.86	439.14	442.44	445.74
270	449.06	452.40	455.74	459.10	462.47	465.85	469.24	472.65	476.07	479.50
280	482.94	486.40	489.87	493.35	496.84	500.35	503.86	507.39	510.94	514.49
290	518.06	521.64	525.23	528.83	532.45	536.07	539.72	543.37	547.03	550.71
300	554.40	558.10	561.82	565.54	569.28	573.03	576.80	580.57	584.36	588.16
310	591.98	595.80	599.64	603.49	607.35	611.23	615.11	619.01	622.92	626.85
320	630.78	634.73	638.69	642.67	646.65	650.65	654.66	658.68	662.72	666.76
330	670.82	674.90	678.98	683.08	687.19	691.31	695.44	699.59	703.74	707.91
340	712.10	716.29	720.50	724.72	728.95	733.19	737.45	741.72	746.00	750.29
350	754.60	758.92	763.25	767.59	771.95	776.31	780.69	785.09	789.49	793.91
360	798.34	802.78	807.23	811.70	816.18	820.67	825.17	829.68	834.21	838.75
370	843.30	847.87	852.45	857.04	861.64	866.25	870.88	875.52	880.17	884.83
380	889.50	894.19	898.89	903.60	908.33	913.07	917.82	922.58	927.35	932.14
390	936.94	941.75	946.57	951.41	956.25	961.11	965.99	970.87	975.77	980.68
400	985.60	990.53	995.48	1000.44	1005.41	1010.39	1015.39	1020.40	1025.42	1030.45
410	1035.50	1040.55	1045.62	1050.71	1055.80	1060.91	1066.02	1071.16	1076.30	1081.46
420	1086.62	1091.80	1097.00	1102.20	1107.42	1112.65	1117.89	1123.15	1128.41	1133.69
430	1138.98	1144.29	1149.60	1154.93	1160.27	1165.63	1170.99	1176.37	1181.76	1187.16
440	1192.58	1198.00	1203.44	1208.89	1214.36	1219.83	1225.32	1230.82	1236.34	1241.86
450	1247.40	1252.95	1258.51	1264.09	1269.67	1275.27	1280.89	1286.51	1292.15	1297.79
460	1303.46	1309.13	1314.82	1320.51	1326.22	1331.95	1337.68	1343.43	1349.19	1354.96
470	1360.74	1366.54	1372.35	1378.17	1384.00	1389.85	1395.71	1401.58	1407.46	1413.36
480	1419.26	1425.18	1431.12	1437.06	1443.02	1448.99	1454.97	1460.96	1466.97	1472.99
490	1479.02	1485.06	1491.11	1497.18	1503.26	1509.35	1515.46	1521.58	1527.70	1533.85
500	1540.00	1546.17	1552.34	1558.54	1564.74	1570.95	1577.18	1583.42	1589.67	1595.94

EXAMPLE 1. Weight of round steel bar, diameter (128 mm) and length 1 m), is 100.93 kgf.

EXAMPLE 2. Weight of round cast iron bar, diameter (128 mm) and length (1 m) is;

100.93 (Weight of steel) ×0.918 (Steel ratio) = 92.65 kgf

5 List of Elements by Symbol and Specific Gravity

Name	Symbol	Specific gravity (20℃) g/cm³	Name	Symbol	Specific gravity (20℃) g/cm³	Name	Symbol	Specific gravity (20℃) g/cm³
Zinc	Zn	7.133 (25°)	Bromine	Br	3.12	Sodium	Na	0.9712
Aluminium	Al	2.699	Zirconium	Zr	6.489	Lead	Pb	11.36
Antimony	Sb	6.62	Mercury	Hg	13.546	Niobium	Nb	8.57
Sulfur	S	2.07	Hydrogen	H	0.0899×10^{-3}	Nickel	Ni	8.902 (25°)
Ytterbium	Yb	6.96	Tin	Sn	7.2984	Platinum	Pt	21.45
Yttrium	Y	4.47	Strontium	Sr	2.60	Vanadium	V	6.1
Iridium	Ir	22.5	Caesium	Cs	1.903 (0°)	Palladium	Pd	12.02
Indium	In	7.31	Cerium	Ce	6.77	Barium	Ba	3.5
Uranium	U	19.07	Selenium	Se	4.79	Arsenic	As	5.72
Chlorine	Cl	3.214×10^{-3}	Bismuth	Bi	9.80	Fluorine	F	1.696×10^{-3}
Cadmium	Cd	8.65	Thallium	Tl	11.85	Plutonium	Pu	19.00 ～ 19.72
Potassium	K	0.86	Tungsten	W	19.3	Beryllium	Be	1.848
Calcium	Ca	1.55	Carbon	C	2.25	Boron	B	2.34
Gold	Au	19.32	Tantalum	Ta	16.6	Magnesium	Mg	1.74
Silver	Ag	10.49	Titanium	Ti	4.507	Manganese	Mn	7.43
Chlorine	Cr	7.19	Nitrogen	N	1.250×10^{-3}	Molybdenum	Mo	10.22
Silicon	Si	2.33 (25°)	Iron	Fe	7.87	Iodine	I	4.94
Germanium	Ge	5.323 (25°)	Tellurium	Te	6.24	Radium	Ra	5.0
Cobalt	Co	8.85	Copper	Cu	8.96	Lithium	Li	0.534
Oxygen	O	1.429×10^{-3}	Thorium	Th	11.66	Phosphorus	P	1.83

✱The Specific Gravity of the Main Gear Materials (Reference)

Material	Major Materials	Specific gravity (gf/cm³)
Steel	S45C	7.85
Alloy steel	SCM415	7.85
	SCM440	7.85
Stainless steel	SUS304	7.81
	SUS303	7.80
MC Nylon	MC901	1.16
	MC602ST	1.23
Duracon	M90-44	1.41
	M25-44	1.41
Free-cutting brass	C3604	8.50
Aluminium bronze	CAC702 (AlBC2)	7.60
Phosphor bronze	CAC502 (PBC2)	8.80
Cast iron	FC200	7.21

6 Hardness Comparison Table

Approximate conversion values against Rockwell C hardness of steel materials (NOTE 1)

HRC	HV	HB		HRA	HRB	HRD	HR15N	HR30N	HR45N	HS		HRC	
Rockwell C hardness (NOTE 3)	Vickers hardness	Brinell hardness 10mm Ball·Load 3000kgf: Standard ball	Brinell hardness 10mm Ball·Load 3000kgf: Tungsten-carbide ball	Rockwell hardness (NOTE 3): A Scale Load 60kgf brale	Rockwell hardness (NOTE 3): B Scale Load 100kgf Dia. 1/16in Ball	Rockwell hardness (NOTE 3): D Scale Load 100kgf brale indenter	Rockwell hardness Superficial Hardness Conical diamond indenter: 15-N Scale Load 15kgf	Rockwell hardness Superficial Hardness Conical diamond indenter: 30-N Scale Load 30kgf	Rockwell hardness Superficial Hardness Conical diamond indenter: 45-N Scale Load 45kgf	Shore hardness	Tensile Strength Aprox. value MPa (NOTE 2.)	Approx. hardness of principal materials (NOTE 3.)	Approx. hardness of principal materials
68	940	-	-	85.6	-	76.9	93.2	84.4	75.4	97	-	68	
67	900	-	-	85.0	-	76.1	92.9	83.6	74.2	95	-	67	
66	865	-	-	84.5	-	75.4	92.5	82.8	73.3	92	-	66	
65	832	-	(739)	83.9	-	74.5	92.2	81.9	72.0	91	-	65	
64	800	-	(722)	83.4	-	73.8	91.8	81.1	71.0	88	-	64	
63	772	-	(705)	82.8	-	73.0	91.4	80.1	69.9	87	-	63	
62	746	-	(688)	82.3	-	72.2	91.1	79.3	68.8	85	-	62	
61	720	-	(670)	81.8	-	71.5	90.7	78.4	67.7	83	-	61	
60	697	-	(654)	81.2	-	70.7	90.2	77.5	66.6	81	-	60	SCM415 case hardening surface hardness
59	674	-	(634)	80.7	-	69.9	89.8	76.6	65.5	80	-	59	
58	653	-	615	80.1	-	69.2	89.3	75.7	64.3	78	-	58	
57	633	-	595	79.6	-	68.5	88.9	74.8	63.2	76	-	57	SCM440 induction hardening surface hardness
56	613	-	577	79.0	-	67.7	88.3	73.9	62.0	75	-	56	
55	595	-	560	78.5	-	66.9	87.9	73.0	60.9	74	2075	55	
54	577	-	543	78.0	-	66.1	87.4	72.0	59.8	72	2015	54	
53	560	-	525	77.4	-	65.4	86.9	71.2	58.6	71	1950	53	S45C induction hardening surface hardness
52	544	(500)	512	76.8	-	64.6	86.4	70.2	57.4	69	1880	52	
51	528	(487)	496	76.3	-	63.8	85.9	69.4	56.1	68	1820	51	
50	513	(475)	481	75.9	-	63.1	85.5	68.5	55.0	67	1760	50	
49	498	(464)	469	75.2	-	62.1	85.0	67.6	53.8	66	1695	49	
48	484	451	455	74.7	-	61.4	84.5	66.7	52.5	64	1635	48	
47	471	442	443	74.1	-	60.8	83.9	65.8	51.4	63	1580	47	
46	458	432	432	73.6	-	60.0	83.5	64.8	50.3	62	1530	46	
45	446	421	421	73.1	-	59.2	83.0	64.0	49.0	60	1480	45	
44	434	409	409	72.5	-	58.5	82.5	63.1	47.8	58	1435	44	
43	423	400	400	72.0	-	57.7	82.0	62.2	48.7	57	1385	43	
42	412	390	390	71.5	-	56.9	81.5	61.3	45.5	56	1340	42	
41	402	381	381	70.9	-	56.2	80.9	60.4	44.3	55	1295	41	
40	392	371	371	70.4	-	55.4	80.4	59.5	43.1	54	1250	40	
39	382	362	362	69.9	-	54.6	79.9	58.6	41.9	52	1215	39	
38	372	353	353	69.4	-	53.8	79.4	57.7	40.8	51	1180	38	
37	363	344	344	68.9	-	53.1	78.8	56.8	39.6	50	1160	37	
36	354	336	336	68.4	(109.0)	52.3	78.3	55.9	38.4	49	1115	36	
35	345	327	327	67.9	(108.5)	51.5	77.7	55.0	37.2	48	1080	35	
34	336	319	319	67.4	(108.0)	50.8	77.2	54.2	36.1	47	1055	34	SCM415 Case hardening core hardness
33	327	311	311	66.8	(107.5)	50.0	76.6	53.3	34.9	46	1025	33	
32	318	301	301	66.3	(107.0)	49.2	76.1	52.1	33.7	44	1000	32	
31	310	294	294	65.8	(106.0)	48.4	75.6	51.3	32.5	43	980	31	
30	302	286	286	65.3	(105.5)	47.7	75.0	50.4	31.3	42	950	30	
29	294	279	279	64.7	(104.5)	47.0	74.5	49.5	30.1	41	930	29	
28	286	271	271	64.3	(104.0)	46.1	73.9	48.6	28.9	41	910	28	
27	279	264	264	63.8	(103.0)	45.2	73.3	47.7	27.8	40	880	27	S45C Thermal refining core hardness
26	272	258	258	63.3	(102.5)	44.6	72.8	46.8	26.7	38	860	26	
25	266	253	253	62.8	(101.5)	43.8	72.2	45.9	25.5	38	840	25	
24	260	247	247	62.4	(101.0)	43.1	71.6	45.0	24.3	37	825	24	
23	254	243	243	62.0	100.0	42.1	71.0	44.0	23.1	36	805	23	SCM440 Thermal refining Core hardness
22	248	237	237	61.5	99.0	41.6	70.5	43.2	22.0	35	785	22	
21	243	231	231	61.0	98.5	40.9	69.9	42.3	20.7	35	770	21	
20	238	226	226	60.5	97.8	40.1	69.4	41.5	19.6	34	760	20	
(18)	230	219	219	-	96.7	-	-	-	-	33	730	(18)	
(16)	222	212	212	-	95.5	-	-	-	-	32	705	(16)	
(14)	213	203	203	-	93.9	-	-	-	-	31	675	(14)	S45C Material hardness
(12)	204	194	194	-	92.3	-	-	-	-	29	650	(12)	
(10)	196	187	187	-	90.7	-	-	-	-	28	620	(10)	
(8)	188	179	179	-	89.5	-	-	-	-	27	600	(8)	SUS303 Material hardness
(6)	180	171	171	-	87.1	-	-	-	-	26	580	(6)	
(4)	173	165	165	-	85.5	-	-	-	-	25	550	(4)	
(2)	166	158	158	-	83.5	-	-	-	-	24	530	(2)	
(0)	160	152	152	-	81.7	-	-	-	-	24	515	(0)	

NOTE 1. The boldfaced figures are based on ASTM E 140 Table 3 (SAE-ASM-ASTM)

NOTE 2. 1MPa = 1N/mm²

NOTE 3. The parenthesized values in the table are not used so frequently.

7 Table of Comparative Gear Pitch

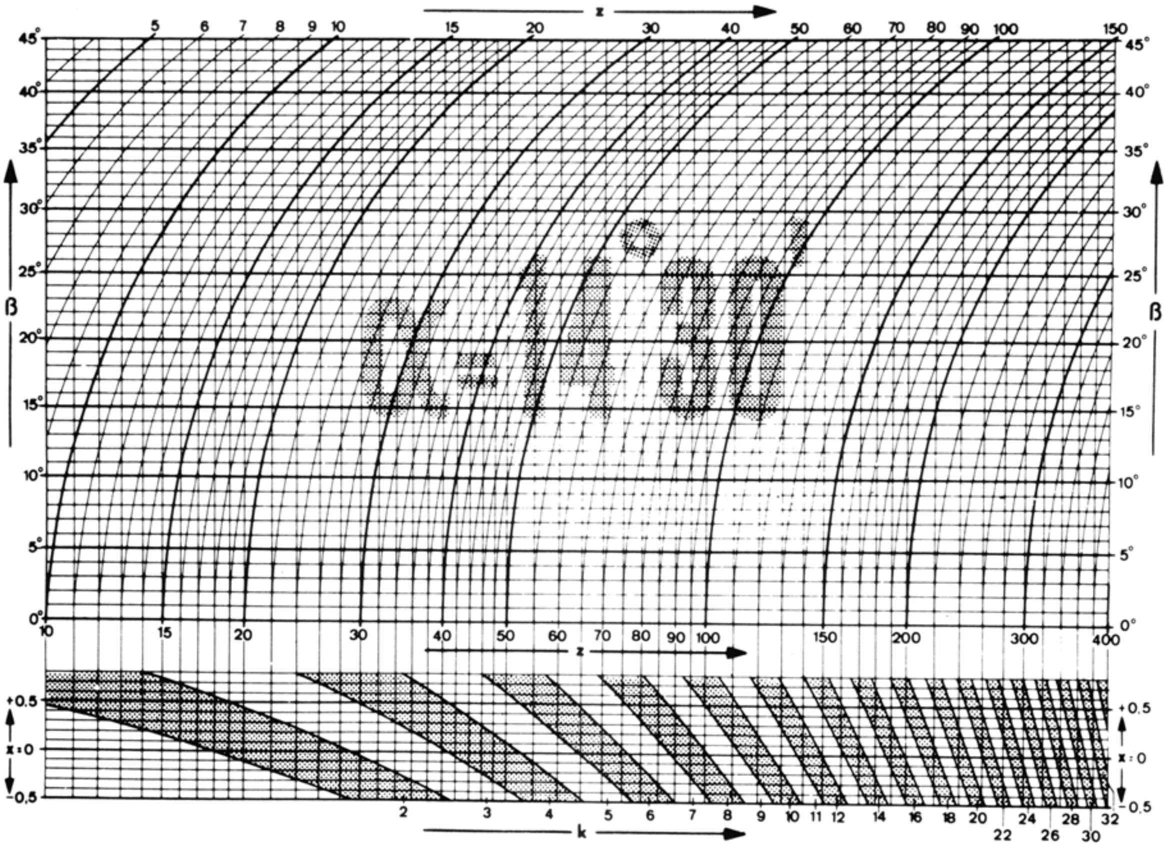
Diametral pitch	Pitch		Module m
	in	mm	
1	3.1416	79.796	25.4000
1.0053	$3\frac{1}{8}$	79.375	25.2658
1.0160	3.0921	78.540	25
1.0472	3	76.200	24.2550
1.0583	2.9684	75.398	24
1.0640	2.9528	75	23.8732
1.0927	$2\frac{7}{8}$	73.025	23.2446
1.1399	2.7559	70	22.2817
1.1424	$2\frac{3}{4}$	69.850	22.2339
1.1545	2.7211	69.115	22
1.1968	$2\frac{5}{8}$	66.675	21.2233
1.2276	2.5591	65	20.6901
1.25	2.5133	63.837	20.3200
1.2566	$2\frac{1}{2}$	63.500	20.2127
1.27	2.4737	62.832	20
1.3228	$2\frac{3}{8}$	60.325	19.2020
1.3299	2.3622	60	19.0986
1.3963	$2\frac{1}{4}$	57.150	18.1914
1.4111	2.2263	56.549	18
1.4508	2.1654	55	17.5070
1.4784	$2\frac{1}{8}$	53.975	17.1808
1.5	2.0944	53.198	16.9333
1.5708	2	50.8	16.1701
1.5875	1.9790	50.265	16
1.5959	1.9685	50	15.9155
1.6755	$1\frac{7}{8}$	47.625	15.1595
1.6933	1.8553	47.124	15
1.75	1.7952	45.598	14.5143
1.7733	1.7717	45	14.3239
1.7952	$1\frac{3}{4}$	44.45	14.1489
1.8143	1.7316	43.982	14
1.9333	$1\frac{5}{8}$	41.275	13.1382
1.9538	1.6079	40.841	13
1.9949	1.5748	40	12.7324
2	1.5708	39.898	12.7000
2.0944	$1\frac{1}{2}$	38.1	12.1276
2.0999	1.4961	38	12.0958
2.1167	1.4842	37.699	12
2.1855	$1\frac{7}{16}$	36.513	11.6223
2.2166	1.4173	36	11.4592
2.25	1.3963	35.465	11.2889
2.2848	$1\frac{3}{8}$	34.925	11.1170
2.3091	1.3606	34.559	11
2.3470	1.3386	34	10.8225
2.3936	$1\frac{5}{16}$	33.338	10.6117
2.4936	1.2598	32	10.1859
2.5	1.2566	31.919	10.1600
2.5133	$1\frac{1}{4}$	31.750	10.1063
2.54	1.2369	31.416	10
2.6456	$1\frac{3}{16}$	30.163	9.6010
2.6599	1.1811	30	9.5493
2.75	1.1424	29.017	9.2364
2.7925	$1\frac{1}{8}$	28.575	9.0957
2.8222	1.1132	28.274	9
2.8499	1.1024	28	8.9127

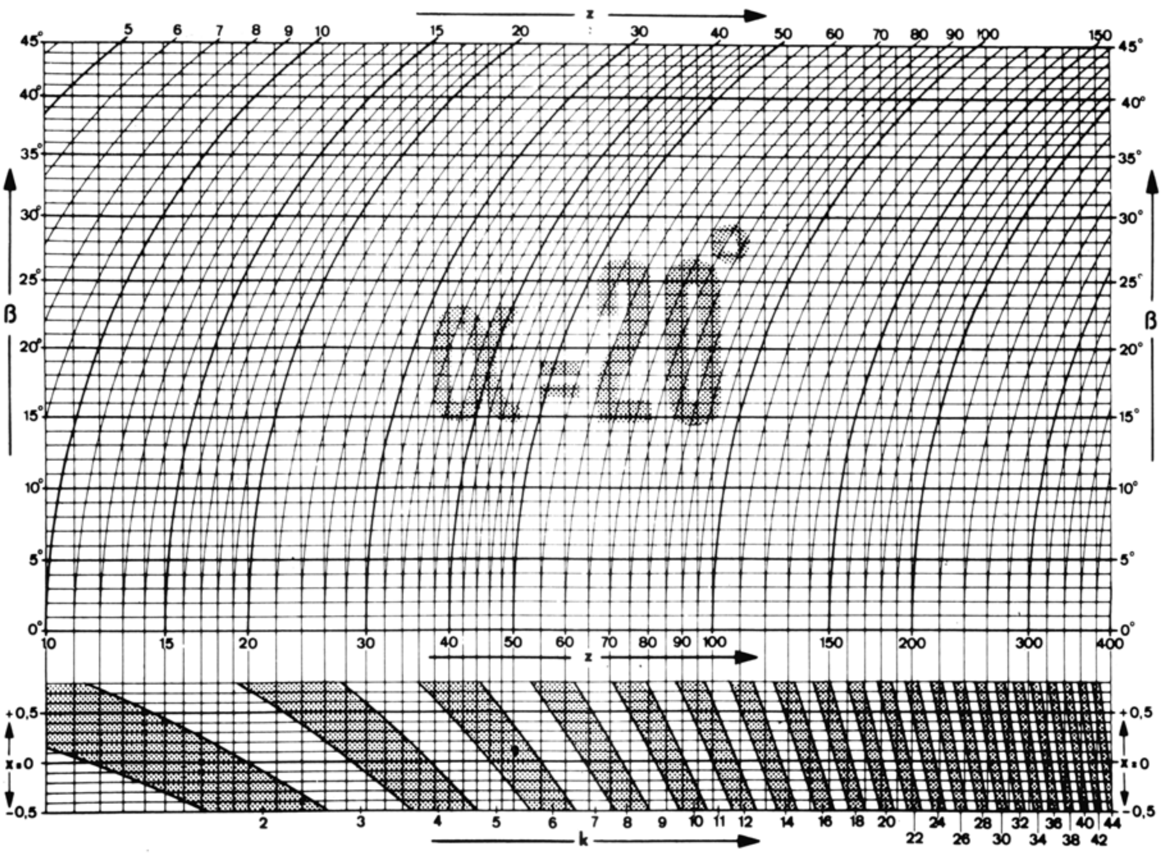
Diametral pitch	Pitch		Module m
	in	mm	
2.9568	$1\frac{1}{16}$	26.988	8.5904
3	1.0472	26.599	8.4667
3.0691	1.0236	26	8.2761
3.1416	1	25.4	8.0851
3.175	.9895	25.133	8
3.1919	.9843	25	7.9577
3.25	.9666	24.553	7.8154
3.3249	.9449	24	7.6394
3.3510	$\frac{15}{16}$	23.813	7.5798
3.5	.8976	22.799	7.2571
3.5904	$\frac{7}{8}$	22.225	7.0744
3.6271	.8661	22	7.0028
3.6286	.8658	21.991	7
3.75	.8378	21.279	6.7733
3.8666	$\frac{13}{16}$	20.638	6.5691
3.9898	.7874	20	6.3662
4	.7854	19.949	6.3500
4.1888	$\frac{3}{4}$	19.05	6.0638
4.1998	.7480	19	6.0479
4.2333	.7421	18.850	6
4.4331	.7087	18	5.7296
4.5	.6981	17.733	5.6444
4.5696	$\frac{11}{16}$	17.463	5.5585
4.6182	.6803	17.279	5.5
4.6939	.6693	17	5.4113
4.9873	.6299	16	5.0930
5	.6283	15.959	5.0800
5.0265	$\frac{5}{8}$	15.875	5.0532
5.08	.6184	15.708	5
5.3198	.5906	15	4.7746
5.5	.5712	14.508	4.6182
5.5851	$\frac{9}{16}$	14.288	4.5479
5.6444	.5566	14.137	4.5
5.6997	.5512	14	4.4563
6	.5236	13.299	4.2333
6.1382	.5118	13	4.1380
6.2832	$\frac{1}{2}$	12.7	4.0425
6.35	.4947	12.566	4
6.5	.4833	12.276	3.9077
6.6497	.4724	12	3.8197
7	.4488	11.399	3.6286
7.1808	$\frac{7}{16}$	11.113	3.5372
7.2542	.4331	11	3.5014
7.2571	.4329	10.996	3.5
7.9796	.3937	10	3.1831
8	.3927	9.975	3.175
8.3776	$\frac{3}{8}$	9.525	3.0319
8.3996	.3740	9.5	3.0239
8.4667	.3711	9.425	3
8.8663	.3543	9	2.8648
9	.3491	8.866	2.8222
9.2364	.3401	8.639	2.75
9.3878	.3346	8.5	2.7056
9.9746	.3150	8	2.5465
10	.3142	7.980	2.54

Diametral pitch	Pitch		Module m
	in	mm	
10.0531	$\frac{5}{16}$	7.938	2.5266
10.16	.3092	7.854	2.5
10.6395	.2953	7.5	2.3873
11	.2856	7.254	2.3091
11.2889	.2783	7.069	2.25
11.3995	.2756	7	2.2282
12	.2618	6.650	2.1167
12.2764	.2559	6.5	2.0690
12.5664	$\frac{1}{4}$	6.35	2.0213
12.7	.2474	6.283	2
13	.2417	6.138	1.9538
13.2994	.2362	6	1.9099
14	.2244	5.700	1.8143
14.5084	.2165	5.5	1.7507
14.5143	.2164	5.498	1.75
15	.2094	5.320	1.6933
15.9593	.1969	5	1.5915
16	.1963	4.987	1.5875
16.7552	$\frac{3}{16}$	4.763	1.5160
16.9333	.1855	4.712	1.5
17.7325	.1772	4.5	1.4324
18	.1745	4.433	1.4111
19.9491	.1575	4	1.2732
20	.1571	3.990	1.27
20.32	.1546	3.927	1.25
22	.1428	3.627	1.1545
22.7990	.1378	3.5	1.1141
23	.1366	3.469	1.1043
24	.1309	3.325	1.0583
25	.1257	3.192	1.016
25.1327	$\frac{1}{8}$	3.175	1.0106
25.4	.1237	3.142	1
26	.1208	3.069	.9769
26.5988	.1181	3	.9549
28	.1122	2.850	.9071
29	.1083	2.752	.8759
30	.1047	2.660	.8467
31.75	.0989	2.513	.8
31.9186	.0984	2.5	.7958
32	.0982	2.494	.7938
33.8667	.0928	2.356	.75
34	.0924	2.347	.7471
36	.0873	2.217	.7056
38	.0827	2.100	.6684
39.8982	.0787	2	.6366
40	.0785	1.995	.635
45	.0698	1.773	.5644
50	.0628	1.596	.5080
50.2655	$\frac{1}{16}$	1.588	.5053
50.8	.0618	1.571	.5
53.1976	.0591	1.5	.4775
63.5	.0495	1.256	.4
79.7965	.0394	1	.3183
84.6667	.0371	.942	.3
127	.0247	.628	.2

8 Charts Indicating Span Number of Teeth of Spur and Helical Gears

(Maag's data)





9 Span Measurement Over k Teeth of Standard Spur Gear

W (*x* = 0) *m* = 1 (*α* = 20°)

z	k	W	z	k	W	z	k	W	z	k	W
			61	7	20.0432	121	14	41.5484	181	21	63.0537
			62	7	20.0572	122	14	41.5625	182	21	63.0677
			63	8	23.0233	123	14	41.5765	183	21	63.0817
			64	8	23.0373	124	14	41.5905	184	21	63.0957
5	2	4.4982	65	8	23.0513	125	14	41.6045	185	21	63.1097
6	2	4.5122	66	8	23.0654	126	15	44.5706	186	21	63.1237
7	2	4.5262	67	8	23.0794	127	15	44.5846	187	21	63.1377
8	2	4.5402	68	8	23.0934	128	15	44.5986	188	21	63.1517
9	2	4.5542	69	8	23.1074	129	15	44.6126	189	22	66.1179
10	2	4.5683	70	8	23.1214	130	15	44.6266	190	22	66.1319
11	2	4.5823	71	8	23.1354	131	15	44.6406	191	22	66.1459
12	2	4.5963	72	9	26.1015	132	15	44.6546	192	22	66.1599
13	2	4.6103	73	9	26.1155	133	15	44.6686	193	22	66.1739
14	2	4.6243	74	9	26.1295	134	15	44.6826	194	22	66.1879
15	2	4.6383	75	9	26.1435	135	16	47.6488	195	22	66.2019
16	2	4.6523	76	9	26.1575	136	16	47.6628	196	22	66.2159
17	2	4.6663	77	9	26.1715	137	16	47.6768	197	22	66.2299
18	3	7.6324	78	9	26.1855	138	16	47.6908	198	23	69.1961
19	3	7.6464	79	9	26.1996	139	16	47.7048	199	23	69.2101
20	3	7.6604	80	9	26.2136	140	16	47.7188	200	23	69.2241
21	3	7.6744	81	10	29.1797	141	16	47.7328	201	23	69.2381
22	3	7.6885	82	10	29.1937	142	16	47.7468	202	23	69.2521
23	3	7.7025	83	10	29.2077	143	16	47.7608	203	23	69.2661
24	3	7.7165	84	10	29.2217	144	17	50.7270	204	23	69.2801
25	3	7.7305	85	10	29.2357	145	17	50.7410	205	23	69.2941
26	3	7.7445	86	10	29.2497	146	17	50.7550	206	23	69.3081
27	4	10.7106	87	10	29.2637	147	17	50.7690	207	24	72.2742
28	4	10.7246	88	10	29.2777	148	17	50.7830	208	24	72.2882
29	4	10.7386	89	10	29.2917	149	17	50.7970	209	24	72.3022
30	4	10.7526	90	11	32.2579	150	17	50.8110	210	24	72.3163
31	4	10.7666	91	11	32.2719	151	17	50.8250	211	24	72.3303
32	4	10.7806	92	11	32.2859	152	17	50.8390	212	24	72.3443
33	4	10.7946	93	11	32.2999	153	18	53.8051	213	24	72.3583
34	4	10.8086	94	11	32.3139	154	18	53.8192	214	24	72.3723
35	4	10.8227	95	11	32.3279	155	18	53.8332	215	24	72.3863
36	5	13.7888	96	11	32.3419	156	18	53.8472	216	25	75.3524
37	5	13.8028	97	11	32.3559	157	18	53.8612	217	25	75.3664
38	5	13.8168	98	11	32.3699	158	18	53.8752	218	25	75.3804
39	5	13.8308	99	12	35.3361	159	18	53.8892	219	25	75.3944
40	5	13.8448	100	12	35.3501	160	18	53.9032	220	25	75.4084
41	5	13.8588	101	12	35.3641	161	18	53.9172	221	25	75.4224
42	5	13.8728	102	12	35.3781	162	19	56.8833	222	25	75.4364
43	5	13.8868	103	12	35.3921	163	19	56.8973	223	25	75.4505
44	5	13.9008	104	12	35.4061	164	19	56.9113	224	25	75.4645
45	6	16.8670	105	12	35.4201	165	19	56.9253	225	26	78.4306
46	6	16.8810	106	12	35.4341	166	19	56.9394	226	26	78.4446
47	6	16.8950	107	12	35.4481	167	19	56.9534	227	26	78.4586
48	6	16.9090	108	13	38.4142	168	19	56.9674	228	26	78.4726
49	6	16.9230	109	13	38.4282	169	19	56.9814	229	26	78.4866
50	6	16.9370	110	13	38.4423	170	19	56.9954	230	26	78.5006
51	6	16.9510	111	13	38.4563	171	20	59.9615	231	26	78.5146
52	6	16.9650	112	13	38.4703	172	20	59.9755	232	26	78.5286
53	6	16.9790	113	13	38.4843	173	20	59.9895	233	26	78.5426
54	7	19.9452	114	13	38.4983	174	20	60.0035	234	27	81.5088
55	7	19.9592	115	13	38.5123	175	20	60.0175	235	27	81.5228
56	7	19.9732	116	13	38.5263	176	20	60.0315	236	27	81.5368
57	7	19.9872	117	14	41.4924	177	20	60.0455	237	27	81.5508
58	7	20.0012	118	14	41.5064	178	20	60.0595	238	27	81.5648
59	7	20.0152	119	14	41.5204	179	20	60.0736	239	27	81.5788
60	7	20.0292	120	14	41.5344	180	21	63.0397	240	27	81.5928

$m = 1$ ($\alpha = 20°$)

z	k	W	z	k	W	z	k	W	z	k	W
241	27	81.6068	301	34	103.1121	361	41	124.6173	421	47	143.1704
242	27	81.6208	302	34	103.1261	362	41	124.6313	422	47	143.1844
243	28	84.5870	303	34	103.1401	363	41	124.6453	423	48	146.1506
244	28	84.6010	304	34	103.1541	364	41	124.6593	424	48	146.1646
245	28	84.6150	305	34	103.1681	365	41	124.6733	425	48	146.1786
246	28	84.6290	306	35	106.1342	366	41	124.6874	426	48	146.1926
247	28	84.6430	307	35	106.1482	367	41	124.7014	427	48	146.2066
248	28	84.6570	308	35	106.1622	368	41	124.7154	428	48	146.2206
249	28	84.6710	309	35	106.1762	369	42	127.6815	429	48	146.2346
250	28	84.6850	310	35	106.1903	370	42	127.6955	430	48	146.2486
251	28	84.6990	311	35	106.2043	371	42	127.7095	431	48	146.2626
252	29	87.6651	312	35	106.2183	372	42	127.7235	432	49	149.2288
253	29	87.6791	313	35	106.2323	373	42	127.7375	433	49	149.2428
254	29	87.6932	314	35	106.2463	374	42	127.7515	434	49	149.2568
255	29	87.7072	315	36	109.2124	375	42	127.7655	435	49	149.2708
256	29	87.7212	316	36	109.2264	376	42	127.7795	436	49	149.2848
257	29	87.7352	317	36	109.2404	377	42	127.7935	437	49	149.2988
258	29	87.7492	318	36	109.2544	378	43	130.7597	438	49	149.3128
259	29	87.7632	319	36	109.2684	379	43	130.7737	439	49	149.3268
260	29	87.7772	320	36	109.2824	380	43	130.7877	440	49	149.3408
261	30	90.7433	321	36	109.2964	381	43	130.8017	441	50	152.3069
262	30	90.7573	322	36	109.3104	382	43	130.8157	442	50	152.3210
263	30	90.7713	323	36	109.3245	383	43	130.8297	443	50	152.3350
264	30	90.7853	324	37	112.2906	384	43	130.8437	444	50	152.3490
265	30	90.7993	325	37	112.3046	385	43	130.8577	445	50	152.3630
266	30	90.8134	326	37	112.3186	386	43	130.8717	446	50	152.3770
267	30	90.8274	327	37	112.3326	387	44	133.8379	447	50	152.3910
268	30	90.8414	328	37	112.3466	388	44	133.8519	448	50	152.4050
269	30	90.8554	329	37	112.3606	389	44	133.8659	449	50	152.4190
270	31	93.8215	330	37	112.3746	390	44	133.8799	450	51	155.3851
271	31	93.8355	331	37	112.3886	391	44	133.8939	451	51	155.3991
272	31	93.8495	332	37	112.4026	392	44	133.9079	452	51	155.4131
273	31	93.8635	333	38	115.3688	393	44	133.9219	453	51	155.4271
274	31	93.8775	334	38	115.3828	394	44	133.9359	454	51	155.4412
275	31	93.8915	335	38	115.3968	395	44	133.9499	455	51	155.4552
276	31	93.9055	336	38	115.4108	396	45	136.9160	456	51	155.4692
277	31	93.9195	337	38	115.4248	397	45	136.9300	457	51	155.4832
278	31	93.9335	338	38	115.4388	398	45	136.9441	458	51	155.4972
279	32	96.8997	339	38	115.4528	399	45	136.9581	459	52	158.4633
280	32	96.9137	340	38	115.4668	400	45	136.9721	460	52	158.4773
281	32	96.9277	341	38	115.4808	401	45	136.9861	461	52	158.4913
282	32	96.9417	342	39	118.4470	402	45	137.0001	462	52	158.5053
283	32	96.9557	343	39	118.4610	403	45	137.0141	463	52	158.5193
284	32	96.9697	344	39	118.4750	404	45	137.0281	464	52	158.5333
285	32	96.9837	345	39	118.4890	405	46	139.9942	465	52	158.5473
286	32	96.9977	346	39	118.5030	406	46	140.0082	466	52	158.5614
287	32	97.0117	347	39	118.5170	407	46	140.0222	467	52	158.5754
288	33	99.9779	348	39	118.5310	408	46	140.0362	468	53	161.5415
289	33	99.9919	349	39	118.5450	409	46	140.0502	469	53	161.5555
290	33	100.0059	350	39	118.5590	410	46	140.0643	470	53	161.5695
291	33	100.0199	351	40	121.5251	411	46	140.0783	471	53	161.5835
292	33	100.0339	352	40	121.5391	412	46	140.0923	472	53	161.5975
293	33	100.0479	353	40	121.5531	413	46	140.1063	473	53	161.6115
294	33	100.0619	354	40	121.5672	414	47	143.0724	474	53	161.6255
295	33	100.0759	355	40	121.5812	415	47	143.0864	475	53	161.6395
296	33	100.0899	356	40	121.5952	416	47	143.1004	476	53	161.6535
297	34	103.0560	357	40	121.6092	417	47	143.1144	477	54	164.6197
298	34	103.0701	358	40	121.6232	418	47	143.1284	478	54	164.6337
299	34	103.0841	359	40	121.6372	419	47	143.1424	479	54	164.6477
300	34	103.0981	360	41	124.6033	420	47	143.1564	480	54	164.6617

10 Span Measurement Over k Teeth of Standard Spur Gear

$W\ (x = 0)$ $m = 1\ (\alpha = 14.5°)$

z	k	W	z	k	W	z	k	W	z	k	W
			61	5	14.0143	121	10	29.5440	181	15	45.0738
			62	5	14.0197	122	10	29.5494	182	15	45.0791
			63	6	17.0666	123	10	29.5548	183	15	45.0845
			64	6	17.0720	124	10	29.5602	184	15	45.0899
			65	6	17.0773	125	11	32.6070	185	15	45.0952
			66	6	17.0827	126	11	32.6124	186	15	45.1006
7	2	4.5999	67	6	17.0881	127	11	32.6178	187	16	48.1475
8	2	4.6052	68	6	17.0934	128	11	32.6232	188	16	48.1529
9	2	4.6106	69	6	17.0988	129	11	32.6285	189	16	48.1582
10	2	4.6160	70	6	17.1042	130	11	32.6339	190	16	48.1636
11	2	4.6213	71	6	17.1095	131	11	32.6393	191	16	48.1690
12	2	4.6267	72	6	17.1149	132	11	32.6446	192	16	48.1743
13	2	4.6321	73	6	17.1203	133	11	32.6500	193	16	48.1797
14	2	4.6374	74	6	17.1256	134	11	32.6554	194	16	48.1851
15	2	4.6428	75	7	20.1725	135	11	32.6607	195	16	48.1904
16	2	4.6482	76	7	20.1779	136	11	32.6661	196	16	48.1958
17	2	4.6535	77	7	20.1833	137	12	35.7130	197	16	48.2012
18	2	4.6589	78	7	20.1886	138	12	35.7184	198	16	48.2066
19	2	4.6643	79	7	20.1940	139	12	35.7237	199	17	51.2534
20	2	4.6697	80	7	20.1994	140	12	35.7291	200	17	51.2588
21	2	4.6750	81	7	20.2047	141	12	35.7345	201	17	51.2642
22	2	4.6804	82	7	20.2101	142	12	35.7398	202	17	51.2696
23	2	4.6858	83	7	20.2155	143	12	35.7452	203	17	51.2749
24	2	4.6911	84	7	20.2208	144	12	35.7506	204	17	51.2803
25	3	7.7380	85	7	20.2262	145	12	35.7559	205	17	51.2857
26	3	7.7434	86	7	20.2316	146	12	35.7613	206	17	51.2910
27	3	7.7488	87	8	23.2785	147	12	35.7667	207	17	51.2964
28	3	7.7541	88	8	23.2838	148	12	35.7720	208	17	51.3018
29	3	7.7595	89	8	23.2892	149	13	38.8189	209	17	51.3071
30	3	7.7649	90	8	23.2946	150	13	38.8243	210	17	51.3125
31	3	7.7702	91	8	23.2999	151	13	38.8297	211	17	51.3179
32	3	7.7756	92	8	23.3053	152	13	38.8350	212	18	54.3648
33	3	7.7810	93	8	23.3107	153	13	38.8404	213	18	54.3701
34	3	7.7863	94	8	23.3161	154	13	38.8458	214	18	54.3755
35	3	7.7917	95	8	23.3214	155	13	38.8511	215	18	54.3809
36	3	7.7971	96	8	23.3268	156	13	38.8565	216	18	54.3862
37	3	7.8024	97	8	23.3322	157	13	38.8619	217	18	54.3916
38	4	10.8493	98	8	23.3375	158	13	38.8672	218	18	54.3970
39	4	10.8547	99	8	23.3429	159	13	38.8726	219	18	54.4023
40	4	10.8601	100	9	26.3898	160	13	38.8780	220	18	54.4077
41	4	10.8654	101	9	26.3952	161	13	38.8834	221	18	54.4131
42	4	10.8708	102	9	26.4005	162	14	41.9302	222	18	54.4184
43	4	10.8762	103	9	26.4059	163	14	41.9356	223	18	54.4238
44	4	10.8815	104	9	26.4113	164	14	41.9410	224	19	57.4707
45	4	10.8869	105	9	26.4166	165	14	41.9464	225	19	57.4761
46	4	10.8923	106	9	26.4220	166	14	41.9517	226	19	57.4814
47	4	10.8976	107	9	26.4274	167	14	41.9571	227	19	57.4868
48	4	10.9030	108	9	26.4327	168	14	41.9625	228	19	57.4922
49	4	10.9084	109	9	26.4381	169	14	41.9678	229	19	57.4975
50	5	13.9553	110	9	26.4435	170	14	41.9732	230	19	57.5029
51	5	13.9606	111	9	26.4488	171	14	41.9786	231	19	57.5083
52	5	13.9660	112	10	29.4957	172	14	41.9839	232	19	57.5137
53	5	13.9714	113	10	29.5011	173	14	41.9893	233	19	57.5190
54	5	13.9767	114	10	29.5065	174	15	45.0362	234	19	57.5244
55	5	13.9821	115	10	29.5118	175	15	45.0416	235	19	57.5298
56	5	13.9875	116	10	29.5172	176	15	45.0469	236	20	60.5766
57	5	13.9929	117	10	29.5226	177	15	45.0523	237	20	60.5820
58	5	13.9982	118	10	29.5279	178	15	45.0577	238	20	60.5874
59	5	14.0036	119	10	29.5333	179	15	45.0630	239	20	60.5928
60	5	14.0090	120	10	29.5387	180	15	45.0684	240	20	60.5981

$m = 1$ ($\alpha = 14.5°$)

z	k	W	z	k	W	z	k	W	z	k	W
241	20	60.6035	301	25	76.1332	361	30	91.6629	421	34	104.1511
242	20	60.6089	302	25	76.1386	362	30	91.6683	422	34	104.1565
243	20	60.6142	303	25	76.1439	363	30	91.6737	423	35	107.2034
244	20	60.6196	304	25	76.1493	364	30	91.6790	424	35	107.2088
245	20	60.6250	305	25	76.1547	365	30	91.6844	425	35	107.2141
246	20	60.6303	306	25	76.1601	366	30	91.6898	426	35	107.2195
247	20	60.6357	307	25	76.1654	367	30	91.6951	427	35	107.2249
248	20	60.6411	308	25	76.1708	368	30	91.7005	428	35	107.2302
249	21	63.6880	309	25	76.1762	369	30	91.7059	429	35	107.2356
250	21	63.6933	310	25	76.1815	370	30	91.7112	430	35	107.2410
251	21	63.6987	311	26	79.2284	371	30	91.7166	431	35	107.2463
252	21	63.7041	312	26	79.2338	372	30	91.7220	432	35	107.2517
253	21	63.7094	313	26	79.2392	373	31	94.7689	433	35	107.2571
254	21	63.7148	314	26	79.2445	374	31	94.7742	434	35	107.2624
255	21	63.7202	315	26	79.2499	375	31	94.7796	435	36	110.3093
256	21	63.7255	316	26	79.2553	376	31	94.7850	436	36	110.3147
257	21	63.7309	317	26	79.2606	377	31	94.7903	437	36	110.3201
258	21	63.7363	338	26	79.2660	378	31	94.7957	438	36	110.3254
259	21	63.7416	319	26	79.2714	379	31	94.8011	439	36	110.3308
260	21	63.7470	320	26	79.2767	380	31	94.8065	440	36	110.3362
261	22	66.7939	321	26	79.2821	381	31	94.8118	441	36	110.3415
262	22	66.7993	322	26	79.2875	382	31	94.8172	442	36	110.3469
263	22	66.8046	323	27	82.3344	383	31	94.8226	443	36	110.3523
264	22	66.8100	324	27	82.3397	384	31	94.8279	444	36	110.3576
265	22	66.8154	325	27	82.3451	385	32	97.8748	445	36	110.3630
266	22	66.8207	326	27	82.3505	386	32	97.8802	446	36	110.3684
267	22	66.8261	327	27	82.3558	387	32	97.8856	447	37	113.4153
268	22	66.8315	328	27	82.3612	388	32	97.8909	448	37	113.4206
269	22	66.8369	329	27	82.3666	389	32	97.8963	449	37	113.4260
270	22	66.8422	330	27	82.3719	390	32	97.9017	450	37	113.4314
271	22	66.8476	331	27	82.3773	391	32	97.9070	451	37	113.4368
272	22	66.8530	332	27	82.3827	392	32	97.9124	452	37	113.4421
273	22	66.8583	333	27	82.3880	393	32	97.9178	453	37	113.4475
274	23	69.9052	334	27	82.3934	394	32	97.9231	454	37	113.4529
275	23	69.9106	335	27	82.3988	395	32	97.9285	455	37	113.4582
276	23	69.9160	336	28	85.4457	396	32	97.9339	456	37	113.4636
277	23	69.9213	337	28	85.4510	397	32	97.9392	457	37	113.4690
278	23	69.9267	338	28	85.4564	398	33	100.9861	458	37	113.4743
279	23	69.9321	339	28	85.4618	399	33	100.9915	459	37	113.4797
280	23	69.9374	340	28	85.4671	400	33	100.9969	460	38	116.5266
281	23	69.9428	341	28	85.4725	401	33	101.0022	461	38	116.5320
282	23	69.9482	342	28	85.4779	402	33	101.0076	462	38	116.5373
283	23	69.9535	343	28	85.4833	403	33	101.0130	463	38	116.5427
284	23	69.9589	344	28	85.4886	404	33	101.0183	464	38	116.5481
285	23	69.9643	345	28	85.4940	405	33	101.0237	465	38	116.5534
286	24	73.0112	346	28	85.4994	406	33	101.0291	466	38	116.5588
287	24	73.0165	347	28	85.5047	407	33	101.0344	467	38	116.5642
288	24	73.0219	348	29	88.5516	408	33	101.0398	468	38	116.5695
289	24	73.0273	349	29	88.5570	409	33	101.0452	469	38	116.5749
290	24	73.0326	350	29	88.5624	410	34	104.0921	470	38	116.5803
291	24	73.0380	351	29	88.5677	411	34	104.0974	471	38	116.5856
292	24	73.0434	352	29	88.5731	412	34	104.1028	472	39	119.6325
293	24	73.0487	353	29	88.5785	413	34	104.1082	473	39	119.6379
294	24	73.0541	354	29	88.5838	414	34	104.1136	474	39	119.6433
295	24	73.0595	355	29	88.5892	415	34	104.1189	475	39	119.6486
296	24	73.0648	356	29	88.5946	416	34	104.1243	476	39	119.6540
297	24	73.0702	357	29	88.5999	417	34	104.1297	477	39	119.6594
298	25	76.1171	358	29	88.6053	418	34	104.1350	478	39	119.6647
299	25	76.1225	359	29	88.6107	419	34	104.1404	479	39	119.6701
300	25	76.1278	360	30	91.6576	420	34	104.1458	480	39	119.6755

11 Inverse Involute Function

The inverse involute function is the formula to determine the value α from invα, the involute function (invα = tanα − α).

(1) Inverse Involute Function Calculation

The following shows the formula and the calculation examples for Inverse Involute Function.

✻ The Calculation

This calculation is done in radians. Beginning from the calculation for 1 rad....

$$a_1 = 1 + (\mathrm{inv}\alpha - \tan 1 + 1) \div \tan 1^2$$
$$a_2 = a_1 + (\mathrm{inv}\alpha - \tan a_1 + a_1) \div \tan a_1^2$$
$$a_3 = a_2 + (\mathrm{inv}\alpha - \tan a_2 + a_2) \div \tan a_2^2$$
$$\vdots$$
$$a_x = a_{x-1} + (\mathrm{inv}\alpha - \tan a_{x-1} + a_{x-1}) \div \tan a_{x-1}^2$$

At the end, it is converted from radians to degrees,
$\alpha = a_x \times 180 \div \pi$
The calculation must continue until the value converges.

✻ Example

Inverse Involute Function Calculation (If invα = 0.014904384)

No.	Formula	Symbol	Unit	Example
1		invα	rad	0.014904384
2	$a_1 = 1 + (\mathrm{inv}\alpha - \tan 1 + 1) \div \tan 1^2$	a_1		0.776335135
3	$a_2 = a_1 + (\mathrm{inv}\alpha - \tan a_1 + a_1) \div \tan a_1^2$	a_2		0.578494316
4	$a_3 = a_2 + (\mathrm{inv}\alpha - \tan a_2 + a_2) \div \tan a_2^2$	a_3		0.438683749
5	$a_4 = a_3 + (\mathrm{inv}\alpha - \tan a_3 + a_3) \div \tan a_3^2$	a_4		0.367880815
6	$a_5 = a_4 + (\mathrm{inv}\alpha - \tan a_4 + a_4) \div \tan a_4^2$	a_5		0.350096245
7	$a_6 = a_5 + (\mathrm{inv}\alpha - \tan a_5 + a_5) \div \tan a_5^2$	a_6		0.349069141
8	$a_7 = a_6 + (\mathrm{inv}\alpha - \tan a_6 + a_6) \div \tan a_6^2$	a_7		0.34906585
9	$a_8 = a_7 + (\mathrm{inv}\alpha - \tan a_7 + a_7) \div \tan a_7^2$	a_8		0.34906585
10	$a_9 = a_8 + (\mathrm{inv}\alpha - \tan a_8 + a_8) \div \tan a_8^2$	a_9		0.34906585
11	$\alpha = \dfrac{a_9 \times 180}{\pi}$	α	degree	20

Appendix : Calculator for Involute Functions

Since the involute function involves convergence calculation, it is easier to use a calculating software. The following shows how to use the calculator for the calculation in (1) above, and the example.

✻ The Calculation

Input the involute function in invα

= 1+(invα − TAN(1)+1)/TAN(1)^2

The value calculated above is defined as B. Assign B to the formula below.

= B+(invα−TAN(B)+B)/TAN(B)^2

The calculation above is performed repeatedly while the resultant value converges, and then B is converted into radians.

= DEGREES(B)

✻ Example

Actual calculations are performed as follows.
(If invα = 0.014904384)

	A	B
1	0.014904384	
2	=1 + (A1- TAN(1) + 1) / TAN(1) ^ 2	=DEGREES(A2)
3	=A2 + (A1- TAN(A2) + A2) / TAN(A2) ^ 2	=DEGREES(A3)
4	=A3 + (A1- TAN(A3) + A3) / TAN(A3) ^ 2	=DEGREES(A4)
5	=A4 + (A1- TAN(A4) + A4) / TAN(A4) ^ 2	=DEGREES(A5)
6	=A5 + (A1- TAN(A5) + A5) / TAN(A5) ^ 2	=DEGREES(A6)
7	=A6 + (A1- TAN(A6) + A6) / TAN(A6) ^ 2	=DEGREES(A7)
8	=A7 + (A1- TAN(A7) + A7) / TAN(A7) ^ 2	=DEGREES(A8)
9	=A8 + (A1- TAN(A8) + A8) / TAN(A8) ^ 2	=DEGREES(A9)
10	=A9 + (A1- TAN(A9) + A9) / TAN(A9) ^ 2	=DEGREES(A10)

	A	B
1	0.014904384	
2	0.776335135	44.4807267
3	0.578494316	33.14528275
4	0.438683749	25.13472737
5	0.367880815	21.07801808
6	0.350096245	20.05903727
7	0.349069141	20.00018856
8	0.34906585	20
9	0.34906585	20
10	0.34906585	20

← The value converges at a constant.

12 Involute Function Table

$\mathrm{inv}\,\alpha = \tan\alpha - \alpha$

min. (′)	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°
0	0.00001418	0.00004790	0.0001136	0.0002222	0.0003845	0.0006115	0.0009145	0.001305	0.001794	0.002394
1	0.00001454	0.00004871	0.0001151	0.0002244	0.0003877	0.0006159	0.0009203	0.001312	0.001803	0.002405
2	0.00001491	0.00004952	0.0001165	0.0002267	0.0003909	0.0006203	0.0009260	0.001319	0.001812	0.002416
3	0.00001528	0.00005034	0.0001180	0.0002289	0.0003942	0.0006248	0.0009318	0.001327	0.001821	0.002427
4	0.00001565	0.00005117	0.0001194	0.0002312	0.0003975	0.0006292	0.0009377	0.001334	0.001830	0.002438
5	0.00001603	0.00005201	0.0001209	0.0002335	0.0004008	0.0006337	0.0009435	0.001342	0.001840	0.002449
6	0.00001642	0.00005286	0.0001224	0.0002358	0.0004041	0.0006382	0.0009494	0.001349	0.001849	0.002461
7	0.00001682	0.00005372	0.0001239	0.0002382	0.0004074	0.0006427	0.0009553	0.001357	0.001858	0.002472
8	0.00001722	0.00005458	0.0001254	0.0002405	0.0004108	0.0006473	0.0009612	0.001364	0.001867	0.002483
9	0.00001762	0.00005546	0.0001269	0.0002429	0.0004141	0.0006518	0.0009672	0.001372	0.001877	0.002494
10	0.00001804	0.00005634	0.0001285	0.0002452	0.0004175	0.0006564	0.0009732	0.001379	0.001886	0.002506
11	0.00001846	0.00005724	0.0001300	0.0002476	0.0004209	0.0006610	0.0009792	0.001387	0.001895	0.002517
12	0.00001888	0.00005814	0.0001316	0.0002500	0.0004244	0.0006657	0.0009852	0.001394	0.001905	0.002528
13	0.00001931	0.00005906	0.0001332	0.0002524	0.0004278	0.0006703	0.0009913	0.001402	0.001914	0.002540
14	0.00001975	0.00005998	0.0001347	0.0002549	0.0004313	0.0006750	0.0009973	0.001410	0.001924	0.002551
15	0.00002020	0.00006091	0.0001363	0.0002573	0.0004347	0.0006797	0.0010034	0.001417	0.001933	0.002563
16	0.00002065	0.00006186	0.0001380	0.0002598	0.0004382	0.0006844	0.0010096	0.001425	0.001943	0.002574
17	0.00002111	0.00006281	0.0001396	0.0002622	0.0004417	0.0006892	0.0010157	0.001433	0.001952	0.002586
18	0.00002158	0.00006377	0.0001412	0.0002647	0.0004453	0.0006939	0.0010219	0.001441	0.001962	0.002598
19	0.00002205	0.00006474	0.0001429	0.0002673	0.0004488	0.0006987	0.0010281	0.001448	0.001972	0.002609
20	0.00002253	0.00006573	0.0001445	0.0002698	0.0004524	0.0007035	0.0010343	0.001456	0.001981	0.002621
21	0.00002301	0.00006672	0.0001462	0.0002723	0.0004560	0.0007083	0.0010406	0.001464	0.001991	0.002633
22	0.00002351	0.00006772	0.0001479	0.0002749	0.0004596	0.0007132	0.0010469	0.001472	0.002001	0.002644
23	0.00002401	0.00006873	0.0001496	0.0002775	0.0004632	0.0007181	0.0010532	0.001480	0.002010	0.002656
24	0.00002452	0.00006975	0.0001513	0.0002801	0.0004669	0.0007230	0.0010595	0.001488	0.002020	0.002668
25	0.00002503	0.00007078	0.0001530	0.0002827	0.0004706	0.0007279	0.0010659	0.001496	0.002030	0.002680
26	0.00002555	0.00007183	0.0001548	0.0002853	0.0004743	0.0007328	0.0010722	0.001504	0.002040	0.002692
27	0.00002608	0.00007288	0.0001565	0.0002879	0.0004780	0.0007378	0.0010786	0.001512	0.002050	0.002703
28	0.00002662	0.00007394	0.0001583	0.0002906	0.0004817	0.0007428	0.0010851	0.001520	0.002060	0.002715
29	0.00002716	0.00007501	0.0001601	0.0002933	0.0004854	0.0007478	0.0010915	0.001528	0.002069	0.002727
30	0.00002771	0.00007610	0.0001619	0.0002959	0.0004892	0.0007528	0.0010980	0.001536	0.002079	0.002739
31	0.00002827	0.00007719	0.0001637	0.0002986	0.0004930	0.0007579	0.0011045	0.001544	0.002089	0.002751
32	0.00002884	0.00007829	0.0001655	0.0003014	0.0004968	0.0007629	0.0011111	0.001553	0.002100	0.002764
33	0.00002941	0.00007941	0.0001674	0.0003041	0.0005006	0.0007680	0.0011176	0.001561	0.002110	0.002776
34	0.00002999	0.00008053	0.0001692	0.0003069	0.0005045	0.0007732	0.0011242	0.001569	0.002120	0.002788
35	0.00003058	0.00008167	0.0001711	0.0003096	0.0005083	0.0007783	0.0011308	0.001577	0.002130	0.002800
36	0.00003117	0.00008281	0.0001729	0.0003124	0.0005122	0.0007835	0.0011375	0.001586	0.002140	0.002812
37	0.00003178	0.00008397	0.0001748	0.0003152	0.0005161	0.0007887	0.0011441	0.001594	0.002150	0.002825
38	0.00003239	0.00008514	0.0001767	0.0003180	0.0005200	0.0007939	0.0011508	0.001602	0.002160	0.002837
39	0.00003301	0.00008632	0.0001787	0.0003209	0.0005240	0.0007991	0.0011575	0.001611	0.002171	0.002849
40	0.00003364	0.00008751	0.0001806	0.0003237	0.0005280	0.0008044	0.0011643	0.001619	0.002181	0.002862
41	0.00003427	0.00008871	0.0001825	0.0003266	0.0005319	0.0008096	0.0011711	0.001628	0.002191	0.002874
42	0.00003491	0.00008992	0.0001845	0.0003295	0.0005359	0.0008150	0.0011779	0.001636	0.002202	0.002887
43	0.00003556	0.00009114	0.0001865	0.0003324	0.0005400	0.0008203	0.0011847	0.001645	0.002212	0.002899
44	0.00003622	0.00009237	0.0001885	0.0003353	0.0005440	0.0008256	0.0011915	0.001653	0.002223	0.002912
45	0.00003689	0.00009362	0.0001905	0.0003383	0.0005481	0.0008310	0.0011984	0.001662	0.002233	0.002924
46	0.00003757	0.00009487	0.0001925	0.0003412	0.0005522	0.0008364	0.0012053	0.001670	0.002244	0.002937
47	0.00003825	0.00009614	0.0001945	0.0003442	0.0005563	0.0008418	0.0012122	0.001679	0.002254	0.002949
48	0.00003894	0.00009742	0.0001965	0.0003472	0.0005604	0.0008473	0.0012192	0.001688	0.002265	0.002962
49	0.00003964	0.00009870	0.0001986	0.0003502	0.0005645	0.0008527	0.0012262	0.001696	0.002275	0.002975
50	0.00004035	0.00010000	0.0002007	0.0003532	0.0005687	0.0008582	0.0012332	0.001705	0.002286	0.002987
51	0.00004107	0.00010132	0.0002028	0.0003563	0.0005729	0.0008638	0.0012402	0.001714	0.002297	0.003000
52	0.00004179	0.00010264	0.0002049	0.0003593	0.0005771	0.0008693	0.0012473	0.001723	0.002307	0.003013
53	0.00004252	0.00010397	0.0002070	0.0003624	0.0005813	0.0008749	0.0012544	0.001731	0.002318	0.003026
54	0.00004327	0.00010532	0.0002091	0.0003655	0.0005856	0.0008805	0.0012615	0.001740	0.002329	0.003039
55	0.00004402	0.00010668	0.0002113	0.0003686	0.0005898	0.0008861	0.0012687	0.001749	0.002340	0.003052
56	0.00004478	0.00010805	0.0002134	0.0003718	0.0005941	0.0008917	0.0012758	0.001758	0.002350	0.003065
57	0.00004554	0.00010943	0.0002156	0.0003749	0.0005985	0.0008974	0.0012830	0.001767	0.002361	0.003078
58	0.00004632	0.00011082	0.0002178	0.0003781	0.0006028	0.0009031	0.0012903	0.001776	0.002372	0.003091
59	0.00004711	0.00011223	0.0002200	0.0003813	0.0006071	0.0009088	0.0012975	0.001785	0.002383	0.003104
60	0.00004790	0.00011364	0.0002222	0.0003845	0.0006115	0.0009145	0.0013048	0.001794	0.002394	0.003117

Involute Function Table

$\mathrm{inv}\,\alpha = \tan\alpha - \alpha$

min. (')	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°
0	0.003117	0.003975	0.004982	0.006150	0.007493	0.009025	0.010760	0.012715	0.014904	0.017345
1	0.003130	0.003991	0.005000	0.006171	0.007517	0.009052	0.010791	0.012750	0.014943	0.017388
2	0.003143	0.004006	0.005018	0.006192	0.007541	0.009079	0.010822	0.012784	0.014982	0.017431
3	0.003157	0.004022	0.005036	0.006213	0.007565	0.009107	0.010853	0.012819	0.015020	0.017474
4	0.003170	0.004038	0.005055	0.006234	0.007589	0.009134	0.010884	0.012854	0.015059	0.017517
5	0.003183	0.004053	0.005073	0.006255	0.007613	0.009161	0.010915	0.012888	0.015098	0.017560
6	0.003197	0.004069	0.005091	0.006276	0.007637	0.009189	0.010946	0.012923	0.015137	0.017603
7	0.003210	0.004085	0.005110	0.006297	0.007661	0.009216	0.010977	0.012958	0.015176	0.017647
8	0.003223	0.004101	0.005128	0.006318	0.007686	0.009244	0.011008	0.012993	0.015215	0.017690
9	0.003237	0.004117	0.005146	0.006340	0.007710	0.009272	0.011039	0.013028	0.015254	0.017734
10	0.003250	0.004132	0.005165	0.006361	0.007735	0.009299	0.011071	0.013063	0.015293	0.017777
11	0.003264	0.004148	0.005184	0.006382	0.007759	0.009327	0.011102	0.013098	0.015333	0.017821
12	0.003277	0.004164	0.005202	0.006404	0.007784	0.009355	0.011133	0.013134	0.015372	0.017865
13	0.003291	0.004180	0.005221	0.006425	0.007808	0.009383	0.011165	0.013169	0.015411	0.017908
14	0.003305	0.004197	0.005239	0.006447	0.007833	0.009411	0.011196	0.013204	0.015451	0.017952
15	0.003318	0.004213	0.005258	0.006469	0.007857	0.009439	0.011228	0.013240	0.015490	0.017996
16	0.003332	0.004229	0.005277	0.006490	0.007882	0.009467	0.011260	0.013275	0.015530	0.018040
17	0.003346	0.004245	0.005296	0.006512	0.007907	0.009495	0.011291	0.013311	0.015570	0.018084
18	0.003360	0.004261	0.005315	0.006534	0.007932	0.009523	0.011323	0.013346	0.015609	0.018129
19	0.003374	0.004277	0.005334	0.006555	0.007957	0.009552	0.011355	0.013382	0.015649	0.018173
20	0.003387	0.004294	0.005353	0.006577	0.007982	0.009580	0.011387	0.013418	0.015689	0.018217
21	0.003401	0.004310	0.005372	0.006599	0.008007	0.009608	0.011419	0.013454	0.015729	0.018262
22	0.003415	0.004327	0.005391	0.006621	0.008032	0.009637	0.011451	0.013490	0.015769	0.018306
23	0.003429	0.004343	0.005410	0.006643	0.008057	0.009665	0.011483	0.013526	0.015809	0.018351
24	0.003443	0.004359	0.005429	0.006665	0.008082	0.009694	0.011515	0.013562	0.015849	0.018395
25	0.003458	0.004376	0.005448	0.006687	0.008107	0.009722	0.011547	0.013598	0.015890	0.018440
26	0.003472	0.004393	0.005467	0.006709	0.008133	0.009751	0.011580	0.013634	0.015930	0.018485
27	0.003486	0.004409	0.005487	0.006732	0.008158	0.009780	0.011612	0.013670	0.015971	0.018530
28	0.003500	0.004426	0.005506	0.006754	0.008183	0.009808	0.011644	0.013707	0.016011	0.018575
29	0.003514	0.004443	0.005525	0.006776	0.008209	0.009837	0.011677	0.013743	0.016052	0.018620
30	0.003529	0.004459	0.005545	0.006799	0.008234	0.009866	0.011709	0.013779	0.016092	0.018665
31	0.003543	0.004476	0.005564	0.006821	0.008260	0.009895	0.011742	0.013816	0.016133	0.018710
32	0.003557	0.004493	0.005584	0.006843	0.008285	0.009924	0.011775	0.013852	0.016174	0.018755
33	0.003572	0.004510	0.005603	0.006866	0.008311	0.009953	0.011807	0.013889	0.016214	0.018800
34	0.003586	0.004527	0.005623	0.006888	0.008337	0.009982	0.011840	0.013926	0.016255	0.018846
35	0.003600	0.004544	0.005643	0.006911	0.008362	0.010011	0.011873	0.013963	0.016296	0.018891
36	0.003615	0.004561	0.005662	0.006934	0.008388	0.010041	0.011906	0.013999	0.016337	0.018937
37	0.003630	0.004578	0.005682	0.006956	0.008414	0.010070	0.011939	0.014036	0.016379	0.018983
38	0.003644	0.004595	0.005702	0.006979	0.008440	0.010099	0.011972	0.014073	0.016420	0.019028
39	0.003659	0.004612	0.005722	0.007002	0.008466	0.010129	0.012005	0.014110	0.016461	0.019074
40	0.003673	0.004629	0.005742	0.007025	0.008492	0.010158	0.012038	0.014148	0.016502	0.019120
41	0.003688	0.004646	0.005762	0.007048	0.008518	0.010188	0.012071	0.014185	0.016544	0.019166
42	0.003703	0.004664	0.005782	0.007071	0.008544	0.010217	0.012105	0.014222	0.016585	0.019212
43	0.003718	0.004681	0.005802	0.007094	0.008571	0.010247	0.012138	0.014259	0.016627	0.019258
44	0.003733	0.004698	0.005822	0.007117	0.008597	0.010277	0.012172	0.014297	0.016669	0.019304
45	0.003747	0.004716	0.005842	0.007140	0.008623	0.010307	0.012205	0.014334	0.016710	0.019350
46	0.003762	0.004733	0.005862	0.007163	0.008650	0.010336	0.012239	0.014372	0.016752	0.019397
47	0.003777	0.004751	0.005882	0.007186	0.008676	0.010366	0.012272	0.014409	0.016794	0.019443
48	0.003792	0.004768	0.005903	0.007209	0.008702	0.010396	0.012306	0.014447	0.016836	0.019490
49	0.003807	0.004786	0.005923	0.007233	0.008729	0.010426	0.012340	0.014485	0.016878	0.019536
50	0.003822	0.004803	0.005943	0.007256	0.008756	0.010456	0.012373	0.014523	0.016920	0.019583
51	0.003838	0.004821	0.005964	0.007280	0.008782	0.010486	0.012407	0.014560	0.016962	0.019630
52	0.003853	0.004839	0.005984	0.007303	0.008809	0.010517	0.012441	0.014598	0.017004	0.019676
53	0.003868	0.004856	0.006005	0.007327	0.008836	0.010547	0.012475	0.014636	0.017047	0.019723
54	0.003883	0.004874	0.006025	0.007350	0.008863	0.010577	0.012509	0.014674	0.017089	0.019770
55	0.003898	0.004892	0.006046	0.007374	0.008889	0.010608	0.012543	0.014713	0.017132	0.019817
56	0.003914	0.004910	0.006067	0.007397	0.008916	0.010638	0.012578	0.014751	0.017174	0.019864
57	0.003929	0.004928	0.006087	0.007421	0.008943	0.010669	0.012612	0.014789	0.017217	0.019912
58	0.003944	0.004946	0.006108	0.007445	0.008970	0.010699	0.012646	0.014827	0.017259	0.019959
59	0.003960	0.004964	0.006129	0.007469	0.008998	0.010730	0.012681	0.014866	0.017302	0.020006
60	0.003975	0.004982	0.006150	0.007493	0.009025	0.010760	0.012715	0.014904	0.017345	0.020054

Involute Function Table

inv $\alpha = \tan \alpha - \alpha$

min. (')	22°	23°	24°	25°	26°	27°	28°	29°	30°	31°
0	0.020054	0.023049	0.026350	0.029975	0.033947	0.038287	0.043017	0.048164	0.053751	0.059809
1	0.020101	0.023102	0.026407	0.030039	0.034016	0.038362	0.043100	0.048253	0.053849	0.059914
2	0.020149	0.023154	0.026465	0.030102	0.034086	0.038438	0.043182	0.048343	0.053946	0.060019
3	0.020197	0.023207	0.026523	0.030166	0.034155	0.038514	0.043264	0.048432	0.054043	0.060124
4	0.020244	0.023259	0.026581	0.030229	0.034225	0.038589	0.043347	0.048522	0.054140	0.060230
5	0.020292	0.023312	0.026639	0.030293	0.034294	0.038666	0.043430	0.048612	0.054238	0.060335
6	0.020340	0.023365	0.026697	0.030357	0.034364	0.038742	0.043513	0.048702	0.054336	0.060441
7	0.020388	0.023418	0.026756	0.030420	0.034434	0.038818	0.043596	0.048792	0.054433	0.060547
8	0.020436	0.023471	0.026814	0.030484	0.034504	0.038894	0.043679	0.048883	0.054531	0.060653
9	0.020484	0.023524	0.026872	0.030549	0.034574	0.038971	0.043762	0.048973	0.054629	0.060759
10	0.020533	0.023577	0.026931	0.030613	0.034644	0.039047	0.043845	0.049063	0.054728	0.060866
11	0.020581	0.023631	0.026989	0.030677	0.034714	0.039124	0.043929	0.049154	0.054826	0.060972
12	0.020629	0.023684	0.027048	0.030741	0.034785	0.039201	0.044012	0.049245	0.054924	0.061079
13	0.020678	0.023738	0.027107	0.030806	0.034855	0.039278	0.044096	0.049336	0.055023	0.061186
14	0.020726	0.023791	0.027166	0.030870	0.034926	0.039355	0.044180	0.049427	0.055122	0.061292
15	0.020775	0.023845	0.027225	0.030935	0.034996	0.039432	0.044264	0.049518	0.055221	0.061400
16	0.020824	0.023899	0.027284	0.031000	0.035067	0.039509	0.044348	0.049609	0.055320	0.061507
17	0.020873	0.023952	0.027343	0.031065	0.035138	0.039586	0.044432	0.049701	0.055419	0.061614
18	0.020921	0.024006	0.027402	0.031130	0.035209	0.039664	0.044516	0.049792	0.055518	0.061721
19	0.020970	0.024060	0.027462	0.031195	0.035280	0.039741	0.044601	0.049884	0.055617	0.061829
20	0.021019	0.024114	0.027521	0.031260	0.035352	0.039819	0.044685	0.049976	0.055717	0.061937
21	0.021069	0.024169	0.027581	0.031325	0.035423	0.039897	0.044770	0.050068	0.055817	0.062045
22	0.021118	0.024223	0.027640	0.031390	0.035494	0.039974	0.044855	0.050160	0.055916	0.062153
23	0.021167	0.024277	0.027700	0.031456	0.035566	0.040052	0.044939	0.050252	0.056016	0.062261
24	0.021217	0.024332	0.027760	0.031521	0.035637	0.040131	0.045024	0.050344	0.056116	0.062369
25	0.021266	0.024386	0.027820	0.031587	0.035709	0.040209	0.045110	0.050437	0.056217	0.062478
26	0.021316	0.024441	0.027880	0.031653	0.035781	0.040287	0.045195	0.050529	0.056317	0.062586
27	0.021365	0.024495	0.027940	0.031718	0.035853	0.040366	0.045280	0.050622	0.056417	0.062695
28	0.021415	0.024550	0.028000	0.031784	0.035925	0.040444	0.045366	0.050715	0.056518	0.062804
29	0.021465	0.024605	0.028060	0.031850	0.035997	0.040523	0.045451	0.050808	0.056619	0.062913
30	0.021514	0.024660	0.028121	0.031917	0.036069	0.040602	0.045537	0.050901	0.056720	0.063022
31	0.021564	0.024715	0.028181	0.031983	0.036142	0.040680	0.045623	0.050994	0.056821	0.063131
32	0.021614	0.024770	0.028242	0.032049	0.036214	0.040759	0.045709	0.051087	0.056922	0.063241
33	0.021665	0.024825	0.028302	0.032116	0.036287	0.040838	0.045795	0.051181	0.057023	0.063350
34	0.021715	0.024881	0.028363	0.032182	0.036359	0.040918	0.045881	0.051274	0.057124	0.063460
35	0.021765	0.024936	0.028424	0.032249	0.036432	0.040997	0.045967	0.051368	0.057226	0.063570
36	0.021815	0.024992	0.028485	0.032315	0.036505	0.041076	0.046054	0.051462	0.057328	0.063680
37	0.021866	0.025047	0.028546	0.032382	0.036578	0.041156	0.046140	0.051556	0.057429	0.063790
38	0.021916	0.025103	0.028607	0.032449	0.036651	0.041236	0.046227	0.051650	0.057531	0.063901
39	0.021967	0.025159	0.028668	0.032516	0.036724	0.041316	0.046313	0.051744	0.057633	0.064011
40	0.022018	0.025214	0.028729	0.032583	0.036798	0.041395	0.046400	0.051838	0.057736	0.064122
41	0.022068	0.025270	0.028791	0.032651	0.036871	0.041475	0.046487	0.051933	0.057838	0.064232
42	0.022119	0.025326	0.028852	0.032718	0.036945	0.041556	0.046575	0.052027	0.057940	0.064343
43	0.022170	0.025382	0.028914	0.032785	0.037018	0.041636	0.046662	0.052122	0.058043	0.064454
44	0.022221	0.025439	0.028976	0.032853	0.037092	0.041716	0.046749	0.052217	0.058146	0.064565
45	0.022272	0.025495	0.029037	0.032920	0.037166	0.041797	0.046837	0.052312	0.058249	0.064677
46	0.022324	0.025551	0.029099	0.032988	0.037240	0.041877	0.046924	0.052407	0.058352	0.064788
47	0.022375	0.025608	0.029161	0.033056	0.037314	0.041958	0.047012	0.052502	0.058455	0.064900
48	0.022426	0.025664	0.029223	0.033124	0.037388	0.042039	0.047100	0.052597	0.058558	0.065012
49	0.022478	0.025721	0.029285	0.033192	0.037462	0.042120	0.047188	0.052693	0.058662	0.065123
50	0.022529	0.025778	0.029348	0.033260	0.037537	0.042201	0.047276	0.052788	0.058765	0.065236
51	0.022581	0.025834	0.029410	0.033328	0.037611	0.042282	0.047364	0.052884	0.058869	0.065348
52	0.022632	0.025891	0.029472	0.033397	0.037686	0.042363	0.047452	0.052980	0.058973	0.065460
53	0.022684	0.025948	0.029535	0.033465	0.037761	0.042444	0.047541	0.053076	0.059077	0.065573
54	0.022736	0.026005	0.029598	0.033534	0.037835	0.042526	0.047630	0.053172	0.059181	0.065685
55	0.022788	0.026062	0.029660	0.033602	0.037910	0.042607	0.047718	0.053268	0.059285	0.065798
56	0.022840	0.026120	0.029723	0.033671	0.037985	0.042689	0.047807	0.053365	0.059390	0.065911
57	0.022892	0.026177	0.029786	0.033740	0.038060	0.042771	0.047896	0.053461	0.059494	0.066024
58	0.022944	0.026235	0.029849	0.033809	0.038136	0.042853	0.047985	0.053558	0.059599	0.066137
59	0.022997	0.026292	0.029912	0.033878	0.038211	0.042935	0.048074	0.053655	0.059704	0.066250
60	0.023049	0.026350	0.029975	0.033947	0.038287	0.043017	0.048164	0.053751	0.059809	0.066364

Involute Function Table

$\text{inv}\,\alpha = \tan\alpha - \alpha$

min. (')	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°
0	0.066364	0.073449	0.081097	0.089342	0.098224	0.107782	0.118061	0.129106	0.140968	0.15370
1	0.066478	0.073572	0.081229	0.089485	0.098378	0.107948	0.118238	0.129296	0.141173	0.15392
2	0.066591	0.073695	0.081362	0.089628	0.098531	0.108113	0.118416	0.129488	0.141378	0.15414
3	0.066705	0.073818	0.081494	0.089771	0.098685	0.108279	0.118594	0.129679	0.141583	0.15436
4	0.066819	0.073941	0.081627	0.089914	0.098840	0.108445	0.118772	0.129870	0.141789	0.15458
5	0.066934	0.074064	0.081760	0.090058	0.098994	0.108611	0.118951	0.130062	0.141995	0.15480
6	0.067048	0.074188	0.081894	0.090201	0.099149	0.108777	0.119130	0.130254	0.142201	0.15503
7	0.067163	0.074311	0.082027	0.090345	0.099303	0.108943	0.119309	0.130446	0.142408	0.15525
8	0.067277	0.074435	0.082161	0.090489	0.099458	0.109110	0.119488	0.130639	0.142614	0.15547
9	0.067392	0.074559	0.082294	0.090633	0.099614	0.109277	0.119667	0.130832	0.142821	0.15569
10	0.067507	0.074684	0.082428	0.090777	0.099769	0.109444	0.119847	0.131025	0.143028	0.15591
11	0.067622	0.074808	0.082562	0.090922	0.099924	0.109611	0.120027	0.131218	0.143236	0.15614
12	0.067738	0.074932	0.082697	0.091067	0.100080	0.109779	0.120207	0.131411	0.143443	0.15636
13	0.067853	0.075057	0.082831	0.091211	0.100236	0.109947	0.120387	0.131605	0.143651	0.15658
14	0.067969	0.075182	0.082966	0.091356	0.100392	0.110114	0.120567	0.131798	0.143859	0.15680
15	0.068084	0.075307	0.083100	0.091502	0.100548	0.110283	0.120748	0.131993	0.144067	0.15703
16	0.068200	0.075432	0.083235	0.091647	0.100705	0.110451	0.120929	0.132187	0.144276	0.15725
17	0.068316	0.075557	0.083371	0.091793	0.100862	0.110619	0.121110	0.132381	0.144485	0.15748
18	0.068432	0.075683	0.083506	0.091938	0.101019	0.110788	0.121291	0.132576	0.144694	0.15770
19	0.068549	0.075808	0.083641	0.092084	0.101176	0.110957	0.121473	0.132771	0.144903	0.15793
20	0.068665	0.075934	0.083777	0.092230	0.101333	0.111126	0.121655	0.132966	0.145113	0.15815
21	0.068782	0.076060	0.083913	0.092377	0.101490	0.111295	0.121837	0.133162	0.145323	0.15838
22	0.068899	0.076186	0.084049	0.092523	0.101648	0.111465	0.122019	0.133357	0.145533	0.15860
23	0.069016	0.076312	0.084185	0.092670	0.101806	0.111635	0.122201	0.133553	0.145743	0.15883
24	0.069133	0.076439	0.084321	0.092816	0.101964	0.111805	0.122384	0.133750	0.145954	0.15905
25	0.069250	0.076565	0.084457	0.092963	0.102122	0.111975	0.122567	0.133946	0.146165	0.15928
26	0.069367	0.076692	0.084594	0.093111	0.102280	0.112145	0.122750	0.134143	0.146376	0.15950
27	0.069485	0.076819	0.084731	0.093258	0.102439	0.112316	0.122933	0.134339	0.146587	0.15973
28	0.069602	0.076946	0.084868	0.093406	0.102598	0.112486	0.123116	0.134536	0.146798	0.15996
29	0.069720	0.077073	0.085005	0.093553	0.102757	0.112657	0.123300	0.134734	0.147010	0.16019
30	0.069838	0.077200	0.085142	0.093701	0.102916	0.112829	0.123484	0.134931	0.147222	0.16041
31	0.069956	0.077328	0.085280	0.093849	0.103075	0.113000	0.123668	0.135129	0.147435	0.16064
32	0.070075	0.077455	0.085418	0.093998	0.103235	0.113171	0.123853	0.135327	0.147647	0.16087
33	0.070193	0.077583	0.085555	0.094146	0.103395	0.113343	0.124037	0.135525	0.147860	0.16110
34	0.070312	0.077711	0.085693	0.094295	0.103555	0.113515	0.124222	0.135724	0.148073	0.16133
35	0.070430	0.077839	0.085832	0.094443	0.103715	0.113687	0.124407	0.135923	0.148286	0.16156
36	0.070549	0.077968	0.085970	0.094592	0.103875	0.113860	0.124592	0.136122	0.148500	0.16178
37	0.070668	0.078096	0.086108	0.094742	0.104036	0.114032	0.124778	0.136321	0.148714	0.16201
38	0.070788	0.078225	0.086247	0.094891	0.104196	0.114205	0.124964	0.136520	0.148928	0.16224
39	0.070907	0.078354	0.086386	0.095041	0.104357	0.114378	0.125150	0.136720	0.149142	0.16247
40	0.071026	0.078483	0.086525	0.095190	0.104518	0.114552	0.125336	0.136920	0.149357	0.16270
41	0.071146	0.078612	0.086664	0.095340	0.104680	0.114725	0.125522	0.137120	0.149572	0.16293
42	0.071266	0.078741	0.086804	0.095490	0.104841	0.114899	0.125709	0.137320	0.149787	0.16317
43	0.071386	0.078871	0.086943	0.095641	0.105003	0.115073	0.125895	0.137521	0.150002	0.16340
44	0.071506	0.079000	0.087083	0.095791	0.105165	0.115247	0.126083	0.137722	0.150218	0.16363
45	0.071626	0.079130	0.087223	0.095942	0.105327	0.115421	0.126270	0.137923	0.150433	0.16386
46	0.071747	0.079260	0.087363	0.096093	0.105489	0.115595	0.126457	0.138124	0.150650	0.16409
47	0.071867	0.079390	0.087503	0.096244	0.105652	0.115770	0.126645	0.138326	0.150866	0.16432
48	0.071988	0.079520	0.087644	0.096395	0.105814	0.115945	0.126833	0.138528	0.151083	0.16456
49	0.072109	0.079651	0.087784	0.096546	0.105977	0.116120	0.127021	0.138730	0.151299	0.16479
50	0.072230	0.079781	0.087925	0.096698	0.106140	0.116296	0.127209	0.138932	0.151516	0.16502
51	0.072351	0.079912	0.088066	0.096850	0.106304	0.116471	0.127398	0.139134	0.151734	0.16525
52	0.072473	0.080043	0.088207	0.097002	0.106467	0.116647	0.127587	0.139337	0.151951	0.16549
53	0.072594	0.080174	0.088348	0.097154	0.106631	0.116823	0.127776	0.139540	0.152169	0.16572
54	0.072716	0.080306	0.088490	0.097306	0.106795	0.116999	0.127965	0.139743	0.152388	0.16596
55	0.072838	0.080437	0.088631	0.097459	0.106959	0.117175	0.128155	0.139947	0.152606	0.16619
56	0.072959	0.080569	0.088773	0.097611	0.107123	0.117352	0.128344	0.140151	0.152825	0.16642
57	0.073082	0.080700	0.088915	0.097764	0.107288	0.117529	0.128534	0.140355	0.153043	0.16666
58	0.073204	0.080832	0.089057	0.097917	0.107452	0.117706	0.128725	0.140559	0.153263	0.16689
59	0.073326	0.080964	0.089200	0.098071	0.107617	0.117883	0.128915	0.140763	0.153482	0.16713
60	0.073449	0.081097	0.089342	0.098224	0.107782	0.118061	0.129106	0.140968	0.153702	0.16737

Involute Function Table

inv $\alpha = \tan\alpha - \alpha$

min. (′)	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°
0	0.16737	0.18202	0.19774	0.21460	0.23268	0.25206	0.27285	0.29516	0.31909	0.34478
1	0.16760	0.18228	0.19802	0.21489	0.23299	0.25240	0.27321	0.29554	0.31950	0.34522
2	0.16784	0.18253	0.19829	0.21518	0.23330	0.25273	0.27357	0.29593	0.31992	0.34567
3	0.16807	0.18278	0.19856	0.21548	0.23362	0.25307	0.27393	0.29631	0.32033	0.34611
4	0.16831	0.18304	0.19883	0.21577	0.23393	0.25341	0.27429	0.29670	0.32075	0.34656
5	0.16855	0.18329	0.19910	0.21606	0.23424	0.25374	0.27465	0.29709	0.32116	0.34700
6	0.16879	0.18355	0.19938	0.21635	0.23456	0.25408	0.27501	0.29747	0.32158	0.34745
7	0.16902	0.18380	0.19965	0.21665	0.23487	0.25442	0.27538	0.29786	0.32199	0.34790
8	0.16926	0.18406	0.19992	0.21694	0.23519	0.25475	0.27574	0.29825	0.32241	0.34834
9	0.16950	0.18431	0.20020	0.21723	0.23550	0.25509	0.27610	0.29864	0.32283	0.34879
10	0.16974	0.18457	0.20047	0.21753	0.23582	0.25543	0.27646	0.29903	0.32324	0.34924
11	0.16998	0.18482	0.20075	0.21782	0.23613	0.25577	0.27683	0.29942	0.32366	0.34969
12	0.17022	0.18508	0.20102	0.21812	0.23645	0.25611	0.27719	0.29981	0.32408	0.35014
13	0.17045	0.18534	0.20130	0.21841	0.23676	0.25645	0.27755	0.30020	0.32450	0.35059
14	0.17069	0.18559	0.20157	0.21871	0.23708	0.25679	0.27792	0.30059	0.32492	0.35104
15	0.17093	0.18585	0.20185	0.21900	0.23740	0.25713	0.27828	0.30098	0.32534	0.35149
16	0.17117	0.18611	0.20212	0.21930	0.23772	0.25747	0.27865	0.30137	0.32576	0.35194
17	0.17142	0.18637	0.20240	0.21960	0.23803	0.25781	0.27902	0.30177	0.32618	0.35240
18	0.17166	0.18662	0.20268	0.21989	0.23835	0.25815	0.27938	0.30216	0.32661	0.35285
19	0.17190	0.18688	0.20296	0.22019	0.23867	0.25849	0.27975	0.30255	0.32703	0.35330
20	0.17214	0.18714	0.20323	0.22049	0.23899	0.25883	0.28012	0.30295	0.32745	0.35376
21	0.17238	0.18740	0.20351	0.22079	0.23931	0.25918	0.28048	0.30334	0.32787	0.35421
22	0.17262	0.18766	0.20379	0.22108	0.23963	0.25952	0.28085	0.30374	0.32830	0.35467
23	0.17286	0.18792	0.20407	0.22138	0.23995	0.25986	0.28122	0.30413	0.32872	0.35512
24	0.17311	0.18818	0.20435	0.22168	0.24027	0.26021	0.28159	0.30453	0.32915	0.35558
25	0.17335	0.18844	0.20463	0.22198	0.24059	0.26055	0.28196	0.30492	0.32957	0.35604
26	0.17359	0.18870	0.20490	0.22228	0.24091	0.26089	0.28233	0.30532	0.33000	0.35649
27	0.17383	0.18896	0.20518	0.22258	0.24123	0.26124	0.28270	0.30572	0.33042	0.35695
28	0.17408	0.18922	0.20546	0.22288	0.24156	0.26159	0.28307	0.30611	0.33085	0.35741
29	0.17432	0.18948	0.20575	0.22318	0.24188	0.26193	0.28344	0.30651	0.33128	0.35787
30	0.17457	0.18975	0.20603	0.22348	0.24220	0.26228	0.28381	0.30691	0.33171	0.35833
31	0.17481	0.19001	0.20631	0.22378	0.24253	0.26262	0.28418	0.30731	0.33213	0.35879
32	0.17506	0.19027	0.20659	0.22409	0.24285	0.26297	0.28455	0.30771	0.33256	0.35925
33	0.17530	0.19053	0.20687	0.22439	0.24317	0.26332	0.28493	0.30811	0.33299	0.35971
34	0.17555	0.19080	0.20715	0.22469	0.24350	0.26367	0.28530	0.30851	0.33342	0.36017
35	0.17579	0.19106	0.20743	0.22499	0.24382	0.26401	0.28567	0.30891	0.33385	0.36063
36	0.17604	0.19132	0.20772	0.22530	0.24415	0.26436	0.28605	0.30931	0.33428	0.36110
37	0.17628	0.19159	0.20800	0.22560	0.24447	0.26471	0.28642	0.30971	0.33471	0.36156
38	0.17653	0.19185	0.20828	0.22590	0.24480	0.26506	0.28680	0.31012	0.33515	0.36202
39	0.17678	0.19212	0.20857	0.22621	0.24512	0.26541	0.28717	0.31052	0.33558	0.36249
40	0.17702	0.19238	0.20885	0.22651	0.24545	0.26576	0.28755	0.31092	0.33601	0.36295
41	0.17727	0.19265	0.20914	0.22682	0.24578	0.26611	0.28792	0.31133	0.33645	0.36342
42	0.17752	0.19291	0.20942	0.22712	0.24611	0.26646	0.28830	0.31173	0.33688	0.36388
43	0.17777	0.19318	0.20971	0.22743	0.24643	0.26682	0.28868	0.31214	0.33731	0.36435
44	0.17801	0.19344	0.20999	0.22773	0.24676	0.26717	0.28906	0.31254	0.33775	0.36482
45	0.17826	0.19371	0.21028	0.22804	0.24709	0.26752	0.28943	0.31295	0.33818	0.36529
46	0.17851	0.19398	0.21056	0.22835	0.24742	0.26787	0.28981	0.31335	0.33862	0.36575
47	0.17876	0.19424	0.21085	0.22865	0.24775	0.26823	0.29019	0.31376	0.33906	0.36622
48	0.17901	0.19451	0.21114	0.22896	0.24808	0.26858	0.29057	0.31417	0.33949	0.36669
49	0.17926	0.19478	0.21142	0.22927	0.24841	0.26893	0.29095	0.31457	0.33993	0.36716
50	0.17951	0.19505	0.21171	0.22958	0.24874	0.26929	0.29133	0.31498	0.34037	0.36763
51	0.17976	0.19532	0.21200	0.22989	0.24907	0.26964	0.29171	0.31539	0.34081	0.36810
52	0.18001	0.19558	0.21229	0.23020	0.24940	0.27000	0.29209	0.31580	0.34125	0.36858
53	0.18026	0.19585	0.21257	0.23050	0.24973	0.27035	0.29247	0.31621	0.34169	0.36905
54	0.18051	0.19612	0.21286	0.23081	0.25006	0.27071	0.29286	0.31662	0.34213	0.36952
55	0.18076	0.19639	0.21315	0.23112	0.25040	0.27107	0.29324	0.31703	0.34257	0.36999
56	0.18101	0.19666	0.21344	0.23143	0.25073	0.27142	0.29362	0.31744	0.34301	0.37047
57	0.18127	0.19693	0.21373	0.23174	0.25106	0.27178	0.29400	0.31785	0.34345	0.37094
58	0.18152	0.19720	0.21402	0.23206	0.25140	0.27214	0.29439	0.31826	0.34389	0.37142
59	0.18177	0.19747	0.21431	0.23237	0.25173	0.27250	0.29477	0.31868	0.34434	0.37189
60	0.18202	0.19774	0.21460	0.23268	0.25206	0.27285	0.29516	0.31909	0.34478	0.37237

Involute Function Table

inv $\alpha = \tan\alpha - \alpha$

min. (′)	52°	53°	54°	55°	56°	57°	58°	59°	60°	61°
0	0.37237	0.40202	0.43390	0.46822	0.50518	0.54503	0.58804	0.63454	0.68485	0.73940
1	0.37285	0.40253	0.43446	0.46881	0.50582	0.54572	0.58879	0.63534	0.68573	0.74034
2	0.37332	0.40305	0.43501	0.46940	0.50646	0.54641	0.58954	0.63615	0.68660	0.74129
3	0.37380	0.40356	0.43556	0.47000	0.50710	0.54710	0.59028	0.63696	0.68748	0.74224
4	0.37428	0.40407	0.43611	0.47060	0.50774	0.54779	0.59103	0.63777	0.68835	0.74319
5	0.37476	0.40459	0.43667	0.47119	0.50838	0.54849	0.59178	0.63858	0.68923	0.74415
6	0.37524	0.40511	0.43722	0.47179	0.50903	0.54918	0.59253	0.63939	0.69011	0.74510
7	0.37572	0.40562	0.43778	0.47239	0.50967	0.54988	0.59328	0.64020	0.69099	0.74606
8	0.37620	0.40614	0.43833	0.47299	0.51032	0.55057	0.59403	0.64102	0.69187	0.74701
9	0.37668	0.40666	0.43889	0.47359	0.51096	0.55127	0.59479	0.64183	0.69275	0.74797
10	0.37716	0.40717	0.43945	0.47419	0.51161	0.55197	0.59554	0.64265	0.69364	0.74893
11	0.37765	0.40769	0.44001	0.47479	0.51226	0.55267	0.59630	0.64346	0.69452	0.74989
12	0.37813	0.40821	0.44057	0.47539	0.51291	0.55337	0.59705	0.64428	0.69541	0.75085
13	0.37861	0.40873	0.44113	0.47599	0.51356	0.55407	0.59781	0.64510	0.69630	0.75181
14	0.37910	0.40925	0.44169	0.47660	0.51421	0.55477	0.59857	0.64592	0.69719	0.75278
15	0.37958	0.40977	0.44225	0.47720	0.51486	0.55547	0.59933	0.64674	0.69808	0.75375
16	0.38007	0.41030	0.44281	0.47780	0.51551	0.55618	0.60009	0.64756	0.69897	0.75471
17	0.38055	0.41082	0.44337	0.47841	0.51616	0.55688	0.60085	0.64839	0.69986	0.75568
18	0.38104	0.41134	0.44393	0.47902	0.51682	0.55759	0.60161	0.64921	0.70075	0.75665
19	0.38153	0.41187	0.44450	0.47962	0.51747	0.55829	0.60237	0.65004	0.70165	0.75762
20	0.38202	0.41239	0.44506	0.48023	0.51813	0.55900	0.60314	0.65086	0.70254	0.75859
21	0.38251	0.41292	0.44563	0.48084	0.51878	0.55971	0.60390	0.65169	0.70344	0.75957
22	0.38299	0.41344	0.44619	0.48145	0.51944	0.56042	0.60467	0.65252	0.70434	0.76054
23	0.38348	0.41397	0.44676	0.48206	0.52010	0.56113	0.60544	0.65335	0.70524	0.76152
24	0.38397	0.41450	0.44733	0.48267	0.52076	0.56184	0.60620	0.65418	0.70614	0.76250
25	0.38446	0.41502	0.44789	0.48328	0.52141	0.56255	0.60697	0.65501	0.70704	0.76348
26	0.38496	0.41555	0.44846	0.48389	0.52207	0.56326	0.60774	0.65585	0.70794	0.76446
27	0.38545	0.41608	0.44903	0.48451	0.52274	0.56398	0.60851	0.65668	0.70885	0.76544
28	0.38594	0.41661	0.44960	0.48512	0.52340	0.56469	0.60929	0.65752	0.70975	0.76642
29	0.38643	0.41714	0.45017	0.48574	0.52406	0.56540	0.61006	0.65835	0.71066	0.76741
30	0.38693	0.41767	0.45074	0.48635	0.52472	0.56612	0.61083	0.65919	0.71157	0.76839
31	0.38742	0.41820	0.45132	0.48697	0.52539	0.56684	0.61161	0.66003	0.71248	0.76938
32	0.38792	0.41874	0.45189	0.48758	0.52605	0.56756	0.61239	0.66087	0.71339	0.77037
33	0.38841	0.41927	0.45246	0.48820	0.52672	0.56828	0.61316	0.66171	0.71430	0.77136
34	0.38891	0.41980	0.45304	0.48882	0.52739	0.56900	0.61394	0.66255	0.71521	0.77235
35	0.38941	0.42034	0.45361	0.48944	0.52805	0.56972	0.61472	0.66340	0.71613	0.77334
36	0.38990	0.42087	0.45419	0.49006	0.52872	0.57044	0.61550	0.66424	0.71704	0.77434
37	0.39040	0.42141	0.45476	0.49068	0.52939	0.57116	0.61628	0.66509	0.71796	0.77533
38	0.39090	0.42194	0.45534	0.49130	0.53006	0.57188	0.61706	0.66593	0.71888	0.77633
39	0.39140	0.42248	0.45592	0.49192	0.53073	0.57261	0.61785	0.66678	0.71980	0.77733
40	0.39190	0.42302	0.45650	0.49255	0.53141	0.57333	0.61863	0.66763	0.72072	0.77833
41	0.39240	0.42355	0.45708	0.49317	0.53208	0.57406	0.61942	0.66848	0.72164	0.77933
42	0.39290	0.42409	0.45766	0.49380	0.53275	0.57479	0.62020	0.66933	0.72256	0.78033
43	0.39340	0.42463	0.45824	0.49442	0.53343	0.57552	0.62099	0.67019	0.72349	0.78134
44	0.39390	0.42517	0.45882	0.49505	0.53410	0.57625	0.62178	0.67104	0.72441	0.78234
45	0.39441	0.42571	0.45940	0.49568	0.53478	0.57698	0.62257	0.67189	0.72534	0.78335
46	0.39491	0.42625	0.45998	0.49630	0.53546	0.57771	0.62336	0.67275	0.72627	0.78436
47	0.39541	0.42680	0.46057	0.49693	0.53613	0.57844	0.62415	0.67361	0.72720	0.78537
48	0.39592	0.42734	0.46115	0.49756	0.53681	0.57917	0.62494	0.67447	0.72813	0.78638
49	0.39642	0.42788	0.46173	0.49819	0.53749	0.57991	0.62574	0.67532	0.72906	0.78739
50	0.39693	0.42843	0.46232	0.49882	0.53817	0.58064	0.62653	0.67618	0.72999	0.78841
51	0.39743	0.42897	0.46291	0.49945	0.53885	0.58138	0.62733	0.67705	0.73093	0.78942
52	0.39794	0.42952	0.46349	0.50009	0.53954	0.58211	0.62812	0.67791	0.73186	0.79044
53	0.39845	0.43006	0.46408	0.50072	0.54022	0.58285	0.62892	0.67877	0.73280	0.79146
54	0.39896	0.43061	0.46467	0.50135	0.54090	0.58359	0.62972	0.67964	0.73374	0.79247
55	0.39947	0.43116	0.46526	0.50199	0.54159	0.58433	0.63052	0.68050	0.73468	0.79350
56	0.39998	0.43171	0.46585	0.50263	0.54228	0.58507	0.63132	0.68137	0.73562	0.79452
57	0.40049	0.43225	0.46644	0.50326	0.54296	0.58581	0.63212	0.68224	0.73656	0.79554
58	0.40100	0.43280	0.46703	0.50390	0.54365	0.58656	0.63293	0.68311	0.73751	0.79657
59	0.40151	0.43335	0.46762	0.50454	0.54434	0.58730	0.63373	0.68398	0.73845	0.79759
60	0.40202	0.43390	0.46822	0.50518	0.54503	0.58804	0.63454	0.68485	0.73940	0.79862

Involute Function Table

$\text{inv}\,\alpha = \tan\alpha - \alpha$

min. (′)	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°
0	0.79862	0.86305	0.93329	1.01004	1.09412	1.18648	1.28826	1.40081	1.52575	1.66503
1	0.79965	0.86417	0.93452	1.01138	1.09559	1.18810	1.29005	1.40279	1.52794	1.66748
2	0.80068	0.86530	0.93574	1.01272	1.09706	1.18972	1.29183	1.40477	1.53015	1.66994
3	0.80172	0.86642	0.93697	1.01407	1.09853	1.19134	1.29362	1.40675	1.53235	1.67241
4	0.80275	0.86755	0.93820	1.01541	1.10001	1.19296	1.29541	1.40874	1.53456	1.67488
5	0.80378	0.86868	0.93943	1.01676	1.10149	1.19459	1.29721	1.41073	1.53678	1.67735
6	0.80482	0.86980	0.94066	1.01811	1.10297	1.19622	1.29901	1.41272	1.53899	1.67983
7	0.80586	0.87094	0.94190	1.01946	1.10445	1.19785	1.30081	1.41472	1.54122	1.68232
8	0.80690	0.87207	0.94313	1.02081	1.10593	1.19948	1.30262	1.41672	1.54344	1.68480
9	0.80794	0.87320	0.94437	1.02217	1.10742	1.20112	1.30442	1.41872	1.54567	1.68730
10	0.80898	0.87434	0.94561	1.02352	1.10891	1.20276	1.30623	1.42073	1.54791	1.68980
11	0.81003	0.87548	0.94685	1.02488	1.11040	1.20440	1.30805	1.42274	1.55014	1.69230
12	0.81107	0.87662	0.94810	1.02624	1.11190	1.20604	1.30986	1.42475	1.55239	1.69481
13	0.81212	0.87776	0.94934	1.02761	1.11339	1.20769	1.31168	1.42677	1.55463	1.69732
14	0.81317	0.87890	0.95059	1.02897	1.11489	1.20934	1.31351	1.42879	1.55688	1.69984
15	0.81422	0.88004	0.95184	1.03034	1.11639	1.21100	1.31533	1.43081	1.55914	1.70236
16	0.81527	0.88119	0.95309	1.03171	1.11790	1.21265	1.31716	1.43284	1.56140	1.70488
17	0.81632	0.88234	0.95434	1.03308	1.11940	1.21431	1.31899	1.43487	1.56366	1.70742
18	0.81738	0.88349	0.95560	1.03446	1.12091	1.21597	1.32083	1.43691	1.56593	1.70995
19	0.81844	0.88464	0.95686	1.03583	1.12242	1.21763	1.32267	1.43895	1.56820	1.71249
20	0.81949	0.88579	0.95812	1.03721	1.12393	1.21930	1.32451	1.44099	1.57047	1.71504
21	0.82055	0.88694	0.95938	1.03859	1.12545	1.22097	1.32635	1.44304	1.57275	1.71759
22	0.82161	0.88810	0.96064	1.03997	1.12697	1.22264	1.32820	1.44509	1.57503	1.72015
23	0.82267	0.88926	0.96190	1.04136	1.12849	1.22432	1.33005	1.44714	1.57732	1.72271
24	0.82374	0.89042	0.96317	1.04274	1.13001	1.22599	1.33191	1.44920	1.57961	1.72527
25	0.82480	0.89158	0.96444	1.04413	1.13154	1.22767	1.33376	1.45126	1.58191	1.72785
26	0.82587	0.89274	0.96571	1.04552	1.13306	1.22936	1.33562	1.45332	1.58421	1.73042
27	0.82694	0.89390	0.96698	1.04692	1.13459	1.23104	1.33749	1.45539	1.58652	1.73300
28	0.82801	0.89507	0.96825	1.04831	1.13613	1.23273	1.33935	1.45746	1.58882	1.73559
29	0.82908	0.89624	0.96953	1.04971	1.13766	1.23442	1.34122	1.45954	1.59114	1.73818
30	0.83015	0.89741	0.97081	1.05111	1.13920	1.23612	1.34310	1.46162	1.59346	1.74077
31	0.83123	0.89858	0.97209	1.05251	1.14074	1.23781	1.34497	1.46370	1.59578	1.74338
32	0.83230	0.89975	0.97337	1.05391	1.14228	1.23951	1.34685	1.46579	1.59810	1.74598
33	0.83338	0.90092	0.97465	1.05532	1.14383	1.24122	1.34874	1.46788	1.60043	1.74859
34	0.83446	0.90210	0.97594	1.05673	1.14537	1.24292	1.35062	1.46997	1.60277	1.75121
35	0.83554	0.90328	0.97722	1.05814	1.14692	1.24463	1.35251	1.47207	1.60511	1.75383
36	0.83662	0.90446	0.97851	1.05955	1.14847	1.24634	1.35440	1.47417	1.60745	1.75646
37	0.83770	0.90564	0.97980	1.06097	1.15003	1.24805	1.35630	1.47627	1.60980	1.75909
38	0.83879	0.90682	0.98110	1.06238	1.15159	1.24977	1.35820	1.47838	1.61215	1.76172
39	0.83987	0.90801	0.98239	1.06380	1.15315	1.25149	1.36010	1.48050	1.61451	1.76436
40	0.84096	0.90919	0.98369	1.06522	1.15471	1.25321	1.36201	1.48261	1.61687	1.76701
41	0.84205	0.91038	0.98499	1.06665	1.15627	1.25494	1.36391	1.48473	1.61923	1.76966
42	0.84314	0.91157	0.98629	1.06807	1.15784	1.25666	1.36583	1.48686	1.62160	1.77232
43	0.84424	0.91276	0.98759	1.06950	1.15941	1.25839	1.36774	1.48898	1.62398	1.77498
44	0.84533	0.91396	0.98890	1.07093	1.16098	1.26013	1.36966	1.49112	1.62636	1.77765
45	0.84643	0.91515	0.99020	1.07236	1.16256	1.26187	1.37158	1.49325	1.62874	1.78032
46	0.84752	0.91635	0.99151	1.07380	1.16413	1.26360	1.37351	1.49539	1.63113	1.78300
47	0.84862	0.91755	0.99282	1.07524	1.16571	1.26535	1.37544	1.49753	1.63352	1.78568
48	0.84972	0.91875	0.99413	1.07667	1.16729	1.26709	1.37737	1.49968	1.63592	1.78837
49	0.85082	0.91995	0.99545	1.07812	1.16888	1.26884	1.37930	1.50183	1.63832	1.79106
50	0.85193	0.92115	0.99677	1.07956	1.17047	1.27059	1.38124	1.50399	1.64072	1.79376
51	0.85303	0.92236	0.99808	1.08100	1.17206	1.27235	1.38318	1.50614	1.64313	1.79647
52	0.85414	0.92357	0.99941	1.08245	1.17365	1.27410	1.38513	1.50831	1.64555	1.79918
53	0.85525	0.92478	1.00073	1.08390	1.17524	1.27586	1.38708	1.51047	1.64797	1.80189
54	0.85636	0.92599	1.00205	1.08536	1.17684	1.27762	1.38903	1.51264	1.65039	1.80461
55	0.85747	0.92720	1.00338	1.08681	1.17844	1.27936	1.39098	1.51482	1.65282	1.80734
56	0.85858	0.92842	1.00471	1.08827	1.18004	1.28116	1.39294	1.51700	1.65525	1.81007
57	0.85970	0.92963	1.00604	1.08973	1.18165	1.28293	1.39490	1.51918	1.65769	1.81280
58	0.86082	0.93085	1.00737	1.09119	1.18326	1.28470	1.39687	1.52136	1.66013	1.81555
59	0.86193	0.93207	1.00871	1.09265	1.18487	1.28648	1.39884	1.52355	1.66258	1.81829
60	0.86305	0.93329	1.01004	1.09412	1.18648	1.28826	1.40081	1.52575	1.66503	1.82105

Involute Function Table

inv $\alpha = \tan \alpha - \alpha$

min. (')	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°
0	1.82105	1.99676	2.19587	2.42305	2.68433	2.98757	3.34327	3.76574	4.27502	4.90003
1	1.82380	1.99988	2.19941	2.42711	2.68902	2.99304	3.34972	3.77345	4.28439	4.91165
2	1.82657	2.00300	2.20296	2.43118	2.69371	2.99852	3.35619	3.78119	4.29379	4.92331
3	1.82934	2.00613	2.20652	2.43525	2.69842	3.00401	3.36267	3.78895	4.30323	4.93502
4	1.83211	2.00926	2.21008	2.43934	2.70314	3.00952	3.36918	3.79673	4.31270	4.94677
5	1.83489	2.01240	2.21366	2.44343	2.70787	3.01504	3.37570	3.80454	4.32220	4.95856
6	1.83768	2.01555	2.21724	2.44753	2.71262	3.02058	3.38224	3.81237	4.33173	4.97040
7	1.84047	2.01871	2.22083	2.45165	2.71737	3.02613	3.38880	3.82023	4.34130	4.98229
8	1.84326	2.02187	2.22442	2.45577	2.72214	3.03170	3.39538	3.82811	4.35090	4.99422
9	1.84607	2.02504	2.22803	2.45990	2.72692	3.03728	3.40197	3.83601	4.36053	5.00620
10	1.84888	2.02821	2.23164	2.46405	2.73171	3.04288	3.40859	3.84395	4.37020	5.01822
11	1.85169	2.03139	2.23526	2.46820	2.73651	3.04849	3.41523	3.85190	4.37990	5.03029
12	1.85451	2.03458	2.23889	2.47236	2.74133	3.05412	3.42188	3.85988	4.38963	5.04240
13	1.85733	2.03777	2.24253	2.47653	2.74616	3.05977	3.42856	3.86789	4.39940	5.05456
14	1.86016	2.04097	2.24617	2.48071	2.75100	3.06542	3.43525	3.87592	4.40920	5.06677
15	1.86300	2.04418	2.24983	2.48491	2.75585	3.07110	3.44197	3.88398	4.41903	5.07902
16	1.86584	2.04740	2.25349	2.48911	2.76071	3.07679	3.44870	3.89206	4.42890	5.09133
17	1.86869	2.05062	2.25716	2.49332	2.76559	3.08249	3.45545	3.90017	4.43880	5.10368
18	1.87154	2.05385	2.26083	2.49754	2.77048	3.08821	3.46222	3.90830	4.44874	5.11608
19	1.87440	2.05708	2.26452	2.50177	2.77538	3.09395	3.46902	3.91646	4.45871	5.12852
20	1.87726	2.06032	2.26821	2.50601	2.78029	3.09970	3.47583	3.92465	4.46872	5.14102
21	1.88014	2.06357	2.27192	2.51027	2.78522	3.10546	3.48266	3.93286	4.47877	5.15356
22	1.88301	2.06683	2.27563	2.51453	2.79016	3.11125	3.48952	3.94110	4.48885	5.16616
23	1.88589	2.07009	2.27935	2.51880	2.79511	3.11704	3.49639	3.94937	4.49896	5.17880
24	1.88878	2.07336	2.28307	2.52308	2.80007	3.12286	3.50328	3.95766	4.50911	5.19149
25	1.89167	2.07664	2.28681	2.52737	2.80505	3.12869	3.51020	3.96598	4.51930	5.20424
26	1.89457	2.07992	2.29055	2.53168	2.81004	3.13453	3.51713	3.97433	4.52952	5.21703
27	1.89748	2.08321	2.29430	2.53599	2.81504	3.14040	3.52408	3.98270	4.53978	5.22987
28	1.90039	2.08651	2.29807	2.54031	2.82006	3.14627	3.53106	3.99110	4.55007	5.24277
29	1.90331	2.08981	2.30184	2.54465	2.82508	3.15217	3.53806	3.99953	4.56041	5.25572
30	1.90623	2.09313	2.30561	2.54899	2.83012	3.15808	3.54507	4.00798	4.57077	5.26871
31	1.90916	2.09645	2.30940	2.55334	2.83518	3.16401	3.55211	4.01646	4.58118	5.28176
32	1.91210	2.09977	2.31319	2.55771	2.84024	3.16995	3.55917	4.02497	4.59162	5.29486
33	1.91504	2.10310	2.31700	2.56208	2.84532	3.17591	3.56625	4.03351	4.60210	5.30802
34	1.91798	2.10644	2.32081	2.56647	2.85041	3.18188	3.57335	4.04207	4.61262	5.32122
35	1.92094	2.10979	2.32463	2.57087	2.85552	3.18788	3.58047	4.05067	4.62318	5.33448
36	1.92389	2.11315	2.32846	2.57527	2.86064	3.19389	3.58762	4.05929	4.63377	5.34780
37	1.92686	2.11651	2.33230	2.57969	2.86577	3.19991	3.59478	4.06794	4.64441	5.36117
38	1.92983	2.11988	2.33615	2.58412	2.87092	3.20595	3.60197	4.07662	4.65508	5.37459
39	1.93281	2.12325	2.34000	2.58856	2.87607	3.21201	3.60918	4.08532	4.66579	5.38806
40	1.93579	2.12664	2.34387	2.59301	2.88125	3.21809	3.61641	4.09406	4.67654	5.40159
41	1.93878	2.13003	2.34774	2.59747	2.88643	3.22418	3.62366	4.10282	4.68733	5.41518
42	1.94178	2.13343	2.35162	2.60194	2.89163	3.23029	3.63094	4.11162	4.69816	5.42882
43	1.94478	2.13683	2.35551	2.60642	2.89684	3.23642	3.63823	4.12044	4.70902	5.44251
44	1.94779	2.14024	2.35941	2.61092	2.90207	3.24257	3.64555	4.12929	4.71993	5.45626
45	1.95080	2.14366	2.36332	2.61542	2.90731	3.24873	3.65289	4.13817	4.73088	5.47007
46	1.95382	2.14709	2.36724	2.61994	2.91256	3.25491	3.66026	4.14708	4.74186	5.48394
47	1.95685	2.15053	2.37117	2.62446	2.91783	3.26110	3.66764	4.15602	4.75289	5.49786
48	1.95988	2.15397	2.37511	2.62900	2.92311	3.26732	3.67505	4.16499	4.76396	5.51184
49	1.96292	2.15742	2.37905	2.63355	2.92840	3.27355	3.68248	4.17399	4.77507	5.52588
50	1.96596	2.16088	2.38300	2.63811	2.93371	3.27980	3.68993	4.18302	4.78622	5.53997
51	1.96901	2.16434	2.38697	2.64268	2.93903	3.28606	3.69741	4.19208	4.79741	5.55413
52	1.97207	2.16781	2.39094	2.64726	2.94437	3.29235	3.70491	4.20118	4.80865	5.56834
53	1.97514	2.17130	2.39492	2.65186	2.94972	3.29865	3.71243	4.21030	4.81992	5.58261
54	1.97821	2.17478	2.39891	2.65646	2.95509	3.30497	3.71998	4.21945	4.83124	5.59694
55	1.98128	2.17828	2.40291	2.66108	2.96046	3.31131	3.72755	4.22863	4.84260	5.61133
56	1.98437	2.18178	2.40692	2.66571	2.96586	3.31767	3.73514	4.23785	4.85400	5.62578
57	1.98746	2.18529	2.41094	2.67034	2.97126	3.32404	3.74275	4.24709	4.86544	5.64030
58	1.99055	2.18881	2.41497	2.67500	2.97669	3.33043	3.75039	4.25637	4.87693	5.65487
59	1.99365	2.19234	2.41901	2.67966	2.98212	3.33684	3.75806	4.26568	4.88846	5.66950
60	1.99676	2.19587	2.42305	2.68433	2.98757	3.34327	3.76574	4.27502	4.90003	5.68420

Technical Reference -Index-